Problem Set Sample Solutions

The Problem Set provides practice with real-world situations involving the additive inverse such as temperature and money. Students also explore more scenarios from the Integer Game to provide a solid foundation for Lesson 2.

For Problems 1 and 2, refer to the Integer Game.

1. **You have two cards with a sum of (-12) in your hand.**

   a. **What two cards could you have?**

   Answers will vary. $(-6$ and $-6)$

   b. **You add two more cards to your hand, but the total sum of the cards remains the same, (-12). Give some different examples of two cards you could choose.**

   Answers will vary, but numbers must be opposites. $(-2$ and $2)$ and $(4$ and $-4)$

2. **Choose one card value and its additive inverse. Choose from the list below to write a real-world story problem that would model their sum.**

   a. **Elevation: above and below sea level**

   Answers will vary. (A scuba diver is 20 feet below sea level. He had to rise 20 feet in order to get back on the boat.)

   b. **Money: credits and debits, deposits and withdrawals**

   Answers will vary. (The bank charges a fee of $\$5$ for replacing a lost debit card. If you make a deposit of $\$5$, what would be the sum of the fee and the deposit?)

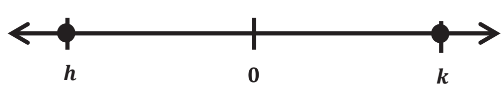
   c. **Temperature: above and below 0 degrees**

   Answers will vary. (The temperature of one room is 5 degrees above 0. The temperature of another room is 5 degrees below zero. What is the sum of both temperatures?)

   d. **Football: loss and gain of yards**

   Answers will vary. (A football player gained 25 yards on the first play. On the second play, he lost 25 yards. What is his net yardage after both plays?)

3. **On the number line below, the numbers h and k are the same distance from 0. Write an equation to express the value of $h + k$. Explain.**



$h + k = 0$ because their absolute values are equal, but their directions are opposite. k is the additive inverse of h, and h is the additive inverse of k because they are the same distance from 0. Therefore, the sum of k and h is 0, because additive inverses have a sum of 0.

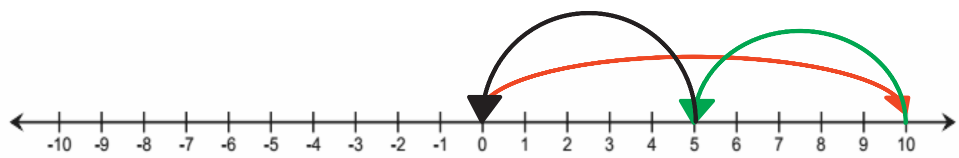
4. **During a football game, Kevin gained five yards on the first play. Then he lost seven yards on the second play. How many yards does Kevin need on the next play to get the team back to where they were when they started? Show your work.**

   He has to gain 2 yards.

   $5 + (-7) + 2 = 0$, $5 + (-7) = -2$, *and* $-2 + 2 = 0$.

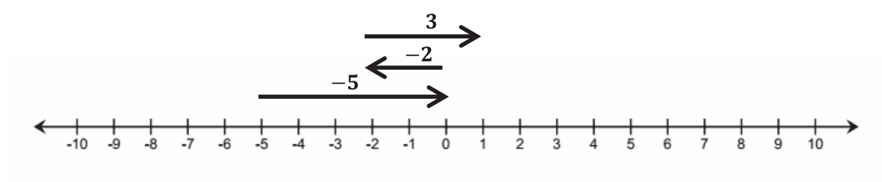
5. **Write an addition number sentence that corresponds to the arrows below.**

   $10 + (-5) + (-5) = 0$



Exit Ticket Sample Solutions

Jessica made the addition model below of the expression $(-5) + (-2) + 3$.



a. **Do the arrows correctly represent the numbers that Jessica is using in her expression?**

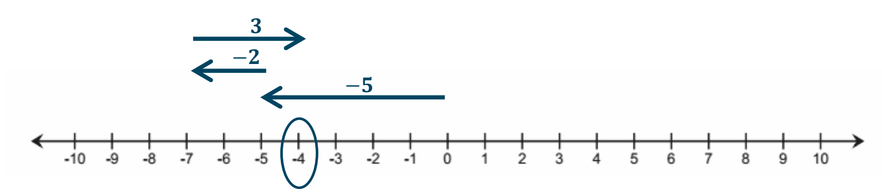
No. Jessica started her first arrow at -5 instead of 0. Negative numbers should be shown as counting down, so the arrow should have started at 0 and pointed left, ending on -5. The other arrows are drawn correctly, but they are in the wrong places because the starting arrow is in the wrong place.

b. **Jessica used the number line diagram above to conclude that the sum of the three numbers is 1. Is she correct?**

Jessica is incorrect.

c. **If she is incorrect, find the sum, and draw the correct model.**

The sum should be -4. $-5 + (-2) + 3 = -4$.



d. **Write a real-world situation that would represent the sum.**

Answers will vary. A football team lost 5 yards on the first play. On the second play, the team lost another 2 yards. Then, the team gained 3 yards. After three plays, the team had a total yardage of -4 yards.

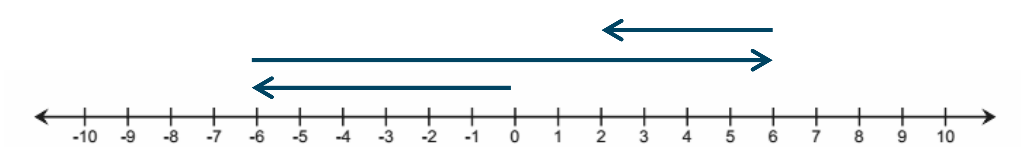
Problem Set Sample Solutions

The Problem Set provides students practice with integer addition using the Integer Game, number lines, and story problems. Students should show their work with accuracy in order to demonstrate mastery.

Represent Problems 1–3 using both a number line diagram and an equation.

1. **David and Victoria are playing the Integer Card Game. David drew three cards, -6, 12, and -4. What is the sum of the cards in his hand? Model your answer on the number line below.**

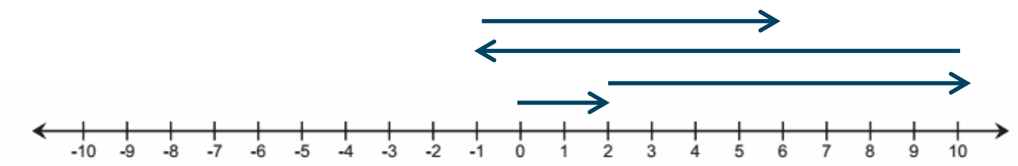
$(-6) + 12 + (-4) = 2$



2. In the Integer Card Game, you drew the cards, 2, 8, and -11. Your partner gave you a 7 from his hand.

   a. What is your total? Model your answer on the number line below.

