Exit Ticket Sample Solutions

Watch the video clip of Tillman the English bulldog, the Guinness World Record holder for Fastest Dog on a Skateboard.

1. **At the conclusion of the video, your classmate takes out his or her calculator and says, "Wow that was amazing! That means the dog went about 5 meters in 1 second!" Is your classmate correct, and how do you know?**

   Yes, the classmate is correct. The dog traveled at an average rate of 100 meters in 19.678 seconds, or an associated rate of $\frac{100}{19.678}$ meters per second, giving a unit rate of approximately 5.08.

2. **After seeing this video, another dog owner trained his dog, Lightning, to try to break Tillman's skateboarding record. Lightning's fastest recorded time was on a 75-meter stretch where it took him 15.5 seconds. Based on these data, did Lightning break Tillman's record for fastest dog? Explain how you know.**

   No, Lightning did not break Tillman's record. Tillman traveled at an average rate of 5.08 meters per second (calculated from an associated rate of $\frac{75}{15.5}$ meters per second), and Lightning traveled at an average rate of 4.84 meters per second (about $\frac{1}{4}$ of a meter slower per second), making Tillman the faster skateboarder.

Problem Set Sample Solutions

1. **Find each rate and unit rate.**

   a. 420 miles in 7 hours

      Rate: 60 miles per hour; Unit Rate: 60

   b. 360 customers in 30 days

      Rate: 12 customers per day; Unit Rate: 12

   c. 40 meters in 16 seconds

      $\frac{40}{16} = 2.5$

      The rate is 2.5 meters per second. The unit rate is 2.5.

   d. $\$7.96$ for 5 pounds

      $\frac{7.96}{5} = 1.592$

      The rate is 1.592 dollars per pound, or approximately $\$1.59$ per pound. The unit rate is 1.592.

2. **Write three ratios that are equivalent to the one given: The ratio of right-handed students to left-handed students is $18:4$.**

   Sample response: The ratio of right-handed students to left-handed students is $9:2$. The ratio of right-handed students to left-handed students is $36:8$. The ratio of right-handed students to left-handed students is $27:6$.

3. Mr. Rowley has 16 homework papers and 14 exit tickets to return. Ms. Rivera has 64 homework papers and 60 exit tickets to return. For each teacher, write a ratio to represent the number of homework papers to number of exit tickets they have to return. Are the ratios equivalent? Explain.

Mr. Rowley's ratio of homework papers to exit tickets is $16:14$. Ms. Rivera's ratio of homework papers to exit tickets is $64:60$. The ratios are not equivalent because Mr. Rowley's unit rate is $\frac{8}{7}$, or approximately 1.14, and Ms. Rivera's unit rate is $\frac{16}{15}$, or approximately 1.07.

4. Jonathan's parents told him that for every 5 hours of homework or reading he completes, he would be able to play 3 hours of video games. His friend Lucas's parents told their son that he could play 30 minutes for every hour of homework or reading time he completes. If both boys spend the same amount of time on homework and reading this week, which boy gets more time playing video games? How do you know?

If both boys spend 5 hours on homework and reading, Jonathan will be able to play 3 hours of video games, and Lucas will be able to play 2.5 hours of video games. Jonathan gets more time playing video games. Jonathan gets 0.6 hours (36 minutes) for every 1 hour of homework and reading time, whereas Lucas gets only 30 minutes for every 1 hour of homework or reading time.

5. Of the 30 girls who tried out for the lacrosse team at Euclid Middle School, 12 were selected. Of the 40 boys who tried out, 16 were selected. Are the ratios of the number of students on the team to the number of students trying out the same for both boys and girls? How do you know?

Yes, the ratios are the same: girls—12 to 30 or 2 to 5; boys—16 to 40 or 2 to 5. The value of each ratio is $\frac{2}{5}$.

6. Devon is trying to find the unit price on a 6-pack of drinks on sale for $\$2.99$. His sister says that at that price, each drink would cost just over $\$2.00$. Is she correct, and how do you know? If she is not, how would Devon's sister find the correct price?

Devon's sister is not correct. She divided the number of drinks by the cost, and to correctly find the unit price, she needs to divide the price by the number of drinks. $\frac{2.99}{6}$, or approximately 0.50, is the correct unit price. The cost is approximately 0.50 dollars per drink.

7. Each year Lizzie's school purchases student agenda books, which are sold in the school store. This year, the school purchased 350 books at a cost of $\$1,137.50$. If the school would like to make a profit of $\$1,500$ to help pay for field trips and school activities, what is the least amount they can charge for each agenda book? Explain how you found your answer.

The unit price per book the school paid is 3.25. To make $\$1,500$, you would need to make a profit of $1500 \div 350 = 4.29$ per book. $3.25 + 4.29$ is the cost per book or $\$7.54$. $\$7.54 \cdot 350$ generates a revenue of $\$2,639$, and $\$2,639$ minus the initial cost of the books, $\$1,137.50$ (expense), gives $\$1,501.50$ of profit.

Exit Ticket Sample Solutions

Ms. Albero decided to make juice to serve along with the pizza at the Student Government party. The directions said to mix 2 scoops of powdered drink mix with a half gallon of water to make each pitcher of juice. One of Ms. Albero's students said she will mix 8 scoops with 2 gallons of water to make 4 pitchers. How can you use the concept of proportional relationships to decide whether the student is correct?

Amount of Powdered Drink Mix (scoops)	1	2	4	8
Amount of Water (gallons)	$\frac{1}{4}$	$\frac{1}{2}$	1	2

As long as the amount of water is proportional to the number of scoops of drink mix, then the second quantity, amount of water, can be determined by multiplying the first quantity by the same constant. In this case, if the amount of powdered drink mix is represented by x, and the gallons of water are represented by y, then $y = \frac{1}{4}x$. To determine any of the measures of water, you will multiply the number of scoops by $\frac{1}{4}$.

Problem Set Sample Solutions

1. **A cran-apple juice blend is mixed in a ratio of cranberry to apple of 3 to 5.**
   a. **Complete the table to show different amounts that are proportional.**

Amount of Cranberry	3	6	9
Amount of Apple	5	10	15

2. **Why are these quantities proportional?**

   The amount of apple is proportional to the amount of cranberry since there exists a constant number, $\frac{5}{3}$, that when multiplied by any of the given measures for the amount of cranberry always produces the corresponding amount of apple. If the amount of cranberry is represented by x, and the amount of apple is represented by y, then each pair of quantities satisfies the equation $y = \frac{5}{3}x$. A similar true relationship could be derived by comparing the amount of cranberry to the amount of apple. In the case where x is the amount of apple and y is the amount of cranberry, the equation would be $y = \frac{3}{5}x$.

3. **John is filling a bathtub that is 18 inches deep. He notices that it takes two minutes to fill the tub with three inches of water. He estimates it will take 10 more minutes for the water to reach the top of the tub if it continues at the same rate. Is he correct? Explain.**

   Yes. In 10 more minutes, the tub will reach 18 inches. At that time, the ratio of time to height may be expressed as 12 to 18, which is equivalent to 2 to 3. The height of the water in the bathtub increases $1\frac{1}{2}$ inches every minute.

Time (minutes)	1	2	12
Bathtub Water Height (inches)	$1\frac{1}{2}$	3	18

Exit Ticket Sample Solutions

The table below shows the price, in dollars, for the number of roses indicated.

Number of Roses	3	6	9	12	15
Price (Dollars)	9	18	27	36	45

1. **Is the price proportional to the number of roses? How do you know?**

   The quantities are proportional to one another because there is a constant of 3 such that when the number of roses is multiplied by the constant, the result is the corresponding price.

2. **Find the cost of purchasing 30 roses.**

   $30 \times 3 = 90$

   If there are 30 roses, then the cost would be $\$90$.

Problem Set Sample Solutions

In each table, determine if y is proportional to x. Explain why or why not.

1.

x	y
3	12
5	20
2	8
8	32

2.

x	y
3	15
4	17
5	19
6	21

3.

x	y
6	4
9	6
12	8
3	2

1. *Yes, y is proportional to x because the values of all ratios of $\frac{y}{x}$ are equivalent to 4. Each measure of x multiplied by this constant of 4 gives the corresponding measure in y.*
2. *No, y is not proportional to x because the values of all the ratios of $\frac{y}{x}$ are not equivalent. There is not a constant where every measure of x multiplied by the constant gives the corresponding measure in y. The values of the ratios are 5, 4.25, 3.8, and 3.5.*
3. *Yes, y is proportional to x because a constant value of $\frac{2}{3}$ exists where each measure of x multiplied by this constant gives the corresponding measure in y.*

4. **Kayla made observations about the selling price of a new brand of coffee that sold in three different-sized bags. She recorded those observations in the following table:**

Ounces of Coffee	6	8	16
Price in Dollars	2.10	2.80	5.60

   a. **Is the price proportional to the amount of coffee? Why or why not?**

   Yes, the price is proportional to the amount of coffee because a constant value of 0.35 exists where each measure of x multiplied by this constant gives the corresponding measure in y.

   b. **Use the relationship to predict the cost of a 20 oz. bag of coffee.**

   20 ounces will cost $\$7$.

5. You and your friends go to the movies. The cost of admission is $\$9.50$ per person. Create a table showing the relationship between the number of people going to the movies and the total cost of admission.

   Explain why the cost of admission is proportional to the amount of people.

Number of People	*Cost (dollars)*
1	9.50
2	19
3	28.50
4	38

The cost is proportional to the number of people because a constant value of 9.50 exists where each measure of the number of people multiplied by this constant gives the corresponding measure in y.

6. For every 5 pages Gil can read, his daughter can read 3 pages. Let g represent the number of pages Gil reads, and let d represent the number of pages his daughter reads. Create a table showing the relationship between the number of pages Gil reads and the number of pages his daughter reads.

   Is the number of pages Gil's daughter reads proportional to the number of pages he reads? Explain why or why not.

g	d
5	3
10	6
15	9

Yes, the number of pages Gil's daughter reads is proportional to the number of pages Gil reads because all the values of the ratios are equivalent to 0.6. When I divide the number of pages Gil's daughter reads by the number of pages Gil reads, I always get the same quotient. Therefore, every measure of the number of pages Gil reads multiplied by the constant 0.6 gives the corresponding values of the number of pages Gil's daughter's reads.

7. The table shows the relationship between the number of parents in a household and the number of children in the same household. Is the number of children proportional to the number of parents in the household? Explain why or why not.

Number of Parents	Number of Children
0	0
1	3
1	5
2	4
2	1

No, there is not a proportional relationship because there is no constant such that every measure of the number of parents multiplied by the constant would result in the corresponding values of the number of children. When I divide the number of children by the corresponding number of parents, I do not get the same quotient every time. Therefore, the values of the ratios of children to parents are not equivalent. They are 3, 5, 2, and 0.5.

8. **The table below shows the relationship between the number of cars sold and the amount of money earned by the car salesperson. Is the amount of money earned, in dollars, proportional to the number of cars sold? Explain why or why not.**

Number of Cars Sold	Money Earned (in dollars)
1	250
2	600
3	950
4	$1,076$
5	$1,555$

No, there is no constant such that every measure of the number of cars sold multiplied by the constant would result in the corresponding values of the earnings because the ratios of money earned to number of cars sold are not equivalent; the values of the ratios are 250, 300, $316\frac{2}{3}$, 269, *and* 311.

9. **Make your own example of a relationship between two quantities that is NOT proportional. Describe the situation, and create a table to model it. Explain why one quantity is not proportional to the other.**

Answers will vary but should include pairs of numbers that do not always have the same value $\frac{B}{A}$.

Exit Ticket Sample Solutions

The table below shows the relationship between the side lengths of a regular octagon and its perimeter.

Side Lengths, s (inches)	Perimeter, P (inches)
1	8
2	16
3	24
4	32
9	72
12	96

Complete the table.

If Gabby wants to make a regular octagon with a side length of 20 inches using wire, how much wire does she need? Justify your reasoning with an explanation of whether perimeter is proportional to the side length.

$20(8) = 160$

Gabby would need 160 inches of wire to make a regular octagon with a side length of 20 inches. This table shows that the perimeter is proportional to the side length because the constant is 8, and when all side lengths are multiplied by the constant, the corresponding perimeter is obtained. Since the perimeter is found by adding all 8 side lengths together (or multiplying the length of 1 side by 8), the two numbers must always be proportional.

Problem Set Sample Solutions

1. Joseph earns $\$15$ for every lawn he mows. Is the amount of money he earns proportional to the number of lawns he mows? Make a table to help you identify the type of relationship.

Number of Lawns Mowed	1	2	3	4
Earnings ($)	15	30	45	60

The table shows that the earnings are proportional to the number of lawns mowed. The value of each ratio is 15. The constant is 15.

2. At the end of the summer, Caitlin had saved $\$120$ from her summer job. This was her initial deposit into a new savings account at the bank. As the school year starts, Caitlin is going to deposit another $\$5$ each week from her allowance. Is her account balance proportional to the number of weeks of deposits? Use the table below. Explain your reasoning.

Time (in weeks)	0	1	2	3
Account Balance ($)	120	125	130	135

Caitlin's account balance is not proportional to the number of weeks because there is no constant such that any time in weeks can be multiplied to get the corresponding balance. In addition, the ratio of the balance to the time in weeks is different for each column in the table.

$120:0$ *is not the same as* $125:1$.

3. Lucas and Brianna read three books each last month. The table shows the number of pages in each book and the length of time it took to read the entire book.

Pages Lucas Read	208	156	234
Time (hours)	8	6	9

Pages Brianna Read	168	120	348
Time (hours)	6	4	12

a. Which of the tables, if any, represent a proportional relationship?

The table shows Lucas's number of pages read to be proportional to the time because when the constant of 26 is multiplied by each measure of time, it gives the corresponding values for the number of pages read.

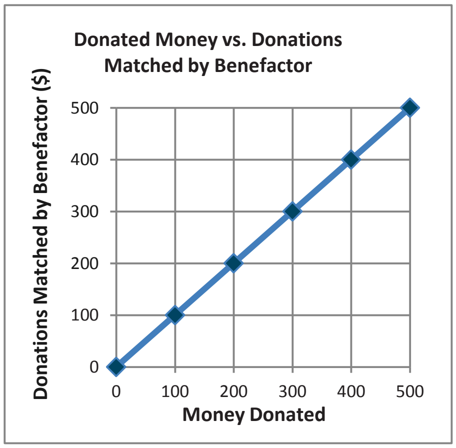
b. Both Lucas and Brianna had specific reading goals they needed to accomplish. What different strategies did each person employ in reaching those goals?

Lucas read at a constant rate throughout the summer, 26 pages per hour, whereas Brianna's reading rate was not the same throughout the summer.

Problem Set Sample Solutions

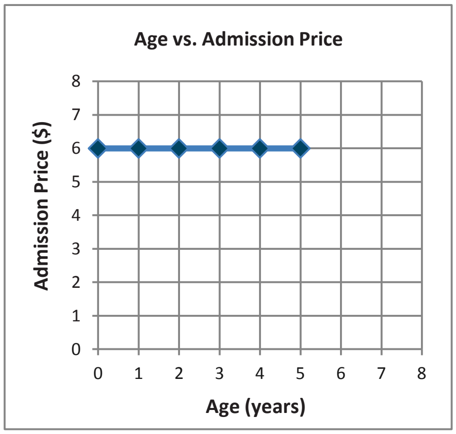
1. **Determine whether or not the following graphs represent two quantities that are proportional to each other. Explain your reasoning.**

a.



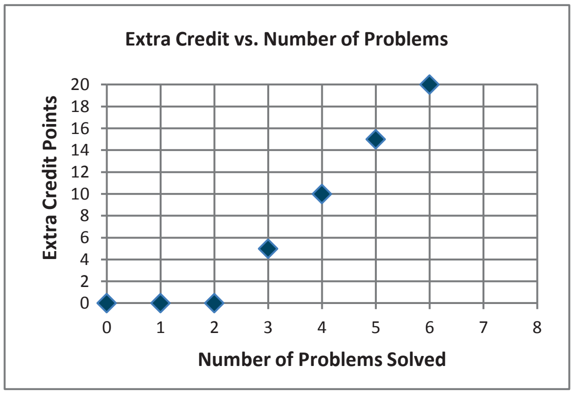
This graph represents two quantities that are proportional to each other because the points appear on a line, and the line that passes through the points would also pass through the origin.

b.



Even though the points appear on a line, the line does not go through the origin. Therefore, this graph does not represent a proportional relationship.

c.

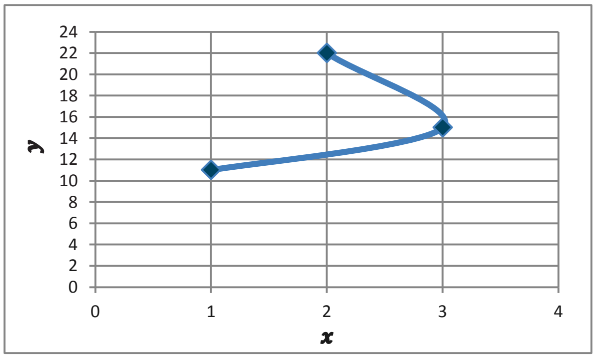


Even though it goes through the origin, this graph does not show a proportional relationship because the points do not appear on one line.

2. Create a table and a graph for the ratios $2:22$, 3 to 15, and $1:11$. Does the graph show that the two quantities are proportional to each other? Explain why or why not.

This graph does not because the points do not appear on a line that goes through the origin.

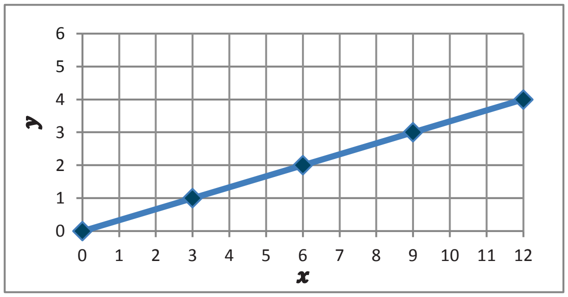
x	y
2	22
3	15
1	11



3. Graph the following tables, and identify if the two quantities are proportional to each other on the graph. Explain why or why not.

a.

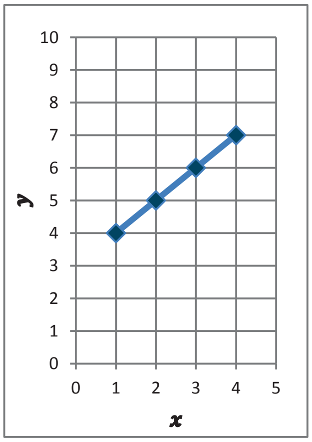
x	y
3	1
6	2
9	3
12	4



Yes, because the graph of the relationship is a straight line that passes through the origin.

b.

x	y
1	4
2	5
3	6
4	7



No, because the graph does not pass through the origin.

Exit Ticket Sample Solutions

1. **Which graphs in the art gallery walk represented proportional relationships, and which did not? List the group number.**

<u>Proportional Relationship</u>	<u>Non-Proportional Relationship</u>	
Group 2	*Group 1*	*Group 5*
Group 7	*Group 3*	*Group 6*
	Group 4	*Group 8*

2. **What are the characteristics of the graphs that represent proportional relationships?**

   Graphs of groups 2 and 7 appear on a line and go through the origin.

3. **For the graphs representing proportional relationships, what does $(0, 0)$ mean in the context of the situation?**

   For zero books sold, the library received zero dollars in donations.

Problem Set Sample Solutions

Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent the number of years to the amount of money in the savings account.

- **After one year, the interest accumulated, and the total in Sally's account was $\$312$.**
- **After three years, the total was $\$340$. After six years, the total was $\$380$.**
- **After nine years, the total was $\$430$. After 12 years, the total amount in Sally's savings account was $\$480$.**

Using the same four-fold method from class, create a table and a graph, and explain whether the amount of money accumulated and time elapsed are proportional to each other. Use your table and graph to support your reasoning.

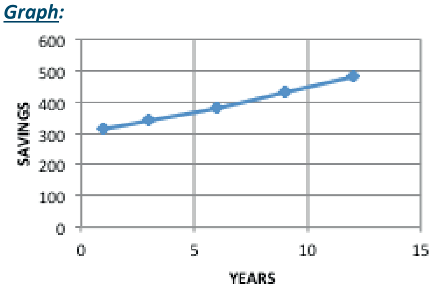
<u>Problem</u>:

Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping the money in the bank. The ratios below represent the number of years to the amount of money in the savings account. Create a table and a graph, and explain whether the quantities are proportional to each other.

<u>Table</u>:

Years	*Savings* ($\$$)
1	312
3	340
6	380
9	430
12	480

<u>Graph</u>:



<u>Explanation</u>:

The graph is not a graph of a proportional relationship because, although the data appears to be a line, it is not a line that goes through the origin. The amount of interest collected is not the same every year.

Exit Ticket Sample Solutions

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will determine the total cost of the sodas. Who is right, and why?

Susan is correct. The table below shows that if you multiply the unit price, say 0.50, by the number of people, say 12, you will determine the total cost of the soda. I created a table to model the proportional relationship. I used a unit price of 0.50 to make the comparison.

Susan

Number of People	2	3	4	12
Total Cost of Soda (in dollars)	1	1.50	2	6

I used the same values to compare to John. $\dfrac{\textit{total cost}}{12\ \textit{people}} = ?$

The total cost is $\$6$, and there 12 people. $\dfrac{6}{12} = \dfrac{1}{2} = 0.50$*, which is the unit price, not the total cost.*

Problem Set Sample Solutions

For each of the following problems, define the constant of proportionality to answer the follow-up question.

1. Bananas are $\$0.59$/pound.
   a. What is the constant of proportionality, or k?

      The constant of proportionality, k, is 0.59.

   b. How much will 25 pounds of bananas cost?

      $(25)(0.59) = 14.75$

      The bananas would cost $\$14.75$.

2. The dry cleaning fee for 3 pairs of pants is $\$18$.
   a. What is the constant of proportionality?

      $\dfrac{18}{3} = 6$*, so k is 6.*

   b. How much will the dry cleaner charge for 11 pairs of pants?

      $(6)(11) = 66$

      The dry cleaner would charge $\$66$.

3. For every $\$5$ that Micah saves, his parents give him $\$10$.
   a. What is the constant of proportionality?

      $\dfrac{10}{5} = 2$*, so k is 2.*

b. If Micah saves $\$150$, how much money will his parents give him?

$(2)(150) = 300$

Micah's parents will give him $\$300$.

4. Each school year, the seventh graders who study Life Science participate in a special field trip to the city zoo. In 2010, the school paid $\$1,260$ for 84 students to enter the zoo. In 2011, the school paid $\$1,050$ for 70 students to enter the zoo. In 2012, the school paid $\$1,395$ for 93 students to enter the zoo.

a. Is the price the school pays each year in entrance fees proportional to the number of students entering the zoo?

Number of Students	*Price* ($\$$)	
84	$1,260$	$\frac{1,260}{84} = 15$
70	$1,050$	$\frac{1,050}{70} = 15$
93	$1,395$	$\frac{1,395}{93} = 15$

$\boxed{\text{YES}}$

b. Explain why or why not.

The price is proportional to the number of students because the ratio of the entrance fee paid per student was the same.

$$\frac{1,260}{84} = 15$$

c. Identify the constant of proportionality and explain what it means in the context of this situation.

The constant of proportionality (k) is 15. This represents the price per student.

d. What would the school pay if 120 students entered the zoo?

$(120)(15) = 1,800$

The school would pay $\$1,800$ if 120 students entered the zoo.

e. How many students would enter the zoo if the school paid $\$1,425$?

$$\frac{1,425}{15} = 95$$

If the school paid $\$1,425$, then 95 students would enter the zoo.

Problem Set Sample Solutions

Write an equation that will model the proportional relationship given in each real-world situation.

1. **There are 3 cans that store 9 tennis balls. Consider the number of balls per can.**

   a. **Find the constant of proportionality for this situation.**

   $\frac{9 \text{ balls}}{3 \text{ cans}} = 3 \frac{\text{balls}}{\text{can}}$

   The constant of proportionality is 3.

   b. **Write an equation to represent the relationship.**

   $B = 3C$, where C represents the number of cans, and B represents the number of balls.

2. **In 25 minutes, Li can run 10 laps around the track. Determine the number of laps she can run per minute.**

   a. **Find the constant of proportionality in this situation.**

   $\frac{10 \text{ laps}}{25 \text{ minutes}} = \frac{2}{5} \frac{\text{laps}}{\text{minute}}$

   The constant of proportionality is $\frac{2}{5}$.

   b. **Write an equation to represent the relationship.**

   $L = \frac{2}{5}M$, where M represents the time in minutes, and L represents the number of laps.