   $2 + 8 + (-11) + 7 = 6$



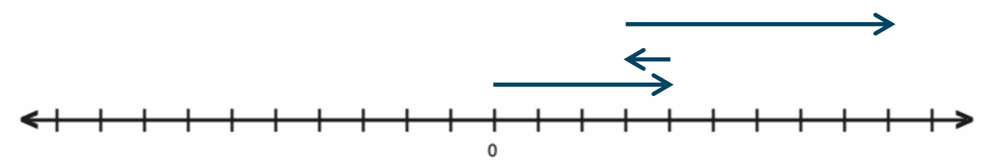
   b. What card(s) would you need to get your score back to zero? Explain. Use and explain the term *additive inverse* in your answer.

   You would need any combination of cards that sum to -6 because the additive inverse of 6 is -6.

   $6 + (-6) = 0$

3. If a football player gains 40 yards on a play, but on the next play, he loses 10 yards, what would his total yards be for the game if he ran for another 60 yards? What did you count by to label the units on your number line?

   90 yards because $40 + (-10) + 60 = 90$. Student answers may vary, but they should not choose to count by 1.



4. Find the sums.

   a. $-2 + 9$

   7

   b. $-8 + -8$

   -16

   c. $-4 + (-6) + 10$

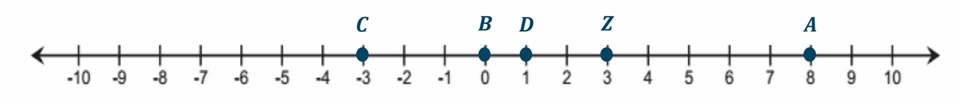
   0

   d. $5 + 7 + (-11)$

   1

5. Mark an integer between 1 and 5 on a number line, and label it point Z. Then, locate and label each of the following points by finding the sums.

Answers will vary. Sample student response below.



a. Point A: $Z + 5$

Point A: $3 + 5 = 8$

b. Point B: $Z + (-3)$

Point B: $3 + (-3) = 0$

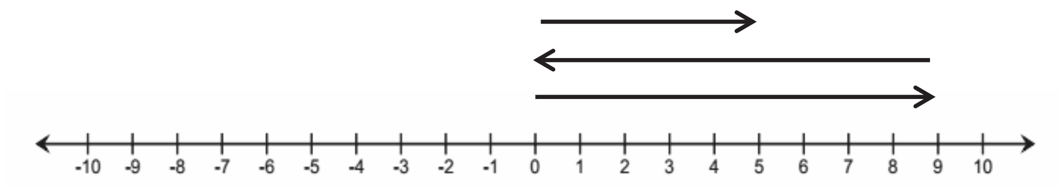
c. Point C: $(-4) + (-2) + Z$

Point C: $(-4) + (-2) + 3 = -3$

d. Point D: $-3 + Z + 1$

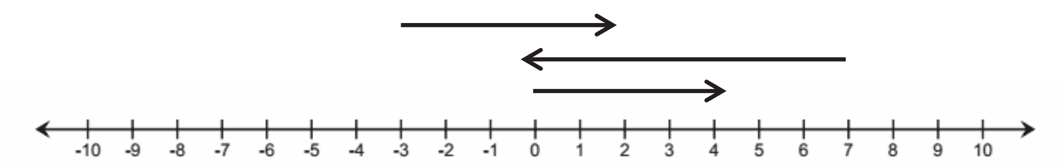
Point D: $-3 + 3 + 1 = 1$

6. Write a story problem that would model the sum of the arrows in the number diagram below.

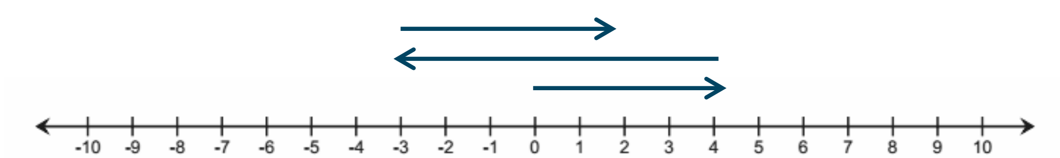


Answers will vary. Jill got on an elevator and went to the 9^{th} floor. She accidently pressed the down button and went back to the lobby. She pressed the button for the 5^{th} floor and got off the elevator.

7. Do the arrows correctly represent the equation $4 + (-7) + 5 = 2$? If not, draw a correct model below.



No, the arrows are incorrect. The correct model is shown.

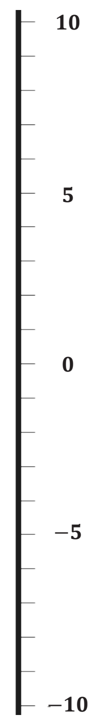


Problem Set Sample Solutions

Practice problems help students build fluency and improve accuracy when adding integers with and without the use of a number line. Students need to be comfortable with using vectors to represent integers on the number line, including the application of absolute value to represent the length of a vector.

1. Below is a table showing the change in temperature from morning to afternoon for one week.

a. Use the vertical number line to help you complete the table. As an example, the first row is completed for you.



Change in Temperatures from Morning to Afternoon

Morning Temperature	Change	Afternoon Temperature	Equation
$1°C$	Rise of $3°C$	$4°C$	$1 + 3 = 4$
$2°C$	Rise of $8°C$	$10°C$	$2 + 8 = 10$
$-2°C$	Fall of $6°C$	$-8°C$	$-2 + (-6) = -8$
$-4°C$	Rise of $7°C$	$3°C$	$-4 + 7 = 3$
$6°C$	Fall of $9°C$	$-3°C$	$6 + (-9) = -3$
$-5°C$	Fall of $5°C$	$-10°C$	$-5 + (-5) = -10$
$7°C$	Fall of $7°C$	$0°C$	$7 + (-7) = 0$

b. Do you agree or disagree with the following statement: "A rise of $-7°C$" means "a fall of $7°C$?" Explain. (Note: No one would ever say, "A rise of -7 degrees"; however, mathematically speaking, it is an equivalent phrase.)

Sample response: I agree with this statement because a rise of -7 is the opposite of a rise of 7. The opposite of a rise of 7 is a fall of 7.

For Problems 2–3, refer to the Integer Game.

2. Terry selected two cards. The sum of her cards is -10.

a. Can both cards be positive? Explain why or why not.

No. In order for the sum to be -10, at least one of the addends would have to be negative. If both cards are positive, then Terry would count up twice going to the right. Negative integers are to the left of 0.

b. Can one of the cards be positive and the other be negative? Explain why or why not.

Yes. Since both cards cannot be positive, this means that one can be positive and the other negative. She could have -11 and 1 or -12 and 2. The card with the greatest absolute value would have to be negative.

c. Can both cards be negative? Explain why or why not.