3. **Jennifer is shopping with her mother. They pay $\$2$ per pound for tomatoes at the vegetable stand.**

   a. **Find the constant of proportionality in this situation.**

   $\frac{2 \text{ dollars}}{1 \text{ pound}} = 2 \frac{\text{dollars}}{\text{pound}}$

   The constant of proportionality is 2.

   b. **Write an equation to represent the relationship.**

   $D = 2P$, where P represents the number of pounds of tomatoes, and D represents the total cost in dollars.

4. **It costs $\$15$ to send 3 packages through a certain shipping company. Consider the number of packages per dollar.**

   a. **Find the constant of proportionality for this situation.**

   $\frac{3 \text{ packages}}{15 \text{ dollars}} = \frac{3}{15} \frac{\text{packages}}{\text{dollar}}$

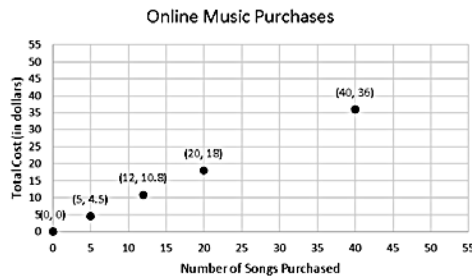
   The constant of proportionality is $\frac{1}{5}$.

   b. **Write an equation to represent the relationship.**

   $P = \frac{1}{5}D$, where D represents the cost in dollars, and P represents the number of packages.

5. On average, Susan downloads 60 songs per month. An online music vendor sells package prices for songs that can be downloaded onto personal digital devices. The graph below shows the package prices for the most popular promotions. Susan wants to know if she should buy her music from this company or pay a flat fee of $\$58.00$ per month offered by another company. Which is the better buy?

Number of Songs Purchased (S)	*Total Cost (C)*	*Constant of Proportionality*
40	36	$\frac{36}{40}=\frac{9}{10}=0.9$
20	18	$\frac{18}{20}=\frac{9}{10}=0.9$
12	10.80	$\frac{10.80}{12}=\frac{9}{10}=0.9$
5	4.50	$\frac{4.50}{5}=\frac{9}{10}=0.9$



a. Find the constant of proportionality for this situation.

The constant of proportionality, k, is 0.9.

b. Write an equation to represent the relationship.

$C = 0.9S$, where S represents the number of songs purchased, and C represents the total cost in dollars.

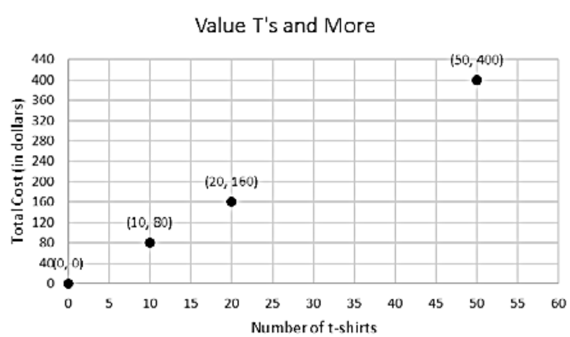
c. Use your equation to find the answer to Susan's question above. Justify your answer with mathematical evidence and a written explanation.

Compare the flat fee of $\$58$ per month to $\$0.90$ per song. If $C = 0.9S$ and we substitute S with 60 (the number of songs), then the result is $C = 0.9(60) = 54$. She would spend $\$54$ on songs if she bought 60 songs. If she maintains the same number of songs, the charge of $\$0.90$ per song would be cheaper than the flat fee of $\$58$ per month.

6. Allison's middle school team has designed t-shirts containing their team name and color. Allison and her friend Nicole have volunteered to call local stores to get an estimate on the total cost of purchasing t-shirts. Print-o-Rama charges a set-up fee, as well as a fixed amount for each shirt ordered. The total cost is shown below for the given number of shirts. Value T's and More charges $\$8$ per shirt. Which company should they use?

Print-o-Rama

Number of Shirts (S)	Total Cost (C)
10	95
25	
50	375
75	
100	



a. **Does either pricing model represent a proportional relationship between the quantity of t-shirts and the total cost? Explain.**

The unit rate of $\frac{y}{x}$ for Print-o-Rama is not constant. The graph for Value T's and More is proportional since the ratios are equivalent (8) and the graph shows a line through the origin.

b. **Write an equation relating cost and shirts for Value T's and More.**

$C = 8S$, where S represents the number of shirts, and C represents the total cost in dollars.

c. **What is the constant of proportionality of Value T's and More? What does it represent?**

8; the cost of one shirt is $\$8$.

d. **How much is Print-o-Rama's set-up fee?**

The set-up fee is $\$25$.

e. **If you need to purchase 90 shirts, write a proposal to your teacher indicating which company the team should use. Be sure to support your choice. Determine the number of shirts that you need for your team.**

Since we plan on a purchase of 90 shirts, we should choose Print-o-Rama.

Print-o-Rama: $C = 7S + 25$; $C = 7(90) + 25$; $C = 655$

Value T's and More: $C = 8S$; $C = 8(90)$; $C = 720$

Exit Ticket Sample Solutions

Oscar and Maria each wrote an equation that they felt represented the proportional relationship between distance in kilometers and distance in miles. One entry in the table paired 152 km with 95 miles. If k represents the number of kilometers and m represents the number of miles, who wrote the correct equation that would relate kilometers to miles? Explain why.

Oscar wrote the equation $k = 1.6m$, and he said that the unit rate $\frac{1.6}{1}$ represents kilometers per mile.

Maria wrote the equation $k = 0.625m$ as her equation, and she said that 0.625 represents kilometers per mile.

Oscar is correct. Oscar found the unit rate to be 1.6 by dividing kilometers by miles. The unit rate that Oscar used represents the number of kilometers per the number of miles. However, it should be noted that the variables were not well defined. Since we do not know which values are independent or dependent, each equation should include a definition of each variable. For example, Oscar should have defined his variables so that k represented the number of kilometers and m represented the number of miles. For Maria's equation to be correct, she should have stated that k represents the number of miles and m represents the number of kilometers.

Problem Set Sample Solutions

1. A person who weighs 100 pounds on Earth weighs $16.6\text{ lb}.$ on the moon.

a. Which variable is the independent variable? Explain why.

Weight on Earth is the independent variable because most people do not fly to the moon to weigh themselves first. The weight on the moon depends on a person's weight on Earth.

b. What is an equation that relates weight on Earth to weight on the moon?

Let E represents weight on Earth in pounds, and M represents weight on the moon in pounds.

$$M = \left(\frac{16.6}{100}\right)E$$

$$M = 0.166E$$

c. How much would a 185-pound astronaut weigh on the moon? Use an equation to explain how you know.

$$M = 0.166E$$
$$M = 0.166(185)$$
$$M = 30.71$$

The astronaut would weigh $30.71\ lb.$ on the moon.

d. How much would a man who weighs 50 pounds on the moon weigh on Earth?

$301\text{ lb}.$

2. **Use this table to answer the following questions.**

Number of Gallons of Gas	Number of Miles Driven
0	0
2	62
4	124
10	310

a. **Which variable is the dependent variable, and why?**

The number of miles driven is the dependent variable because the number of miles you can drive depends on the number of gallons of gas you have in your tank.

b. **Is the number of miles driven proportionally related to the number of gallons of gas? If so, what is the equation that relates the number of miles driven to the number of gallons of gas?**

Yes, the number of miles driven is proportionally related to the number of gallons of gas because every measure of gallons of gas can be multiplied by 31 to get every corresponding measure of miles driven.

$M = 31G$*, where G represents the number of gallons of gas, and M represents the number of miles driven.*

c. **In any ratio relating the number of gallons of gas and the number of miles driven, will one of the values always be larger? If so, which one?**

Yes, the number of miles will be larger except for the point $(0, 0)$. The point $(0, 0)$ means 0 miles driven uses 0 gallons of gas.

d. **If the number of gallons of gas is known, can you find the number of miles driven? Explain how this value would be calculated.**

Yes, multiply the constant of proportionality, 31, by the number of gallons of gas.

e. **If the number of miles driven is known, can you find the number of gallons of gas used? Explain how this value would be calculated.**

Yes, divide the number of miles driven by the constant of proportionality, 31.

f. **How many miles could be driven with 18 gallons of gas?**

558 *miles*

g. **How many gallons are used when the car has been driven 18 miles?**

$\frac{18}{31}$ *gallons*

h. **How many miles have been driven when half a gallon of gas is used?**

15.5 *miles*

i. **How many gallons have been used when the car has been driven for a half mile?**

$\frac{1}{62}$ *gallons*

3. Suppose that the cost of renting a snowmobile is $\$37.50$ for 5 hours.

   a. If c represents the cost and h represents the hours, which variable is the dependent variable? Explain why.

   c is the dependent variable because the cost of using the snowmobile depends on the number of hours you use it.

   $c = 7.5h$

   b. What would be the cost of renting 2 snowmobiles for 5 hours?

   $\$75$

4. In Katya's car, the number of miles driven is proportional to the number of gallons of gas used. Find the missing value in the table.

Number of Gallons of Gas	Number of Miles Driven
0	0
4	112
6	168
8	224
10	280

   a. Write an equation that will relate the number of miles driven to the number of gallons of gas.

   $M = 28G$, where G is the number of gallons of gas, and M is the number of miles driven.

   b. What is the constant of proportionality?

   28

   c. How many miles could Katya go if she filled her 22-gallon tank?

   616 *miles*

   d. If Katya takes a trip of 600 miles, how many gallons of gas would be needed to make the trip?

   $21\frac{3}{7}$ *gallons*

   e. If Katya drives 224 miles during one week of commuting to school and work, how many gallons of gas would she use?

   8 *gallons*

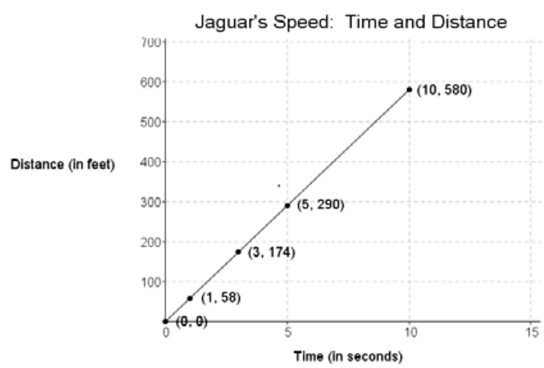
Exit Ticket Sample Solutions

Great Rapids White Water Rafting Company rents rafts for $\$125$ per hour. Explain why the point $(0, 0)$ and $(1, 125)$ are on the graph of the relationship and what these points mean in the context of the problem.

Every graph of a proportional relationship must include the points $(0, 0)$ and $(1, r)$. The point $(0, 0)$ is on the graph because 0 can be multiplied by the constant to determine the corresponding value of 0. The point $(1, 125)$ is on the graph because 125 is the unit rate. On the graph, for every 1 unit change on the horizontal axis, the vertical axis will change by 125 units. The point $(0, 0)$ means 0 hours of renting a raft would cost $\$0$, and $(1, 125)$ means 1 hour of renting the raft would cost $\$125$.

Problem Set Sample Solutions

1. **The graph to the right shows the relationship of the amount of time (in seconds) to the distance (in feet) run by a jaguar.**



   a. **What does the point $(5, 290)$ represent in the context of the situation?**

      In 5 seconds, a jaguar can run 290 feet.

   b. **What does the point $(3, 174)$ represent in the context of the situation?**

      A jaguar can run 174 feet in 3 seconds.

   c. **Is the distance run by the jaguar proportional to the time? Explain why or why not.**

      Yes, the distance run by the jaguar is proportional to the time spent running because the graph shows a line that passes through the origin $(0, 0)$.

   d. **Write an equation to represent the distance run by the jaguar. Explain or model your reasoning.**

      $y = 58x$, where x represents the time in seconds, and y represents the distance run in feet.

      The constant of proportionality, or unit rate of $\frac{y}{x}$, is 58 and can be substituted into the equation $y = kx$ in place of k.

2. **Championship t-shirts sell for $\$22$ each.**

   a. **What point(s) must be on the graph for the quantities to be proportional to each other?**

      $(0, 0)$, $(1, 22)$

   b. **What does the ordered pair $(5, 110)$ represent in the context of this problem?**

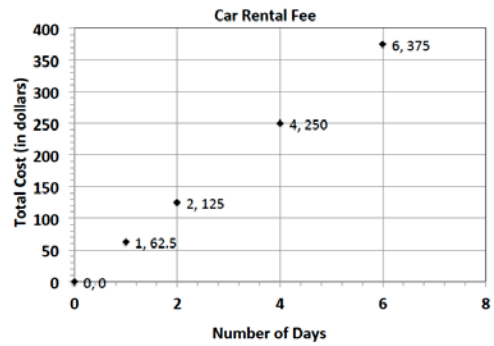
      5 t-shirts will cost $\$110$.

c. How many t-shirts were sold if you spent a total of $\$88$?

$\frac{88}{22} = 4$

Four t-shirts were sold.

3. The graph represents the total cost of renting a car. The cost of renting a car is a fixed amount each day, regardless of how many miles the car is driven.



a. What does the ordered pair $(4, 250)$ represent?

It would cost $\$250$ to rent a car for 4 days.

b. What would be the cost to rent the car for a week? Explain or model your reasoning.

$62.5(7) = 437.50$

Since the unit rate is 62.5, the cost for a week would be $\$437.50$.

4. Jackie is making a snack mix for a party. She is using cashews and peanuts. The table below shows the relationship of the number of packages of cashews she needs to the number of cans of peanuts she needs to make the mix.

Packages of Cashews	Cans of Peanuts
0	0
1	2
2	4
3	6
4	8

a. Write an equation to represent this relationship.

$y = 2x$, where x represents the number of packages of cashews, and y represents the number of cans of peanuts.

b. Describe the ordered pair $(12, 24)$ in the context of the problem.

In the mixture, you will need 12 packages of cashews and 24 cans of peanuts.

5. The following table shows the amount of candy and price paid.

Amount of Candy (in pounds)	2	3	5
Cost (in dollars)	5	7.5	12.5

a. Is the cost of the candy proportional to the amount of candy?

Yes, because there exists a constant, 2.5, such that each measure of the amount of candy multiplied by the constant gives the corresponding measure of cost.

b. Write an equation to illustrate the relationship between the amount of candy and the cost.

$y = 2.5x$, where x represents the amount of candy in pounds, and y represents the cost in dollars.

c. Using the equation, predict how much it will cost for 12 pounds of candy.

$2.5(12) = 30$

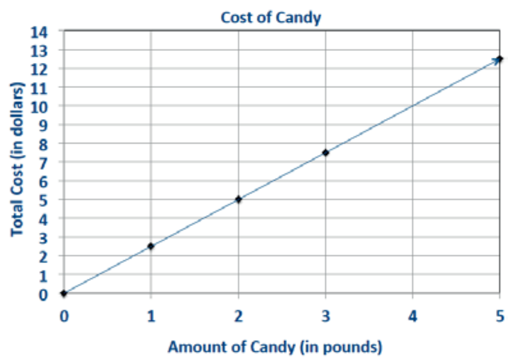
It will cost $\$30$ for 12 pounds of candy.

d. What is the maximum amount of candy you can buy with $\$60$?

$\frac{60}{2.5} = 24$

The maximum amount of candy I can buy with $\$60$ is 24 pounds.

e. Graph the relationship.



Exit Ticket Sample Solutions

Which is the better buy? Show your work and explain your reasoning.

$3\frac{1}{3}$ lb. of turkey for $\$10.50$

$10\frac{1}{2} \div 3\frac{1}{3} = 3.15$

$2\frac{1}{2}$ lb. of turkey for $\$6.25$

$6\frac{1}{4} \div 2\frac{1}{2} = 2.50$

$2\frac{1}{2}$ lb. *is the better buy because the price per pound is cheaper.*

Problem Set Sample Solutions

1. Determine the quotient: $2\frac{4}{7} \div 1\frac{3}{6}$.

   $1\frac{5}{7}$

2. One lap around a dirt track is $\frac{1}{3}$ mile. It takes Bryce $\frac{1}{9}$ hour to ride one lap. What is Bryce's unit rate, in miles, around the track?

   3

3. Mr. Gengel wants to make a shelf with boards that are $1\frac{1}{3}$ feet long. If he has an 18-foot board, how many pieces can he cut from the big board?

   $13\frac{1}{2}$ *boards*

4. The local bakery uses 1.75 cups of flour in each batch of cookies. The bakery used 5.25 cups of flour this morning.
   a. How many batches of cookies did the bakery make?

      3 *batches*

   b. If there are 5 dozen cookies in each batch, how many cookies did the bakery make?

      $5(12) = 60$

      There are 60 *cookies per batch.*

      $60(3) = 180$

      So, the bakery made 180 *cookies.*

5. Jason eats 10 ounces of candy in 5 days.
   a. How many <u>pounds</u> does he eat per day? (Recall: 16 ounces $= 1$ pound)

      $\frac{1}{8}$ lb. *each day*

   b. How long will it take Jason to eat 1 pound of candy?

      8 *days*

Exit Ticket Sample Solutions

If $3\frac{3}{4}$ lb. of candy cost $\$20.25$, how much would 1 lb. of candy cost?

$5\frac{2}{5} = 5.4$

One pound of candy would cost $\$5.40$.

Students may find the unit rate by first converting $\$20.25$ to $\frac{81}{4}$ and then dividing by $\frac{15}{4}$.

Problem Set Sample Solutions

1. You are getting ready for a family vacation. You decide to download as many movies as possible before leaving for the road trip. If each movie takes $1\frac{2}{5}$ hours to download, and you downloaded for $5\frac{1}{4}$ hours, how many movies did you download?

   $3\frac{3}{4}$ movies; however, since you cannot download $\frac{3}{4}$ of a movie, then you downloaded 3 movies.

2. The area of a blackboard is $1\frac{1}{3}$ square yards. A poster's area is $\frac{8}{9}$ square yards. Find the unit rate and explain, in words, what the unit rate means in the context of this problem. Is there more than one unit rate that can be calculated? How do you know?

   $1\frac{1}{2}$. The area of the blackboard is $1\frac{1}{2}$ times the area of the poster.

   Yes. There is another possible unit rate: $\frac{2}{3}$. The area of the poster is $\frac{2}{3}$ the area of the blackboard.

3. A toy jeep is $12\frac{1}{2}$ inches long, while an actual jeep measures $18\frac{3}{4}$ feet long. What is the value of the ratio of the length of the toy jeep to the length of the actual jeep? What does the ratio mean in this situation?

   $$\frac{12\frac{1}{2}}{18\frac{3}{4}} = \frac{\frac{25}{2}}{\frac{75}{4}} = \frac{2}{3}$$

   Every 2 inches in length on the toy jeep corresponds to 3 feet in length on the actual jeep.

4. To make 5 dinner rolls, $\frac{1}{3}$ cup of flour is used.

   a. How much flour is needed to make one dinner roll?

      $\frac{1}{15}$ cup

   b. How many cups of flour are needed to make 3 dozen dinner rolls?

      $2\frac{2}{5}$ cups

   c. How many rolls can you make with $5\frac{2}{3}$ cups of flour?

      85 rolls

Exit Ticket Sample Solutions

The table below shows the combination of a dry prepackaged mix and water to make concrete. The mix says for every 1 gallon of water stir 60 pounds of dry mix. We know that 1 gallon of water is equal to 8 pounds of water. Using the information given in the table, complete the remaining parts of the table.

Dry Mix (pounds)	Water (pounds)	Total (pounds)
60	8	68
75	10	85
$12\frac{1}{2}$	$1\frac{2}{3}$	$14\frac{1}{6}$
$4\frac{1}{2}$	$\frac{3}{5}$	$5\frac{1}{10}$

Problem Set Sample Solutions

1. Students in 6 classes, displayed below, ate the same ratio of cheese pizza slices to pepperoni pizza slices. Complete the following table, which represents the number of slices of pizza students in each class ate.

Slices of Cheese Pizza	Slices of Pepperoni Pizza	Total Slices of Pizza
2	5	7
6	15	21
8	20	28
$5\frac{1}{2}$	$13\frac{3}{4}$	$19\frac{1}{4}$
$3\frac{1}{3}$	$8\frac{1}{3}$	$11\frac{2}{3}$
$\frac{3}{5}$	$1\frac{1}{2}$	$2\frac{1}{10}$

2. To make green paint, students mixed yellow paint with blue paint. The table below shows how many yellow and blue drops from a dropper several students used to make the same shade of green paint.

   a. Complete the table.

Yellow (Y) (mL)	Blue (B) (mL)	Total (mL)
$3\frac{1}{2}$	$5\frac{1}{4}$	$8\frac{3}{4}$
2	3	5
$4\frac{1}{2}$	$6\frac{3}{4}$	$11\frac{1}{4}$
$6\frac{1}{2}$	$9\frac{3}{4}$	$16\frac{1}{4}$

   b. Write an equation to represent the relationship between the amount of yellow paint and blue paint.

   $B = 1.5Y$

3. The ratio of the number of miles run to the number of miles biked is equivalent for each row in the table.

a. Complete the table.

Distance Run (miles)	Distance Biked (miles)	Total Amount of Exercise (miles)
2	4	6
$3\frac{1}{2}$	7	$10\frac{1}{2}$
$2\frac{3}{4}$	$5\frac{1}{2}$	$8\frac{1}{4}$
$2\frac{1}{8}$	$4\frac{1}{4}$	$6\frac{3}{8}$
$1\frac{2}{3}$	$3\frac{1}{3}$	5

b. What is the relationship between distances biked and distances run?

The distances biked were twice as far as the distances run.

4. The following table shows the number of cups of milk and flour that are needed to make biscuits. Complete the table.

Milk (cups)	Flour (cups)	Total (cups)
7.5	9	16.5
$8\frac{3}{4}$	10.5	$19\frac{1}{4}$
12.5	15	27.5
5	6	11

Exit Ticket Sample Solutions

1. A bicycle shop advertised all mountain bikes priced at a $\frac{1}{3}$ discount.

   a. What is the amount of the discount if the bicycle originally costs $\$327$?

   $\frac{1}{3}(\$327) = \$109$. *The discount is* $\$109$.

   b. What is the discount price of the bicycle?

   $\frac{2}{3}(\$327) = \$218$. *The discount price is* $\$218$. *Methods will vary.*

   c. Explain how you found your solution to part (b).

   Answers will vary.

2. A hand-held digital music player was marked down by $\frac{1}{4}$ of the original price.

   a. If the sales price is $\$128.00$, what is the original price?

   Let x represent the original price in dollars.

   $$x - \frac{1}{4}x = 128$$
   $$\frac{3}{4}x = 128$$
   $$x = 170.67$$

   The original price is $\$170.67$.

   b. If the item was marked up by $\frac{1}{2}$ before it was placed on the sales floor, what was the price that the store paid for the digital player?

   Let x represent the price that the store paid in dollars.

   $$x + \frac{1}{2}x = 170.67$$
   $$\frac{3}{2}x = 170.67$$
   $$x = 113.78$$

   The price that the store paid for the digital player was $\$113.78$.

   c. What is the difference between the discount price and the price that the store paid for the digital player?

   $\$128 - \$113.78 = \$14.22$

Problem Set Sample Solutions

1. A salesperson will earn a commission equal to $\frac{1}{32}$ of the total sales. What is the commission earned on sales totaling $\$24,000$?

   $\left(\frac{1}{32}\right)\$24,000 = \$750$

2. DeMarkus says that a store overcharged him on the price of the video game he bought. He thought that the price was marked $\frac{1}{4}$ of the original price, but it was really $\frac{1}{4}$ off the original price. He misread the advertisement. If the original price of the game was $\$48$, what is the difference between the price that DeMarkus thought he should pay and the price that the store charged him?

   $\frac{1}{4}$ of $\$48$ is $\$12$ (the price DeMarkus thought he should pay); $\frac{1}{4}$ off $\$48$ is $\$36$

   Difference between prices: $\$36 - \$12 = \$24$

3. What is the cost of a $\$1,200$ washing machine after a discount of $\frac{1}{5}$ the original price?

   $\left(1-\frac{1}{5}\right)\$1,200 = \$960$ *or* $\$1,200 - \frac{1}{5}(\$1,200) = \$960$

4. If a store advertised a sale that gave customers a $\frac{1}{4}$ discount, what is the fractional part of the original price that the customer will pay?

   $1 - \frac{1}{4} = \frac{3}{4}$

   The customer will pay $\frac{3}{4}$ of the original price.