Yes, both cards could be negative. She could have -8 and -2. On a number line, the sum of two negative integers will be to the left of 0.

3. **When playing the Integer Game, the first two cards you selected were -8 and -10.**

   a. **What is the value of your hand? Write an equation to justify your answer.**

   $-8 + (-10) = -18$

   b. **For part (a), what is the distance of the sum from -8? Does the sum lie to the right or left of -8 on the number line?**

   The distance is 10 units from -8, and it lies to the left of -8 on the number line.

   c. **If you discarded the -10 and then selected a 10, what would be the value of your hand? Write an equation to justify your answer.**

   The value of the hand would be 2. $-8 + 10 = 2$.

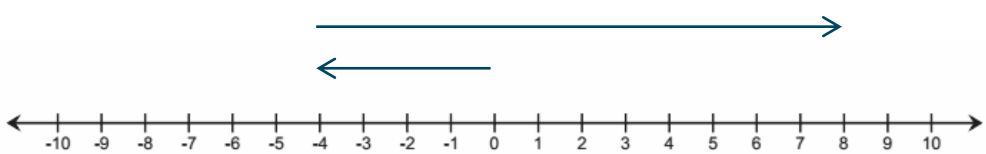
4. **Given the expression $67 + (-35)$, can you determine, without finding the sum, the distance between 67 and the sum? Is the sum to the right or left of 67 on the number line?**

   The distance would be 35 units from 67. The sum is to the left of 67 on the number line.

5. **Use the information given below to write an equation. Then create an *arrow diagram* of this equation on the number line provided below.**

   The sum of -4 and a number is 12 units to the right of -4 on a number line.

   $-4 + 12 = 8$



Exit Ticket Sample Solutions

1. Write an addition problem that has a sum of $-4\frac{4}{5}$ and

   a. The two addends have the same sign.

      Answers will vary. $-1\frac{4}{5} + (-3) = -4\frac{4}{5}$.

   b. The two addends have different signs.

      Answers will vary. $1.8 + (-6.6) = -4.8$.

2. In the Integer Game, what card would you need to draw to get a score of 0 if you have a -16, -35, and 18 in your hand?

   $-16 + (-35) + 18 = -33$, *so I would need to draw a* 33 *because* 33 *is the additive inverse of* -33.
   $-33 + 33 = 0$.

Problem Set Sample Solutions

Students must understand the rules for addition of rational numbers with the same and opposite signs. The Problem Set presents multiple representations of these rules including diagrams, equations, and story problems. Students are expected to show their work or provide an explanation where necessary to justify their answers. Answers can be represented in fraction or decimal form.

1. Find the sum. Show your work to justify your answer.

   a. $4 + 17$

      $4 + 17 = 21$

   b. $-6 + (-12)$

      $-6 + (-12) = -18$

   c. $2.2 + (-3.7)$

      $2.2 + (-3.7) = -1.5$

   d. $-3 + (-5) + 8$

      $-3 + (-5) + 8 = -8 + 8 = 0$

   e. $\frac{1}{3} + \left(-2\frac{1}{4}\right)$

      $\frac{1}{3} + \left(-2\frac{1}{4}\right) = \frac{1}{3} + \left(-\frac{9}{4}\right) = \frac{4}{12} + \left(-\frac{27}{12}\right) = -\frac{23}{12} = -1\frac{11}{12}$

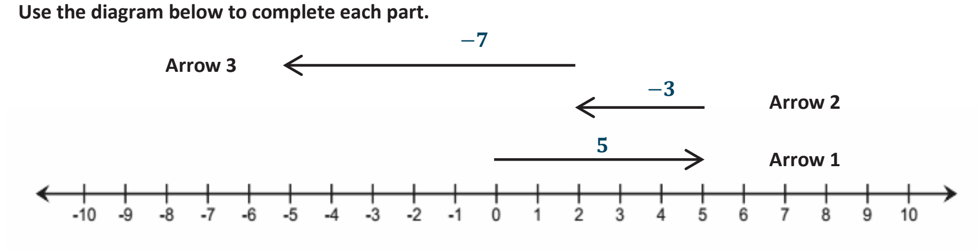
2. Which of these story problems describes the sum $19 + (-12)$? Check all that apply. Show your work to justify your answer.

___*X*___ Jared's dad paid him $\$19$ for raking the leaves from the yard on Wednesday. Jared spent $\$12$ at the movie theater on Friday. How much money does Jared have left?

______ Jared owed his brother $\$19$ for raking the leaves while Jared was sick. Jared's dad gave him $\$12$ for doing his chores for the week. How much money does Jared have now?

___*X*___ Jared's grandmother gave him $\$19$ for his birthday. He bought $\$8$ worth of candy and spent another $\$4$ on a new comic book. How much money does Jared have left over?

3. Use the diagram below to complete each part.



a. Label each arrow with the number the arrow represents.

b. How long is each arrow? What direction does each arrow point?

Arrow	Length	Direction
1	5	*right*
2	3	*left*
3	7	*left*

c. Write an equation that represents the sum of the numbers. Find the sum.

$5 + (-3) + (-7) = -5$

4. Jennifer and Katie were playing the Integer Game in class. Their hands are represented below.

Jennifer's Hand

5 | −8

Katie's Hand

−9 | 7

a. What is the value of each of their hands? Show your work to support your answer.

Jennifer's hand has a value of -3 because $5 + (-8) = -3$. Katie's hand has a value of -2 because $-9 + 7 = -2$.

b. If Jennifer drew two more cards, is it possible for the value of her hand not to change? Explain why or why not.

It is possible for her hand not to change. Jennifer could get any two cards that are the exact opposites such as (-2) and 2. Numbers that are exact opposites are called additive inverses, and they sum to 0. Adding the number 0 to anything will not change the value.

c. **If Katie wanted to win the game by getting a score of 0, what card would she need? Explain.**

Katie would need to draw a 2 because the additive inverse of -2 is 2. $-2+2=0$.

d. **If Jennifer drew -1 and -2, what would be her new score? Show your work to support your answer.**

Jennifer's new score would be -6 because $-3+(-1)+(-2)=-6$.

Exit Ticket Sample Solutions

1. **If a player had the following cards, what is the value of his hand?**

   The current value of the hand is -2. $1+(-7)+4=-2$.