5. Mark bought an electronic tablet on sale for $\frac{1}{4}$ off the original price of $\$825.00$. He also wanted to use a coupon for $\frac{1}{5}$ off the sales price. How much did Mark pay for the tablet?

   $\$825\left(\frac{3}{4}\right) = \$618.75$, *then* $\$618.75\left(\frac{4}{5}\right) = \$495$

6. A car dealer paid a certain price for a car and marked it up by $\frac{7}{5}$ of the price he paid. Later, he sold it for $\$24,000$. What is the original price?

   Let x represent the original price in dollars.

   $$x + \frac{7}{5}x = 24,000$$
   $$\frac{12}{5}x = 24,000$$
   $$x = 10,000$$

   The original price was $\$10,000$.

7. Joanna ran a mile in physical education class. After resting for one hour, her heart rate was 60 beats per minute. If her heart rate decreased by $\frac{2}{5}$, what was her heart rate immediately after she ran the mile?

   Let x represent her heart rate immediately after she ran the mile.

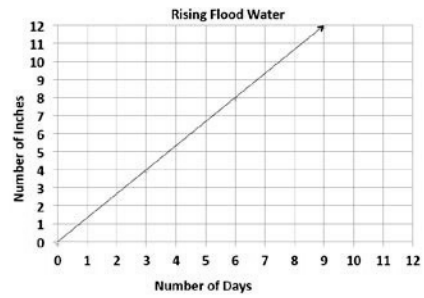
   $$x - \frac{2}{5}x = 60$$
   $$\frac{3}{5}x = 60$$
   $$x = 100$$

   Her heart rate was 100 beats per minute.

Exit Ticket Sample Solutions

1. Describe the relationship that the graph depicts.

   The graph shows that in 3 days the water rose to 4 inches. The water has risen at a constant rate. Therefore, the water has risen $1\frac{1}{3}$ inches per day.



2. Identify two points on the line, and explain what they mean in the context of the problem.

   $(6, 8)$ means that by the 6^{th} day, the water rose 8 inches; $(9, 12)$ means that by the 9^{th} day, the water rose 12 inches.

3. What is the unit rate?

   The unit rate in inches per day is $\frac{4}{3}$.

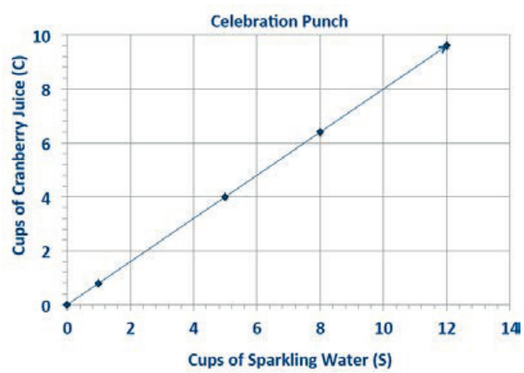
4. What point represents the unit rate?

   The point that shows the unit rate is $\left(1, 1\frac{1}{3}\right)$.

Problem Set Sample Solutions

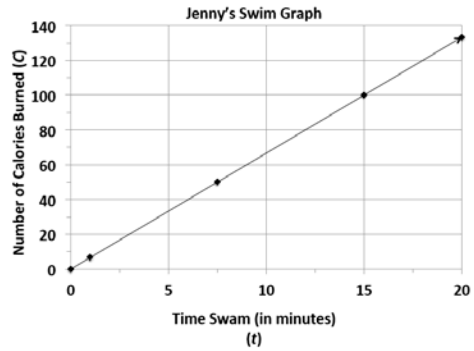
1. Students are responsible for providing snacks and drinks for the Junior Beta Club Induction Reception. Susan and Myra were asked to provide the punch for the 100 students and family members who will attend the event. The chart below will help Susan and Myra determine the proportion of cranberry juice to sparkling water needed to make the punch. Complete the chart, graph the data, and write the equation that models this proportional relationship.

Sparkling Water (S, in cups)	Cranberry Juice (C, in cups)
1	$\frac{4}{5}$
5	4
8	$6\frac{2}{5}$
12	$9\frac{3}{5}$
50	40
100	80



$C = \frac{4}{5}S$, where C represents the number of cups of cranberry juice, and S represents the number of cups of sparkling water.

2. Jenny is a member of a summer swim team.



a. Using the graph, determine how many calories she burns in one minute.

Jenny burns 100 calories every 15 minutes, so she burns $6\frac{2}{3}$ calories each minute.

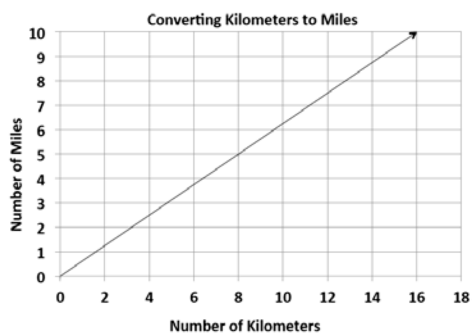
b. Use the graph to determine the equation that models the number of calories Jenny burns within a certain number of minutes.

$C = 6\frac{2}{3}t$, where C represents the number of calories burned, and t represents the time she swims in minutes.

c. How long will it take her to burn off a 480-calorie smoothie that she had for breakfast?

It will take Jenny 72 minutes of swimming to burn off the smoothie she had for breakfast.

3. Students in a world geography class want to determine the distances between cities in Europe. The map gives all distances in kilometers. The students want to determine the number of miles between towns so they can compare distances with a unit of measure with which they are already familiar. The graph below shows the relationship between a given number of kilometers and the corresponding number of miles.



a. Find the constant of proportionality, or the rate of miles per kilometer, for this problem, and write the equation that models this relationship.

The constant of proportionality is $\frac{5}{8}$.

The equation that models this situation is $M = \frac{5}{8}K$, where M represents the number of miles, and K represents the number of kilometers.

b. What is the distance in kilometers between towns that are 5 miles apart?

The distance between towns that are 5 miles apart is 8 km.

c. Describe the steps you would take to determine the distance in miles between two towns that are 200 kilometers apart?

Solve the equation $M = \frac{5}{8}(200)$. To find the number of miles for 200 km, multiply 200 by $\frac{5}{8}$.

$200\left(\frac{5}{8}\right) = 125$. The two towns are 125 miles apart.

4. During summer vacation, Lydie spent time with her grandmother picking blackberries. They decided to make blackberry jam for their family. Her grandmother said that you must cook the berries until they become juice and then combine the juice with the other ingredients to make the jam.

   a. Use the table below to determine the constant of proportionality of cups of juice to cups of blackberries.

Cups of Blackberries	Cups of Juice
0	0
4	$1\frac{1}{3}$
8	$2\frac{2}{3}$
12	4
24	8

   $k = \frac{1}{3}$

   b. Write an equation that models the relationship between the number of cups of blackberries and the number of cups of juice.

   $j = \frac{1}{3}b$, where j represents the number of cups of juice, and b represents the number of cups of blackberries.

   c. How many cups of juice were made from 12 cups of berries? How many cups of berries are needed to make 8 cups of juice?

   4 cups of juice are made from 12 cups of berries.
   24 cups of berries are needed to make 8 cups of juice.

Problem Set Sample Solutions

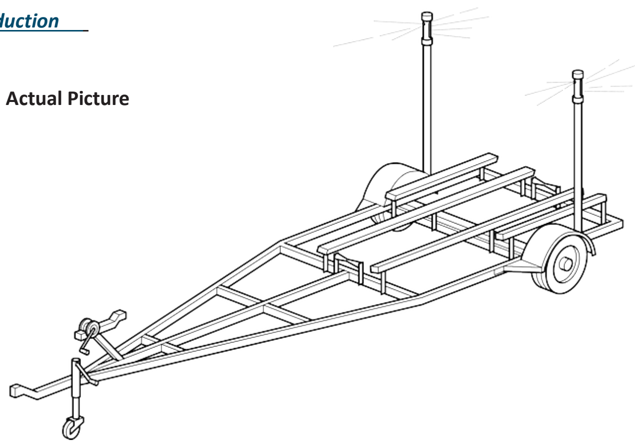
For Problems 1–3, identify if the scale drawing is a reduction or an enlargement of the actual picture.

1. ___*Enlargement*___

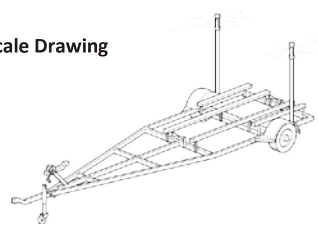
a. **Actual Picture** **b.** **Scale Drawing**

2. ___*Reduction*___

a. **Actual Picture**



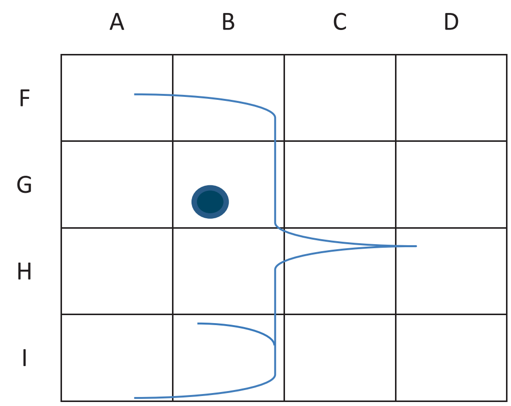
b. **Scale Drawing**



3. ___*Enlargement*___

a. **Actual Picture** **b.** **Scale Drawing**

4. **Using the grid and the abstract picture of a face, answer the following questions:**



a. **On the grid, where is the eye?**

Intersection BG

b. **What is located in DH?**

Tip of the nose

c. **In what part of the square BI is the chin located?**

Bottom right corner

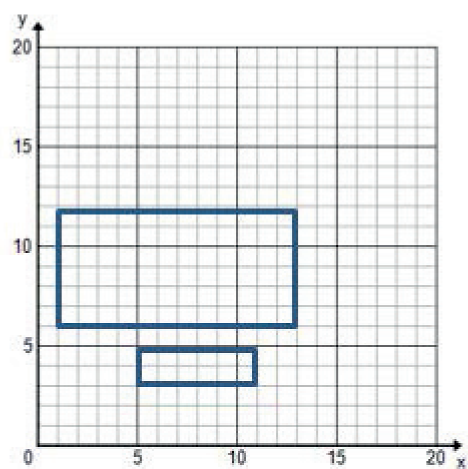
5. **Use the blank graph provided to plot the points and decide if the rectangular cakes are scale drawings of each other.**

Cake 1: $(5, 3)$, $(5, 5)$, $(11, 3)$, $(11, 5)$

Cake 2: $(1, 6)$, $(1, 12)$, $(13, 12)$, $(13, 6)$

How do you know?

These images are not scale drawings of each other because the short length of Cake 2 is three times longer than Cake 1, but the longer length of Cake 2 is only twice as long as Cake 1. Both should either be twice as long or three times as long to have one-to-one correspondence and to be scale drawings of each other.



Exit Ticket Sample Solutions

A rectangular pool in your friend's yard is 150 ft. $\times$ 400 ft. Create a scale drawing with a scale factor of $\frac{1}{600}$. Use a table or an equation to show how you computed the scale drawing lengths.

Actual Length	*Scale Length*
150 ft.	150 ft. *multiplied by* $\frac{1}{600}$ *is* $\frac{1}{4}$ ft., *or* 3 in.
400 ft.	400 ft. *multiplied by* $\frac{1}{600}$ *is* $\frac{2}{3}$ ft., *or* 8 in.

8 in.

3 in.

Problem Set Sample Solutions

1. **Giovanni went to Los Angeles, California, for the summer to visit his cousins. He used a map of bus routes to get from the airport to his cousin's house. The distance from the airport to his cousin's house is 56 km. On his map, the distance was 4 cm. What is the scale factor?**

   The scale factor is $\frac{1}{1{,}400{,}000}$. *I had to change kilometers to centimeters or centimeters to kilometers or both to meters in order to determine the scale factor.*

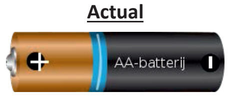
2. **Nicole is running for school president. Her best friend designed her campaign poster, which measured 3 feet by 2 feet. Nicole liked the poster so much, she reproduced the artwork on rectangular buttons that measured 2 inches by $1\frac{1}{3}$ inches. What is the scale factor?**

   The scale factor is $\frac{2}{3}$.