1	-7	4

   a. **Identify two different ways the player could get to a score of 5 by adding or removing only one card. Explain.**

   He could remove the -7 *or add* 7. *If he removes the* -7, *the value of the hand will be* 5, *which is* 7 *larger than* -2. *He could also get a sum of* 5 *by adding* 7 *to the hand. Therefore, removing the* -7 *gives him the same result as adding* 7.

   b. **Write two equations for part (a), one for each of the methods you came up with for arriving at a score of 5.**

   $-2-(-7)=5$ *and* $-2+7=5$

2. **Using the rule of subtraction, rewrite the following subtraction expressions as addition expressions, and find the sums.**

   a. $5-9$

   $5+(-9)=-4$

   b. $-14-(-2)$

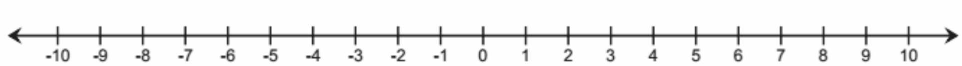
   $-14+2=-12$

Problem Set Sample Solutions

The Problem Set provides students with skill practice and application of the rules for integer subtraction. Students solve problems with and without a number line.

1. **On a number line, find the difference of each number and 4. Complete the table to support your answers. The first example is provided.**

Number	Subtraction Expression	Addition Expression	Answer
10	$10-4$	$10+(-4)$	6
2	$2-4$	$2+(-4)$	-2
-4	$-4-4$	$-4+(-4)$	-8
-6	$-6-4$	$-6+(-4)$	-10
1	$1-4$	$1+(-4)$	-3



2. You and your partner were playing the Integer Game in class. Here are the cards in both hands.

Your hand				*Your partner's hand*			
-8	6	1	-2	9	-5	2	-7

   a. **Find the value of each hand. Who would win based on the current scores? (The score closest to 0 wins.)**

   My hand: $-8+6+1+(-2)=-3$

   Partner's hand: $9+(-5)+2+(-7)=-1$

   My partner would win because -1 *is closer to* 0. *It is* 1 *unit to the left of* 0.

   b. **Find the value of each hand if you discarded the -2 and selected a 5, and your partner discarded the -5 and selected a 5. Show your work to support your answer.**

   My hand: Discard the -2, $-3-(-2)=-1$; *Select a* 5: $-1+5=4$.

   Partner's hand: Discard the -5, $-1-(-5)=4$; *Select a* 5: $4+5=9$.

   c. **Use your score values from part (b) to determine who would win the game now.**

   I would win now because 4 *is closer to zero.*

3. Write the following expressions as a single integer.

   a. $-2+16$

   14

   b. $-2-(-16)$

   14

   c. $18-26$

   -8

   d. $-14-23$

   -37

   e. $30-(-45)$

   75

4. Explain what is meant by the following, and illustrate with an example:

"For any real numbers, p and q, $p - q = p + (-q)$."

Subtracting a number is the same as adding its additive inverse. Examples will vary. A sample response is shown below.

$p = 4$, $q = 6$, $4 - 6$ is the same as $4 + (-6)$ because -6 is the opposite of 6.

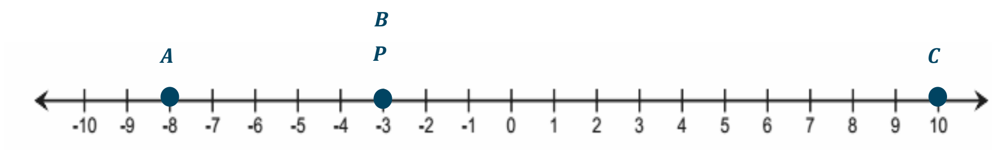
$4 - 6 = -2$

$4 + (-6) = -2$

So, $4 - 6 = 4 + (-6)$ because they both equal -2.

5. Choose an integer between -1 and -5 on the number line, and label it point P. Locate and label the following points on the number line. Show your work.

Answers will vary. A sample response is shown below given that the student chose -3 for P.



a. Point A: $P - 5$

Point A: $-3 - 5 = -8$

b. Point B: $(P - 4) + 4$

Point B: $(-3 - 4) + 4 = -3$ *(same as P)*

c. Point C: $-P - (-7)$

Point C: $-(-3) - (-7) = 3 + 7 = 10$

Challenge Problem:

6. Write two equivalent expressions that represent the situation. What is the difference in their elevations?

An airplane flies at an altitude of $26,000$ feet. A submarine dives to a depth of 700 feet below sea level.

Two equivalent expressions are $26,000 - (-700)$ and $26,000 + 700$. The difference in their elevations is $26,700$ feet.

Exit Ticket Sample Solutions

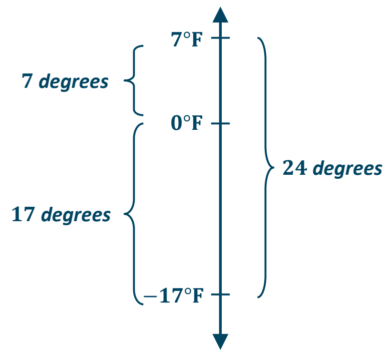
Two Grade 7 students, Monique and Matt, both solved the following math problem:

If the temperature drops from $7°\text{F}$ to $-17°\text{F}$, by how much did the temperature decrease?

The students came up with different answers. Monique said the answer is $24°\text{F}$, and Matt said the answer is $10°\text{F}$. Who is correct? Explain, and support your written response with the use of a formula and a vertical number line diagram.

Monique is correct. If you use the distance formula, you take the absolute value of the difference between 7 and -17 and that equals 24. Using a number line diagram, you can count the number of units between 7 and -17 to get 24.

$|7-(-17)| = |7+17| = |24| = 24$. *There was a $24°\text{F}$ drop in the temperature.*



Problem Set Sample Solutions

1. $|-19-12| = |-19+(-12)| = |-31| = 31$
2. $|19-(-12)| = |19+12| = |31| = 31$
3. $|10-(-43)| = |10+43| = |53| = 53$
4. $|-10-43| = |-10+(-43)| = |-53| = 53$
5. $|-1-(-16)| = |-1+16| = |15| = 15$
6. $|1-16| = |1+(-16)| = |-15| = 15$
7. $|0-(-9)| = |0+9| = |9| = 9$
8. $|0-9| = |0+(-9)| = |-9| = 9$
9. $|-14.5-13| = |-14.5+(-13)| = |-27.5| = 27.5$
10. $|14.5-(-13)| = |14.5+13| = |27.5| = 27.5$
11. **Describe any patterns you see in the answers to the problems in the left- and right-hand columns. Why do you think this pattern exists?**

MP.7

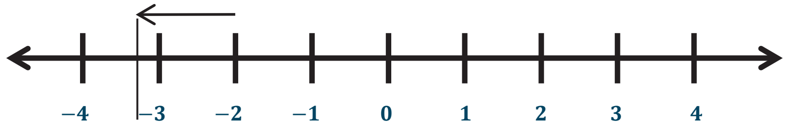
Each problem in the right-hand column has the same answer as the problem across from it in the left-hand column. That is because you are finding the distance between the opposite numbers as compared to the first column. The difference between the opposite numbers is opposite the difference between the original numbers. The absolute values of opposite numbers are the same.