3. **Find the scale factor using the given scale drawings and measurements below.**

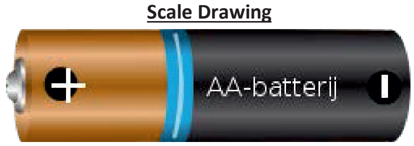
   Scale Factor: $\frac{5}{3}$

Actual



3 cm

Scale Drawing

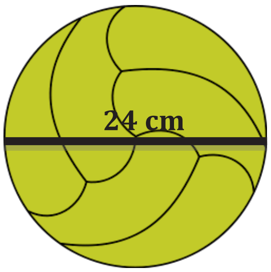


5 cm

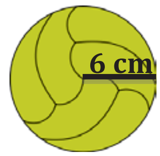
4. Find the scale factor using the given scale drawings and measurements below.

Scale Factor: $\frac{1}{2}$ *****Compare diameter to diameter or radius to radius.***

Actual Picture



Scale Drawing



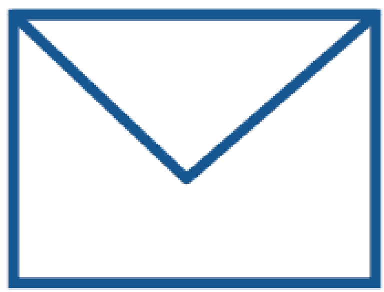
5. Using the given scale factor, create a scale drawing from the actual pictures in centimeters:

a. Scale factor: 3

Small Picture : 1 in.

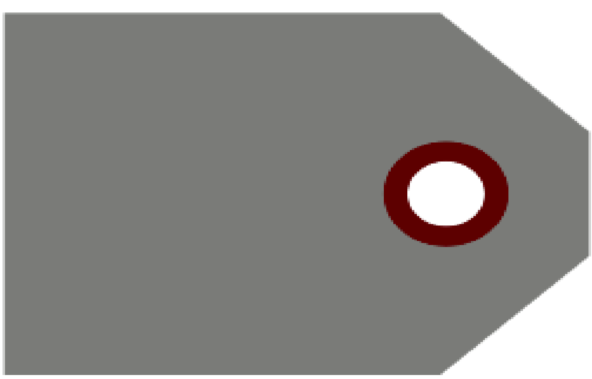
Large Picture: 3 in.

1 in.

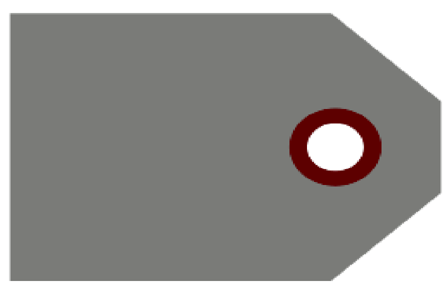


3 in.

b. Scale factor: $\frac{3}{4}$



Actual Drawing Measures: 4 in.



Scale Drawing Measures: 3 in.

6. **Hayden likes building radio-controlled sailboats with her father. One of the sails, shaped like a right triangle, has side lengths measuring 6 inches, 8 inches, and 10 inches. To log her activity, Hayden creates and collects drawings of all the boats she and her father built together. Using the scale factor of $\frac{1}{4}$, create a scale drawing of the sail.**

   A triangle with sides 1.5 inches, 2 inches, and 2.5 inches is drawn.

Scaffolding:

Extension: Students can enlarge an image they want to draw or paint by drawing a grid using a ruler over their reference picture and drawing a grid of equal ratio on their work surface. Direct students to focus on one square at a time until the image is complete. Have students compute the scale factor for the drawing.

Exit Ticket Sample Solutions

A drawing of a surfboard in a catalog shows its length as $8\frac{4}{9}$ inches. Find the actual length of the surfboard if $\frac{1}{2}$ inch length on the drawing corresponds to $\frac{3}{8}$ foot of actual length.

	Scale	*Equivalent Scale Ratio*	*Surfboard*
Scale Drawing Length, x	$\frac{1}{2}$ *inch*	*1 inch*	$8\frac{4}{9}$ *inches*
Actual Length, y	$\frac{3}{8}$ *foot*	$\frac{6}{8}$ ft. *or* $\frac{3}{4}$ ft.	*?*

$\cdot \frac{3}{4}$

$$y = kx$$
$$y = \frac{3}{4}x$$
$$= \frac{3}{4} \cdot 8\frac{4}{9}$$
$$= \frac{3}{4} \cdot \frac{76}{9}$$
$$= \frac{1}{1} \cdot \frac{19}{3}$$
$$= 6\frac{1}{3}$$

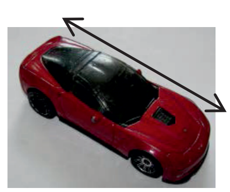
The actual surfboard measures $6\frac{1}{3}$ feet long.

Note: Students could also use an equation where y represents the scale drawing, and x represents the actual measurement, in which case, k would equal $\frac{4}{3}$.

Problem Set Sample Solutions

1. **A toy company is redesigning its packaging for model cars. The graphic design team needs to take the old image shown below and resize it so that $\frac{1}{2}$ inch on the old packaging represents $\frac{1}{3}$ inch on the new package. Find the length of the image on the new package.**

Car image length on old packaging measures 2 inches.

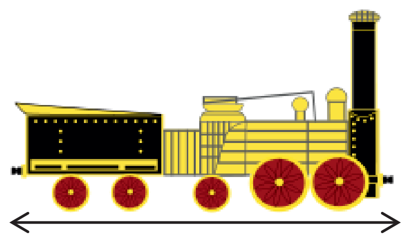


$\frac{4}{3}$ inches; the scale $\frac{1}{2}$ to $\frac{1}{3}$ and the length of the original figure is 2, which is 4 halves, so in the scale drawing the length will be 4 thirds.

2. The city of St. Louis is creating a welcome sign on a billboard for visitors to see as they enter the city. The following picture needs to be enlarged so that $\frac{1}{2}$ inch represents 7 feet on the actual billboard. Will it fit on a billboard that measures 14 feet in height?

Yes, the drawing measures 1 inch in height, which corresponds to 14 feet on the actual billboard.

3. Your mom is repainting your younger brother's room. She is going to project the image shown below onto his wall so that she can paint an enlarged version as a mural. Use a ruler to determine the length of the image of the train. Then determine how long the mural will be if the projector uses a scale where 1 inch of the image represents $2\frac{1}{2}$ feet on the wall.



$2 \times 2.5 = 5$

The scale drawing measures 2 inches, so the image will measure 5 feet long, on the wall.

4. A model of a skyscraper is made so that 1 inch represents 75 feet. What is the height of the actual building if the height of the model is $18\frac{3}{5}$ inches?

$1,395$ *feet*

5. The portrait company that takes little league baseball team photos is offering an option where a portrait of your baseball pose can be enlarged to be used as a wall decal (sticker). Your height in the portrait measures $3\frac{1}{2}$ inches. If the company uses a scale where 1 inch on the portrait represents 20 inches on the wall decal, find the height on the wall decal. Your actual height is 55 inches. If you stand next to the wall decal, will it be larger or smaller than you?

Your height on the wall decal is 70 inches. The wall decal will be larger than your actual height (when you stand next to it).

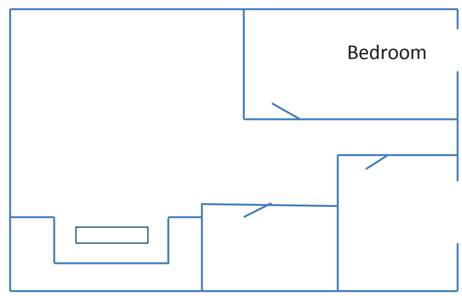
6. The sponsor of a 5K run/walk for charity wishes to create a stamp of its billboard to commemorate the event. If the sponsor uses a scale where 1 inch represents 4 feet, and the billboard is a rectangle with a width of 14 feet and a length of 48 feet, what will be the shape and size of the stamp?

The stamp will be a rectangle measuring $3\frac{1}{2}$ inches by 12 inches.

7. Danielle is creating a scale drawing of her room. The rectangular room measures $20\frac{1}{2}$ ft. by 25 ft. If her drawing uses the scale where 1 inch represents 2 feet of the actual room, will her drawing fit on an $8\frac{1}{2}$ in. by 11 in. piece of paper?

No, the drawing would be $10\frac{1}{4}$ inches by $12\frac{1}{2}$ inches, which is larger than the piece of paper.

8. A model of an apartment is shown below where $\frac{1}{4}$ inch represents 4 feet in the actual apartment. Use a ruler to measure the drawing and find the actual length and width of the bedroom.

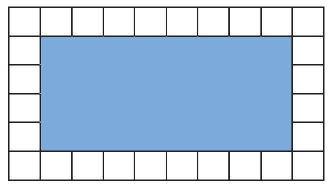


Ruler measurements: $1\frac{1}{8}$ inches by $\frac{9}{16}$ inches.

The actual length would be 18 feet, and the actual width would be 9 feet.

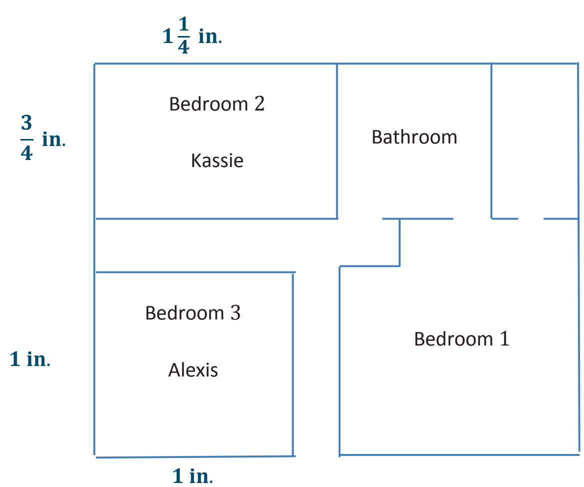
Problem Set Sample Solutions

1. **The shaded rectangle shown below is a scale drawing of a rectangle whose area is 288 square feet. What is the scale factor of the drawing? (Note: Each square on the grid has a length of 1 unit.)**



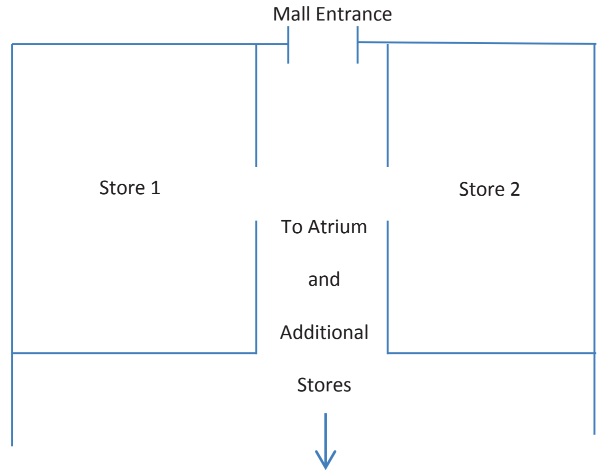
The scale factor is $\frac{1}{3}$.

2. **A floor plan for a home is shown below where $\frac{1}{2}$ inch corresponds to 6 feet of the actual home. Bedroom 2 belongs to 13-year-old Kassie, and Bedroom 3 belongs to 9-year-old Alexis. Kassie claims that her younger sister, Alexis, got the bigger bedroom. Is she right? Explain.**



Bedroom 2 (Kassie) has an area of 135 sq. ft.*, and Bedroom 3 (Alexis) has an area of* 144 sq. ft. *Therefore, the older sister is correct. Alexis got the bigger bedroom by a difference of* 9 *square feet.*

3. On the mall floor plan, $\frac{1}{4}$ inch represents 3 feet in the actual store.



a. Find the actual area of Store 1 and Store 2.

The dimensions of Store 1 measure $1\frac{7}{16}$ inches by $1\frac{13}{16}$ inches. The actual measurements would be $17\frac{1}{4}$ feet by $21\frac{3}{4}$ feet. Store 1 has an area of $375\frac{3}{16}$ square feet. The dimensions of Store 2 measure $1\frac{3}{16}$ inches by $1\frac{13}{16}$ inches. The actual measurements would be $14\frac{1}{4}$ feet by $21\frac{3}{4}$ feet. Store 2 has an area of $309\frac{15}{16}$ square feet.

b. In the center of the atrium, there is a large circular water feature that has an area of $\left(\frac{9}{64}\right)\pi$ square inches on the drawing. Find the actual area in square feet.