Exit Ticket Sample Solutions

At the beginning of the summer, the water level of a pond is 2 feet below its normal level. After an unusually dry summer, the water level of the pond dropped another $1\frac{1}{3}$ feet.

1. **Use a number line diagram to model the pond's current water level in relation to its normal water level.**

   Move $1\frac{1}{3}$ units to the left of -2. $-3\frac{1}{3}$



2. **Write an equation to show how far above or below the normal water level the pond is at the end of the summer.**

   $-2-1\frac{1}{3}=-3\frac{1}{3}$ *or* $-2+\left(-1\frac{1}{3}\right)=-3\frac{1}{3}$

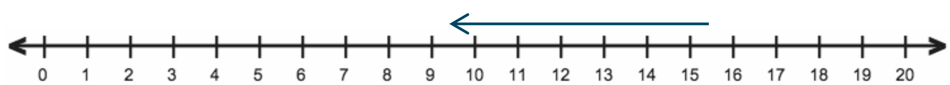
Problem Set Sample Solutions

Represent each of the following problems using both a number line diagram and an equation.

1. **A bird that was perched atop a $15\frac{1}{2}$-foot tree dives down six feet to a branch below. How far above the ground is the bird's new location?**

   $15\frac{1}{2}+(-6)=9\frac{1}{2}$ *or* $15\frac{1}{2}-6=9\frac{1}{2}$

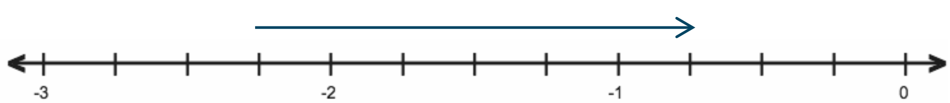
   The bird is $9\frac{1}{2}$ feet above the ground.



2. **Mariah owed her grandfather $\$2.25$ but was recently able to pay him back $\$1.50$. How much does Mariah currently owe her grandfather?**

   $-2.25+1.50=-0.75$

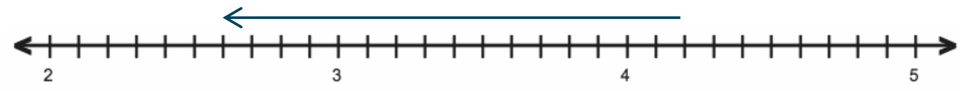
   Mariah owes her grandfather 75 cents.



3. **Jake is hiking a trail that leads to the top of a canyon. The trail is 4.2 miles long, and Jake plans to stop for lunch after he completes 1.6 miles. How far from the top of the canyon will Jake be when he stops for lunch?**

$\mathbf{4.2 - 1.6 = 2.6}$

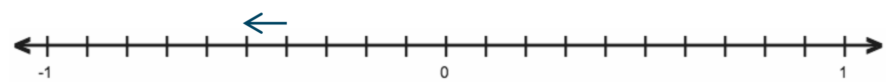
Jake will be 2.6 miles from the top of the canyon.



4. **Sonji and her friend Rachel are competing in a running race. When Sonji is 0.4 miles from the finish line, she notices that her friend Rachel has fallen. If Sonji runs one-tenth of a mile back to help her friend, how far will she be from the finish line?**

$\mathbf{-0.4 + (-0.1) = -0.5}$ ***or*** $\mathbf{-0.4 - 0.1 = -0.5}$

Sonji will be 0.5 miles from the finish line.



5. **Mr. Henderson did not realize his checking account had a balance of $\$200$ when he used his debit card for a $\$317.25$ purchase. What is his checking account balance after the purchase?**

$\mathbf{200 + (-317.25) = -117.25}$ ***or*** $\mathbf{200 - 317.25 = -117.25}$

Mr. Henderson's checking account balance will be $-\$117.25$.

6. **If the temperature is $-3°\text{F}$ at 10:00 p.m., and the temperature falls four degrees overnight, what is the resulting temperature?**

$\mathbf{-3 - 4 = -3 + (-4) = -7}$

The resulting temperature is $-7°\text{F}$.

Exit Ticket Sample Solutions

Mariah and Shane both started to work on a math problem and were comparing their work in math class. Are both of their representations correct? Explain, and finish the math problem correctly to arrive at the correct answer.

<u>Math Problem</u>

Jessica's friend lent her $\$5$. Later that day Jessica gave her friend back $1\frac{3}{4}$ dollars.

Which rational number represents the overall change to the amount of money Jessica's friend has?

Mariah started the problem as follows:

$$-5-\left(-1\frac{3}{4}\right)$$
$$= -5 + 1 - \frac{3}{4}$$

Shane started the problem as follows:

$$-5-\left(-1\frac{3}{4}\right)$$
$$= -5 + \left(1\frac{3}{4}\right)$$
$$= -5 + \left(1 + \frac{3}{4}\right)$$

Shane's method is correct. In Mariah's math work, she only dealt with part of the mixed number. The fractional part should have been positive too because the opposite of $-1\frac{3}{4}$ is $1\frac{3}{4}$, which contains both a positive 1 and a positive $\frac{3}{4}$. The correct work would be

$$-5-\left(-1\frac{3}{4}\right) = -5 + \left(1\frac{3}{4}\right) = -5 + \left(1 + \frac{3}{4}\right) = (-5+1) + \frac{3}{4} = -4 + \frac{3}{4} = -3\frac{1}{4}.$$

The rational number would be $-3\frac{1}{4}$, which means Jessica's friend gave away $3\frac{1}{4}$ dollars, or $\$3.25$.

Problem Set Sample Solutions

1. **Represent each sum as a single rational number.**

   a. $-14 + \left(-\frac{8}{9}\right)$

   $-14\frac{8}{9}$

   b. $7 + \frac{1}{9}$

   $7\frac{1}{9}$

   c. $-3 + \left(-\frac{1}{6}\right)$

   $-3\frac{1}{6}$

Rewrite each of the following to show that *the opposite of a sum is the sum of the opposites.* Problem 2 has been completed as an example.

2. $-(9+8) = -9 + (-8)$

   $-17 = -17$

   Answer provided in student materials.

3. $-\left(\frac{1}{4}+6\right) = -\frac{1}{4}+(-6)$

$$-6\frac{1}{4} = -6\ \frac{1}{4}$$

4. $-(10+(-6)) = -10+6$

$$-4 = -4$$

5. $-\left((-55)+\frac{1}{2}\right) = 55+\left(-\frac{1}{2}\right)$

$$54\frac{1}{2} = 54\frac{1}{2}$$

Use your knowledge of rational numbers to answer the following questions.

6. **Meghan said the opposite of the sum of -12 and 4 is 8. Do you agree? Why or why not?**

   Yes, I agree. The sum of -12 and 4 is -8, and the opposite of -8 is 8.