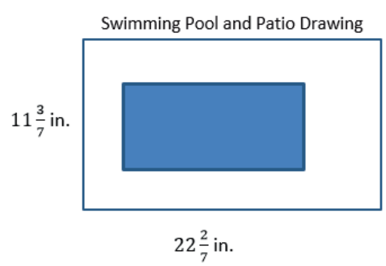
$$\left(\frac{9}{64}\right)\pi \cdot 144 = \left(\frac{81}{4}\right)\pi \approx 63.6$$

The water feature has an area of approximately 63.6 square feet.

4. The greenhouse club is purchasing seed for the lawn in the school courtyard. The club needs to determine how much to buy. Unfortunately, the club meets after school, and students are unable to find a custodian to unlock the door. Anthony suggests they just use his school map to calculate the area that will need to be covered in seed. He measures the rectangular area on the map and finds the length to be 10 inches and the width to be 6 inches. The map notes the scale of 1 inch representing 7 feet in the actual courtyard. What is the actual area in square feet?

$70 \text{ ft.} \times 42 \text{ ft.} = 2,940 \text{ sq. ft.}$

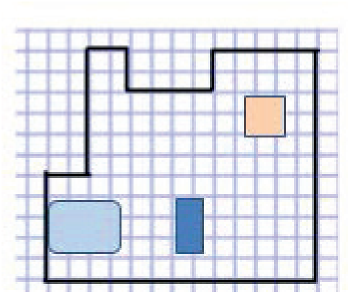
5. The company installing the new in-ground pool in your backyard has provided you with the scale drawing shown below. If the drawing uses a scale of 1 inch to $1\frac{3}{4}$ feet, calculate the total amount of two-dimensional space needed for the pool and its surrounding patio.



Area $=780$ square feet

Exit Ticket Sample Solutions

1. Your sister has just moved into a loft-style apartment in Manhattan and has asked you to be her designer. Indicate the placement of the following objects on the floorplan using the appropriate scale: queen-size bed (60 in. by 80 in.), sofa (36 in. by 64 in.), and dining table (48 in. by 48 in.) In the following scale drawing, 1 cm represents 2 ft. Each square on the grid is 1 cm².



Queen Bed: $60 \div 12 = 5,\ 5 \div 2 = 2\frac{1}{2}$

$80 \div 12 = 6\frac{2}{3},\ 6\frac{2}{3} \div 2 = 3\frac{1}{3}$

The queen bed is $2\frac{1}{2}$ cm *by* $3\frac{1}{3}$ cm *in the scale drawing.*

Sofa: $36 \div 12 = 3,\ 3 \div 2 = 1\frac{1}{2}$

$64 \div 12 = 5\frac{1}{3},\ 5\frac{1}{3} \div 2 = 2\frac{2}{3}$

The sofa is $1\frac{1}{2}$ cm *by* $2\frac{2}{3}$ cm *in the scale drawing.*

Dining Table: $48 \div 12 = 4,\ 4 \div 2 = 2$

The dining table is 2 cm *by* 2 cm *in the scale drawing.*

2. Choose one object and explain the procedure to find the scale lengths.

Take the actual measurements in inches and divide by 12 inches to express the value in feet. Then divide the actual length in feet by 2 since 2 feet represents 1 centimeter. The resulting quotient is the scale length.

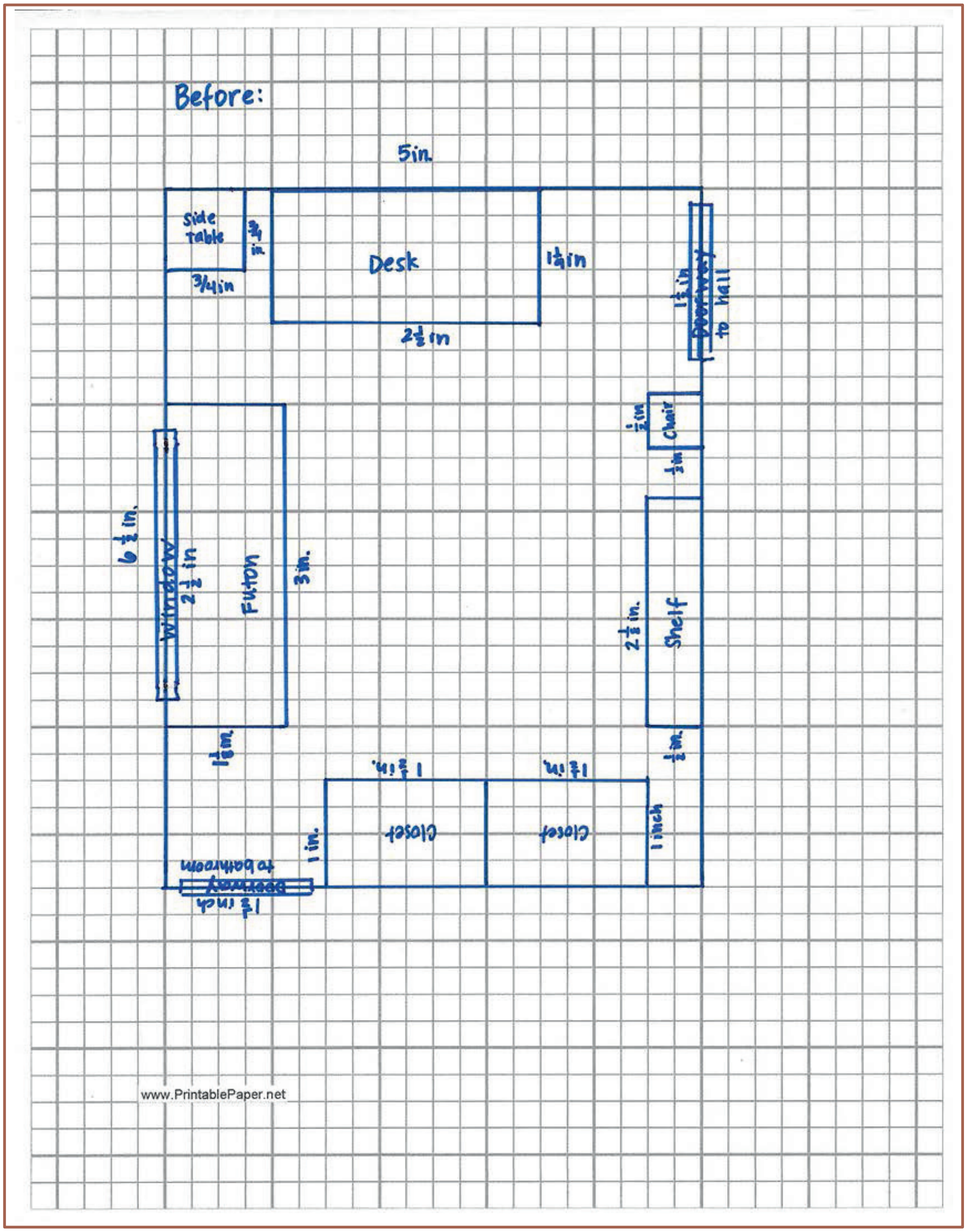
Problem Set Sample Solutions

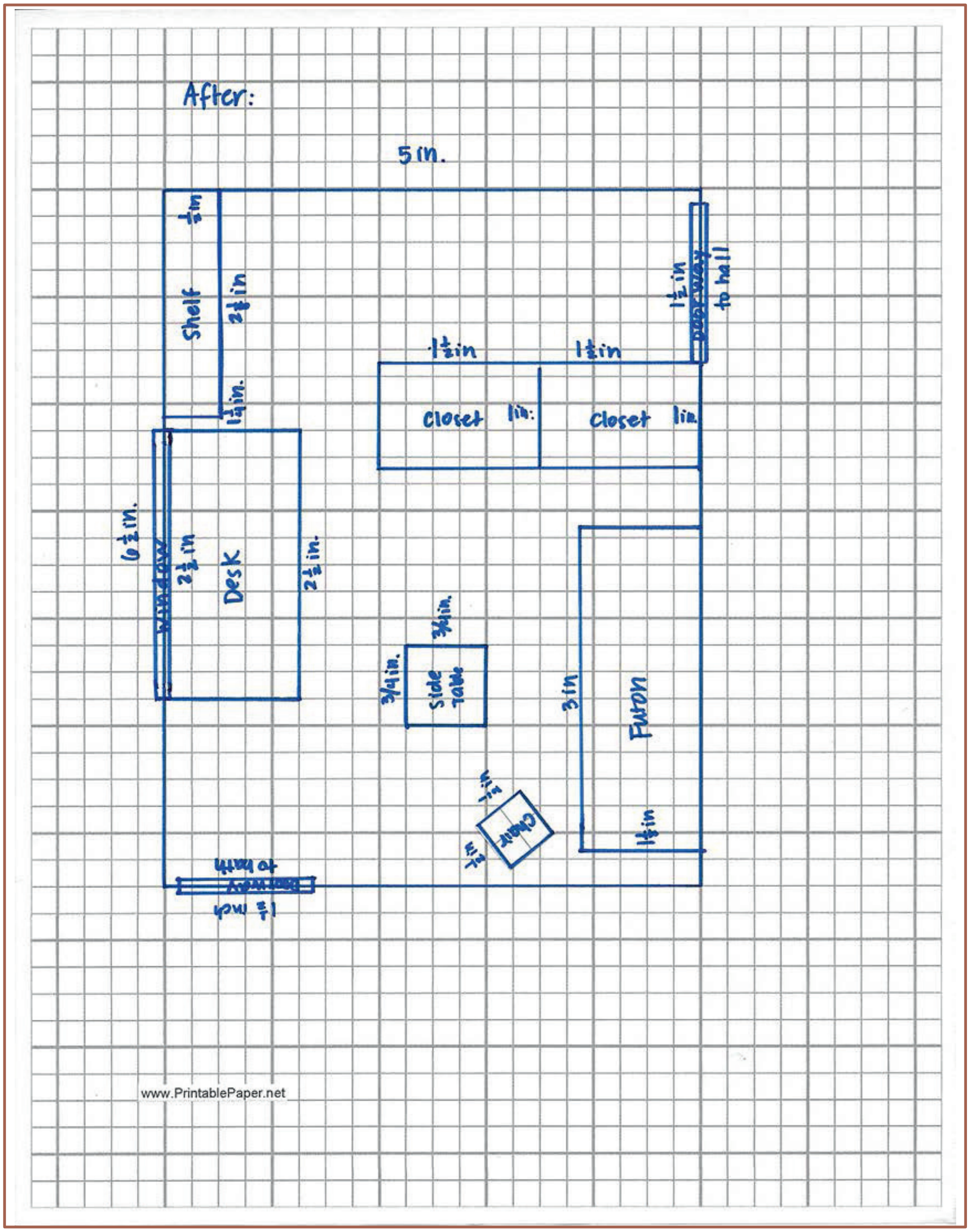
Interior Designer:

You won a spot on a famous interior designing TV show! The designers will work with you and your existing furniture to redesign a room of your choice. Your job is to create a top-view scale drawing of your room and the furniture within it.

- With the scale factor being $\frac{1}{24}$, create a scale drawing of your room or other favorite room in your home on a sheet of 8.5×11-inch graph paper.
- Include the perimeter of the room, windows, doorways, and three or more furniture pieces (such as tables, desks, dressers, chairs, bed, sofa, and ottoman).
- Use the table to record lengths and include calculations of areas.
- Make your furniture “moveable” by duplicating your scale drawing and cutting out the furniture.
- Create a “before” and “after” to help you decide how to rearrange your furniture. Take a photo of your “before.”
- What changed in your furniture plans?
- Why do you like the “after” better than the “before”?

Answers will vary.