7. **Jolene lost her wallet at the mall. It had \$$10$ in it. When she got home, her brother felt sorry for her and gave her \$$5.75$. Represent this situation with an expression involving rational numbers. What is the overall change in the amount of money Jolene has?**

   $-10+5.75 = -4.25$. *The overall change in the amount of money Jolene has is -4.25 dollars.*

8. **Isaiah is completing a math problem and is at the last step: $25-28\frac{1}{5}$. What is the answer? Show your work.**

   $25-28\frac{1}{5} = 25+\left(-28+\left(-\frac{1}{5}\right)\right) = (25+-28)+\left(-\frac{1}{5}\right) = -3\frac{1}{5}$

9. **A number added to its opposite equals zero. What do you suppose is true about *a sum added to its opposite*?**

   Use the following examples to reach a conclusion. Express the answer to each example as a single rational number.

   A sum added to its opposite is zero.

   a. $(3+4)+(-3+\ -4) = 7+(-7) = 0$

   b. $(-8+1)+(8+(-1)) = (-7)+7 = 0$

   c. $\left(-\frac{1}{2}+\left(-\frac{1}{4}\right)\right)+\left(\frac{1}{2}+\frac{1}{4}\right) = \left(-\frac{3}{4}\right)+\frac{3}{4} = 0$

Exit Ticket Sample Solutions

1. Jamie was working on his math homework with his friend, Kent. Jamie looked at the following problem.

$$-9.5-(-8)-6.5$$

He told Kent that he did not know how to subtract negative numbers. Kent said that he knew how to solve the problem using only addition. What did Kent mean by that? Explain. Then, show your work, and represent the answer as a single rational number.

Kent meant that since any subtraction problem can be written as an addition problem by adding the opposite of the number you are subtracting, Jamie can solve the problem by using only addition.

Work Space:

$$\begin{aligned}&-9.5-(-8)-6.5\\&=-9.5+8+(-6.5)\\&=-9.5+(-6.5)+8\\&=-16+8\\&=-8\end{aligned}$$

Answer: -8

2. Use one rational number to represent the following expression. Show your work.

$3+(-0.2)-15\frac{1}{4}$

$=3+(-0.2)+\left(-15+\left(-\frac{1}{4}\right)\right)$

$=3+\left(-0.2+(-15)+(-0.25)\right)$

$=3+(-15.45)$

$=-12.45$

Problem Set Sample Solutions

Show all steps taken to rewrite each of the following as a single rational number.

1. $80+\left(-22\frac{4}{15}\right)$

$=80+\left(-22+\left(-\frac{4}{15}\right)\right)$

$=\left(80+(-22)\right)+\left(-\frac{4}{15}\right)$

$=58+\left(-\frac{4}{15}\right)$

$=57\frac{11}{15}$

2. $10+\left(-3\frac{3}{8}\right)$

$=10+\left(-3+\left(-\frac{3}{8}\right)\right)$

$=\left(10+(-3)\right)+\left(-\frac{3}{8}\right)$

$=7+\left(-\frac{3}{8}\right)$

$=6\frac{5}{8}$

3. $\frac{1}{5}+20.3-\left(-5\frac{3}{5}\right)$

$=\frac{1}{5}+20.3+5\frac{3}{5}$

$=\frac{1}{5}+5\frac{3}{5}+20.3$

$=5\frac{4}{5}+20.3$

$=5\frac{4}{5}+20\frac{3}{10}$

$=5\frac{8}{10}+20\frac{3}{10}$

$=25\frac{11}{10}$

$=26\frac{1}{10}$

4. $\frac{11}{12}-(-10)-\frac{5}{6}$

$=\frac{11}{12}+10+\left(-\frac{5}{6}\right)$

$=\frac{11}{12}+\left(-\frac{5}{6}\right)+10$

$=\frac{11}{12}+\left(-\frac{10}{12}\right)+10$

$=\frac{1}{12}+10$

$=10\frac{1}{12}$

5. **Explain, step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.**

$$1-\frac{3}{4}+\left(-12\frac{1}{4}\right)$$

First, I rewrote the subtraction of $\frac{3}{4}$ *as the addition of its inverse* $-\frac{3}{4}$*:* $=1+\left(-\frac{3}{4}\right)+\left(-12\frac{1}{4}\right)$

Next, I used the associative property of addition to regroup the addend: $=1+\left(\left(-\frac{3}{4}\right)+\left(-12\frac{1}{4}\right)\right)$

Next, I separated $-12\frac{1}{4}$ *into the sum of* -12 *and* $-\frac{1}{4}$*:* $=1+\left(\left(-\frac{3}{4}\right)+(-12)+\left(-\frac{1}{4}\right)\right)$

Next, I used the commutative property of addition: $=1+\left(\left(-\frac{3}{4}\right)+\left(-\frac{1}{4}\right)+(-12)\right)$

Next, I found the sum of $-\frac{3}{4}$ *and* $-\frac{1}{4}$*:* $=1+\left((-1)+(-12)\right)$

Next, I found the sum of -1 *and* -12*:* $=1+(-13)$

Lastly, since the absolute value of 13 *is greater than the absolute value of* 1*, and it is a negative* 13*, the answer will be a negative number. The absolute value of* 13 *minus the absolute value of* 1 *equals* 12*, so the answer is* -12*.* $=-12$

Problem Set Sample Solutions

1. **Describe sets of two or more matching integer cards that satisfy the criteria in each part below:**

   a. **Cards increase the score by eight points.**

   Picking up: eight 1's, four 2's, or two 4's

   OR

   Removing: eight (-1)'s, four (-2)'s, or two (-4)'s

   b. **Cards decrease the score by 9 points.**

   Picking up: nine (-1)'s or three (-3)'s

   OR

   Removing: nine 1's or three 3's

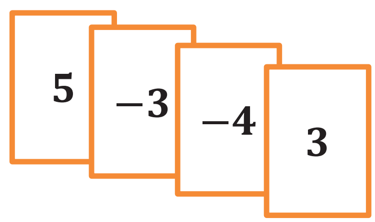
   c. **Removing cards that increase the score by 10 points.**

   Ten (-1)'s, five (-2)'s, or two (-5)'s

   d. **Positive cards that decrease the score by 18 points.**

   Removing eighteen 1's, nine 2's, six 3's, three 6's, or two 9's.

2. **You have the integer cards shown at the right when your teacher tells you to choose a card to multiply four times. If your goal is to get your score as close to zero as possible, which card would you choose? Explain how your choice changes your score.**



   The best choice to multiply is the -3. The cards currently have a score of one. The new score with the -3 multiplied by 4 is -8. The scores where the other cards are multiplied by 4 are 10, -11, and 16, which are all further from zero.

3. **Sherry is playing the Integer Game and is given a chance to discard a set of matching cards. Sherry determines that if she discards one set of cards, her score will increase by 12. If she discards another set, then her score will decrease by eight. If her matching cards make up all six cards in her hand, what cards are in Sherry's hand? Are there any other possibilities?**

   There are two possibilities:

   $2, 2, 2, 2, -6, -6$

   OR

   $-3, -3, -3, -3, 4, 4$

Exit Ticket Sample Solutions

1. Create a real-life example that can be modeled by the expression -2×4, and then state the product.

   Answers will vary. Tobi wants to lose 2 lb. each week for four weeks. Write an integer to represent Tobi's weight change after four weeks. Tobi's weight changes by – 8 lb. after four weeks.

2. Two integers are multiplied, and their product is a positive number. What must be true about the two integers?

   Both integers must be positive numbers, or both integers must be negative numbers.

Problem Set Sample Solutions

1. Complete the problems below. Then, answer the question that follows.

$3 \times 3 = 9$	$3 \times 2 = 6$	$3 \times 1 = 3$	$3 \times 0 = 0$	$3 \times (-1) = -3$	$3 \times (-2) = -6$
$2 \times 3 = 6$	$2 \times 2 = 4$	$2 \times 1 = 2$	$2 \times 0 = 0$	$2 \times (-1) = -2$	$2 \times (-2) = -4$
$1 \times 3 = 3$	$1 \times 2 = 2$	$1 \times 1 = 1$	$1 \times 0 = 0$	$1 \times (-1) = -1$	$1 \times (-2) = -2$
$0 \times 3 = 0$	$0 \times 2 = 0$	$0 \times 1 = 0$	$0 \times 0 = 0$	$0 \times (-1) = 0$	$0 \times (-2) = 0$
$-1 \times 3 = -3$	$-1 \times 2 = -2$	$-1 \times 1 = -1$	$-1 \times 0 = 0$	$-1 \times (-1) = 1$	$-1 \times (-2) = 2$
$-2 \times 3 = -6$	$-2 \times 2 = -4$	$-2 \times 1 = -2$	$-2 \times 0 = 0$	$-2 \times (-1) = 2$	$-2 \times (-2) = 4$
$-3 \times 3 = -9$	$-3 \times 2 = -6$	$-3 \times 1 = -3$	$-3 \times 0 = 0$	$-3 \times (-1) = 3$	$-3 \times (-2) = 6$

   Which row shows the same pattern as the outlined column? Are the problems similar or different? Explain.

   The row outlined shows the same pattern as the outlined column. The problems have the same answers, but the signs of the integer factors are switched. For example, $3 \times (-1) = -3 \times 1$. This shows that the product of two integers with opposite signs is equal to the product of their opposites.

2. Explain why $(-4) \times (-5) = 20$. Use patterns, an example from the Integer Game, or the properties of operations to support your reasoning.

   Answers may vary. Losing four -5 cards will increase a score in the Integer Game by 20 because a negative value decreases a score, but the score increases when it is removed.

3. Each time that Samantha rides the commuter train, she spends $\$4$ for her fare. Write an integer that represents the change in Samantha's money from riding the commuter train to and from work for 13 days. Explain your reasoning.

   If Samantha rides to and from work for 13 days, then she rides the train a total of 26 times. The cost of each ride can be represented by -4. So, the change to Samantha's money is represented by $-4 \times 26 = -104$. The change to Samantha's money after 13 days of riding the train to and from work is $-\$104$.

4. Write a real-world problem that can be modeled by $4 \times (-7)$.

   Answers will vary. Every day, Alec loses 7 pounds of air pressure in a tire on his car. At that rate, what is the change in air pressure in his tire after 4 days?

Challenge:

5. **Use properties to explain why for each integer a, $-a = -1 \times a$. (Hint: What does $(1 + (-1)) \times a$ equal? What is the additive inverse of a?)**

   $\mathbf{0 \times a = 0}$ ***Zero product property***

   $\mathbf{(1 + (-1)) \times a = 0}$ ***Substitution***

   $\mathbf{a + (-1 \times a) = 0}$ ***Distributive property***

   Since a and $(-1 \times a)$ have a sum of zero, they must be additive inverses. By definition, the additive inverse of a is $-a$, so $(-1 \times a) = -a$.

Problem Set Sample Solutions

1. **Find the missing values in each column.**

Column A	Column B	Column C	Column D
$48 \div 4 = 12$	$24 \div 4 = 6$	$63 \div 7 = 9$	$21 \div 7 = 3$
$-48 \div (-4) = 12$	$-24 \div (-4) = 6$	$-63 \div (-7) = 9$	$-21 \div (-7) = 3$
$-48 \div 4 = -12$	$-24 \div 4 = -6$	$-63 \div 7 = -9$	$-21 \div 7 = -3$
$48 \div (-4) = -12$	$24 \div (-4) = -6$	$63 \div (-7) = -9$	$21 \div (-7) = -3$

2. **Describe the pattern you see in each column's answers in Problem 1, relating it to the problems' divisors and dividends. Why is this so?**

   The pattern in the columns' answers is the same two positive values followed by the same two negative values. This is so for the first two problems because the divisor and the dividend have the same signs and absolute values, which yields a positive quotient. This is so for the second two problems because the divisor and dividend have different signs but the same absolute values, which yields a negative quotient.

3. **Describe the pattern you see between the answers for Columns A and B in Problem 1 (e.g., compare the first answer in Column A to the first answer in Column B; compare the second answer in Column A to the second answer in Column B). Why is this so?**

   The answers in Column B are each one-half of the corresponding answers in Column A. That is because the dividend of 48 in Column A is divided by 4, and the dividend of 24 in Column B is divided by 4 (and so on with the same order and same absolute values but different signs). Since 24 is half of 48, the quotient (answer) in Column B will be one-half of the quotient in Column A.

4. **Describe the pattern you see between the answers for Columns C and D in Problem 1. Why is this so?**

   The answers in Column D are each one-third of the corresponding answers in Column C. That is because the dividend of 63 in Column C is divided by 7, and the dividend of 21 in Column D is divided by 7 (and so on with the same order and same absolute values but different signs). Since 21 is one-third of 63, the quotient (answer) in Column D will be one-third of the quotient in Column C.

Exit Ticket Sample Solutions

1. Write 3.0035 as a fraction. Explain your process.

   The right-most decimal place is the ten-thousandths place, so the number in fractional form would be $3\frac{35}{10{,}000}$*. There are common factors of* 5 *in the numerator and denominator, and dividing both by these results in the fraction* $3\frac{7}{2{,}000}$*.*

2. This week is just one of 40 weeks that you spend in the classroom this school year. Convert the fraction $\frac{1}{40}$ to decimal form.

   $$\frac{1}{40}=\frac{1}{2^3\times 5}\times\frac{5^2}{5^2}=\frac{5^2}{2^3\times 5^3}=\frac{25}{1,000}=0.025$$

Scaffolding:

Extend Exit Ticket Problem 2 by asking students to represent this week as a percentage of the school year.