Before:
5in
Side Table
3/4 in
3/4in
Desk
1 1/4 in
2 1/2 in
Doorway to hall
1 1/2 in
1/2 in
Chair
1/2 in
6 1/2 in.
Window
2 1/2 in
Futon
3 in.
2 1/2 in.
Shelf
1 1/8 in.
1/2 in.
1 1/2 in.
1 1/2 in.
Closet
Closet
1 in.
1 inch
to bathroom
Doorway
1 1/2 inch
www.PrintablePaper.net


After:
5 in.
Shelf
Doorway to hall
Closet
Closet
1 in.
Desk
Window
Side Table
Futon
Chair
www.PrintablePaper.net

	Entire Room	Windows	Doors	Desk/Tables	Seating	Storage	Bed	Shelf	Side Table
Actual Length:	10 ft.	5 ft.	3 ft.	5 ft.	1 ft.	3 ft.	6 ft.	$4\frac{1}{4}$ ft.	$1\frac{1}{2}$ ft.
Actual Width:	13 ft.	/	/	$2\frac{5}{12}$ ft.	1 ft.	2 ft.	$2\frac{1}{4}$ ft.	1 ft.	$1\frac{1}{2}$ ft.
Scale Drawing Length:	5 in.	$2\frac{1}{2}$ in.	$1\frac{1}{2}$ in.	$2\frac{1}{2}$ in.	$\frac{1}{2}$ in.	$1\frac{1}{2}$ in.	3 in.	$2\frac{1}{8}$ in.	$\frac{3}{4}$ in.
Scale Drawing Width:	$6\frac{1}{2}$ in.	/	/	*approx.* $1\frac{1}{4}$ in.	$\frac{1}{2}$ in.	1 in.	$1\frac{1}{8}$ in.	$\frac{1}{2}$ in.	$\frac{3}{4}$ in.

	Entire Room Length	Desk/Tables	Seating	Storage	Bed	Shelf	Side Table
Actual Area (ft^2):	10×13 $= 130$	$5 \times 2\frac{5}{12}$ $= 5 \times \frac{29}{12}$ $= \frac{145}{12}$ $= 12\frac{1}{12}$	$1 \times 1 = 1$	$3 \times 2 = 6$	$6 \times 2\frac{1}{4}$ $= 6 \times \frac{9}{4}$ $= \frac{27}{2}$ $= 13\frac{1}{2}$	$4\frac{1}{4} \times 1 = 4\frac{1}{4}$	$1\frac{1}{2} \times 1\frac{1}{2}$ $= \frac{3}{2} \times \frac{3}{2}$ $= \frac{9}{4}$ $= 2\frac{1}{4}$
Scale Drawing Area (in^2):	$5 \times 6\frac{1}{2}$ $= 5 \times \frac{13}{2}$ $= \frac{65}{2}$ $= 32\frac{1}{2}$	$2\frac{1}{2} \times 1\frac{1}{4}$ $= \frac{5}{2} \times \frac{5}{4}$ $= \frac{25}{8}$ $= 3\frac{1}{8}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	$1\frac{1}{2} \times 1 = 1\frac{1}{2}$	$3 \times 1\frac{1}{8}$ $= 3 \times \frac{9}{8}$ $= \frac{27}{8}$ $= 3\frac{3}{8}$	$2\frac{1}{8} \times \frac{1}{2}$ $= \frac{17}{8} \times \frac{1}{2}$ $= \frac{17}{16}$ $= 1\frac{1}{16}$	$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

Lesson Summary

Variations of Scale Drawings with different scale factors are scale drawings of an original scale drawing.

From a scale drawing at a different scale, the scale factor for the original scale drawing can be computed without information of the actual object, figure, or picture.

- **For example, if *scale drawing one* has a scale factor of $\frac{1}{24}$ and *scale drawing two* has a scale factor of $\frac{1}{72}$, then the scale factor relating *scale drawing two* to *scale drawing one* is**

$$\frac{1}{72} \text{ to } \frac{1}{24} = \frac{\frac{1}{72}}{\frac{1}{24}} = \frac{1}{72} \cdot \frac{24}{1} = \frac{1}{3}.$$

Scale drawing two* has lengths that are $\frac{1}{3}$ the size of the lengths of *scale drawing one.

Problem Set Sample Solutions

1. **Jake reads the following problem: If the original scale factor for a scale drawing of a square swimming pool is $\frac{1}{90}$, and the length of the original drawing measured to be 8 inches, what is the length on the new scale drawing if the scale factor of the new scale drawing length to actual length is $\frac{1}{144}$?**

   He works out the problem:

   8 inches $\div \frac{1}{90} = 720$ inches

   720 inches $\times \frac{1}{144} = 5$ inches

   Is he correct? Explain why or why not.

   Jake is correct. He took the original scale drawing length and divided by the original scale factor to get the actual length, 720 inches. To get the new scale drawing length, he takes the actual length, 720, and multiplies by the new scale factor, $\frac{1}{144}$, to get 5 inches.

2. **What is the scale factor of the new scale drawing to the original scale drawing (SD2 to SD1)?**

$$\frac{\frac{1}{144}}{\frac{1}{90}} = \frac{5}{8}$$

3. **Using the scale, if the length of the pool measures 10 cm on the new scale drawing:**

   a. **Using the scale factor from Problem 1, $\frac{1}{144}$, find the actual length of the pool in meters.**

   14.40 m

   b. **What is the surface area of the floor of the actual pool? Rounded to the nearest tenth.**

   $14.4 \text{ m} \times 14.4 \text{ m} = 207.36 \text{ m}^2 \approx 207.4 \text{ m}^2$

c. **If the pool has a constant depth of 1.5 meters, what is the volume of the pool? Rounded to the nearest tenth.**

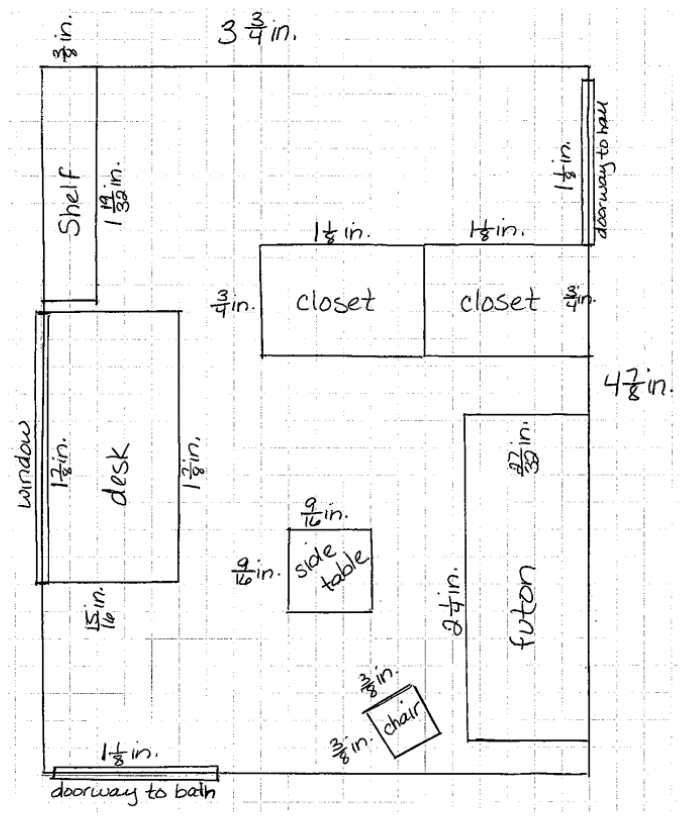
$14.4\text{ m} \times 14.4\text{ m} \times 1.5\text{ m} = 311.04\text{ m}^3 \approx 311.0\text{ m}^3$

d. **If 1 cubic meter of water is equal to 264.2 gallons, how much water will the pool contain when completely filled? Rounded to the nearest unit.**

$311.0\text{ m}^3 \times \dfrac{264.2\text{ gallons}}{1\text{ m}^3} \approx 82{,}166\text{ gallons}$

4. **Complete a new scale drawing of your dream room from the Problem Set in Lesson 20 by either reducing by $\frac{1}{4}$ or enlarging it by 4.**

Scale drawings will vary.



	Entire Room	*Windows*	*Doors*	*Desk*	*Futon*	*Closets*	*Shelf*	*Side Table*	*Chair*
Scale Drawing Length (in.)	$6\frac{1}{2}\times\frac{3}{4}$ $=\frac{13}{2}\times\frac{3}{4}$ $=\frac{39}{8}$ $=4\frac{7}{8}$	$2\frac{1}{2}\times\frac{3}{4}$ $=\frac{5}{2}\times\frac{3}{4}$ $=\frac{15}{8}$ $=1\frac{7}{8}$	$1\frac{1}{2}\times\frac{3}{4}$ $=\frac{3}{2}\times\frac{3}{4}$ $=\frac{9}{8}$ $=1\frac{1}{8}$	$2\frac{1}{2}\times\frac{3}{4}$ $=\frac{5}{2}\times\frac{3}{4}$ $=\frac{15}{8}$ $=1\frac{7}{8}$	$3\times\frac{3}{4}$ $=\frac{3}{1}\times\frac{3}{4}$ $=\frac{9}{4}$ $=2\frac{1}{4}$	$1\frac{1}{2}\times\frac{3}{4}$ $=\frac{3}{2}\times\frac{3}{4}$ $=\frac{9}{8}$ $=1\frac{1}{8}$	$2\frac{1}{8}\times\frac{3}{4}$ $=\frac{17}{8}\times\frac{3}{4}$ $=\frac{51}{32}$ $=1\frac{19}{32}$	$\frac{3}{4}\times\frac{3}{4}$ $=\frac{9}{16}$	$\frac{1}{2}\times\frac{3}{4}$ $=\frac{3}{8}$
Scale Drawing Width (in.)	$5\times\frac{3}{4}$ $=\frac{5}{1}\times\frac{3}{4}$ $=\frac{15}{4}$ $=3\frac{3}{4}$			$1\frac{1}{4}\times\frac{3}{4}$ $=\frac{5}{4}\times\frac{3}{4}$ $=\frac{15}{16}$	$1\frac{1}{8}\times\frac{3}{4}$ $=\frac{9}{8}\times\frac{3}{4}$ $=\frac{27}{32}$	$1\times\frac{3}{4}$ $=\frac{3}{4}$	$\frac{1}{2}\times\frac{3}{4}$ $=\frac{3}{8}$	$\frac{3}{4}\times\frac{3}{4}$ $=\frac{9}{16}$	$\frac{1}{2}\times\frac{3}{4}$ $=\frac{3}{8}$

Exit Ticket Sample Solutions

The school is building a new wheelchair ramp for one of the remodeled bathrooms. The original drawing was created by the contractor, but the principal drew another scale drawing to see the size of the ramp relative to the walkways surrounding it. Find the missing values on the table.

Original Scale Drawing

12 in.

Principal's Scale Drawing

New Scale Factor of SD2 to the actual ramp: $\frac{1}{700}$

3 in.

Scale Factor Table

	Actual Ramp	Original Scale Drawing	Principal's Scale Drawing
Actual Ramp	1	175	700
Original Scale Drawing	$\frac{1}{175}$	1	4
Principal's Scale Drawing	$\frac{1}{700}$	$\frac{1}{4}$	1

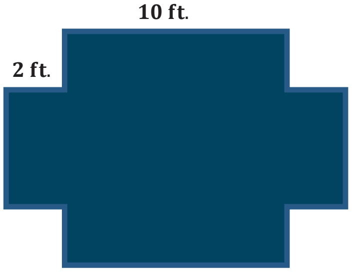
Problem Set Sample Solutions

1. For the scale drawing, the actual lengths are labeled onto the scale drawing. Measure the lengths, in centimeters, of the scale drawing with a ruler, and draw a new scale drawing with a scale factor (SD2 to SD1) of $\frac{1}{2}$.

10 ft.

2 ft.

4 ft.



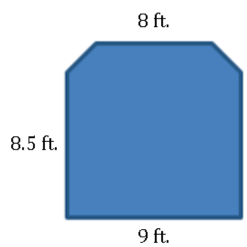
2.25 cm

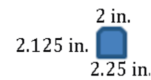
0.4 cm

0.75 cm

2. **Compute the scale factor of the new scale drawing (SD2) to the first scale drawing (SD1) using the information from the given scale drawings.**

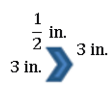
a. **Original Scale Factor:** $\frac{6}{35}$ **New Scale Factor:** $\frac{1}{280}$

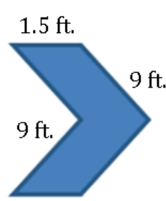




Scale Factor: $\frac{1}{48}$

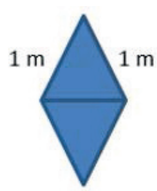
b. **Original Scale Factor:** $\frac{1}{12}$ **New Scale Factor:** 3

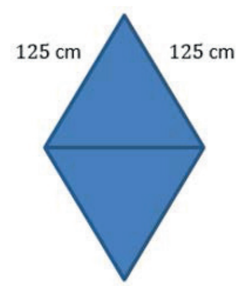




Scale Factor: 36

c. **Original Scale Factor:** 20 **New Scale Factor:** 25





Scale Factor: $\frac{5}{4}$