Answer: 2.5%

Problem Set Sample Solutions

1. Convert each terminating decimal to a fraction in its simplest form.

   a. 0.4

   $0.4=\frac{2}{5}$

   b. 0.16

   $0.16=\frac{4}{25}$

   c. 0.625

   $0.625=\frac{5}{8}$

   d. 0.08

   $0.08=\frac{2}{25}$

   e. 0.012

   $0.012=\frac{3}{250}$

2. Convert each fraction or mixed number to a decimal using an equivalent fraction.

   a. $\frac{4}{5}$

   $\frac{4}{5}=0.8$

   b. $\frac{3}{40}$

   $\frac{3}{40}=0.075$

   c. $\frac{8}{200}$

   $\frac{8}{200}=0.04$

   d. $3\frac{5}{16}$

   $3\frac{5}{16}=3.3125$

3. Tanja is converting a fraction into a decimal by finding an equivalent fraction that has a power of 10 in the denominator. Sara looks at the last step in Tanja's work (shown below) and says that she cannot go any further. Is Sara correct? If she is, explain why. If Sara is incorrect, complete the remaining steps.

   $$\frac{72}{480}=\frac{2^3\cdot 3^2}{2^5\cdot 3\cdot 5}$$

   Tanja can finish the conversion since there is a factor pair of 3*'s in the numerator and denominator that can be divided out with a quotient of* 1*.*

   Remaining Steps:

   $$\frac{720}{480}=\frac{2^3\cdot 3^2}{2^5\cdot 3\cdot 5}=\frac{3}{2^2\cdot 5}\left(\frac{5}{5}\right)=\frac{3\cdot 5}{2^2\cdot 5^2}=\frac{15}{100}$$

   Answer: 0.15

Exit Ticket Sample Solutions

1. What is the decimal value of $\frac{4}{11}$?

   $\frac{4}{11} = 0.\overline{36}$

2. How do you know that $\frac{4}{11}$ is a repeating decimal?

   The prime factor in the denominator is 11. Fractions that correspond with terminating decimals have only factors 2 and 5 in the denominator in simplest form.

3. What causes a repeating decimal in the long division algorithm?

   When a remainder repeats, the division algorithm takes on a cyclic pattern causing a repeating decimal.

Problem Set Sample Solutions

1. Convert each rational number into its decimal form.

$\frac{1}{9} = 0.\overline{1}$

$\frac{1}{6} = 0.1\overline{6}$

$\frac{2}{9} = 0.\overline{2}$

$\frac{1}{3} = 0.\overline{3}$ $\quad$ $\frac{2}{6} = 0.\overline{3}$ $\quad$ $\frac{3}{9} = 0.\overline{3}$

$\frac{4}{9} = 0.\overline{4}$

$\frac{3}{6} = 0.5$

$\frac{5}{9} = 0.\overline{5}$

$\frac{2}{3} = 0.\overline{6}$ $\quad$ $\frac{4}{6} = 0.\overline{6}$ $\quad$ $\frac{6}{9} = 0.\overline{6}$

$\frac{7}{9} = 0.\overline{7}$

$\frac{5}{6} = 0.8\overline{3}$

$\frac{8}{9} = 0.\overline{8}$

One of these decimal representations is not like the others. Why?

$\frac{3}{6}$ in its simplest form is $\frac{1}{2}$ (the common factor of 3 divides out, leaving a denominator of 2, which in decimal form will terminate.

Enrichment:

2. **Chandler tells Aubrey that the decimal value of $-\frac{1}{17}$ is not a repeating decimal. Should Aubrey believe him? Explain.**

   No, Aubrey should not believe Chandler. The divisor 17 is a prime number containing no factors of 2 or 5, and therefore, cannot be written as a terminating decimal. By long division, $-\frac{1}{17} = -0.\overline{0588235294117647}$. The decimal appears as though it is not going to take on a repeating pattern because all 16 possible nonzero remainders appear before the remainder repeats. The seventeenth step produces a repeat remainder causing a cyclical decimal pattern.

3. **Complete the quotients below without using a calculator, and answer the questions that follow.**

   a. **Convert each rational number in the table to its decimal equivalent.**

$\frac{1}{11} = 0.\overline{09}$	$\frac{2}{11} = 0.\overline{18}$	$\frac{3}{11} = 0.\overline{27}$	$\frac{4}{11} = 0.\overline{36}$	$\frac{5}{11} = 0.\overline{45}$
$\frac{6}{11} = 0.\overline{54}$	$\frac{7}{11} = 0.\overline{63}$	$\frac{8}{11} = 0.\overline{72}$	$\frac{9}{11} = 0.\overline{81}$	$\frac{10}{11} = 0.\overline{90}$

   Do you see a pattern? Explain.

   The two digits that repeat in each case have a sum of nine. As the numerator increases by one, the first of the two digits increases by one as the second of the digits decreases by one.

   b. **Convert each rational number in the table to its decimal equivalent.**

$\frac{0}{99} = 0$	$\frac{10}{99} = 0.\overline{10}$	$\frac{20}{99} = 0.\overline{20}$	$\frac{30}{99} = 0.\overline{30}$	$\frac{45}{99} = 0.\overline{45}$
$\frac{58}{99} = 0.\overline{58}$	$\frac{62}{99} = 0.\overline{62}$	$\frac{77}{99} = 0.\overline{7}$	$\frac{81}{99} = 0.\overline{81}$	$\frac{98}{99} = 0.\overline{98}$

   Do you see a pattern? Explain.

   The 2-digit numerator in each fraction is the repeating pattern in the decimal form.

   c. **Can you find other rational numbers that follow similar patterns?**

   Answers will vary.

Exit Ticket Sample Solutions

Harrison made up a game for his math project. It is similar to the Integer Game; however, in addition to integers, there are cards that contain other rational numbers such as -0.5 and -0.25. Write a multiplication or division equation to represent each problem below. Show all related work.

1. **Harrison discards three -0.25 cards from his hand. How does this affect the overall point value of his hand? Write an equation to model this situation.**

   $-3\,(-0.25) = 0.75$

   The point value of Harrison's hand will increase by 0.75 points.

2. **Ezra and Benji are playing the game with Harrison. After Ezra doubles his hand's value, he has a total of -14.5 points. What was his hand's value before he doubled it?**

   $-14.5 \div 2 = -7.25$

   Before Ezra doubled his hand, his hand had a point value of -7.25.

3. **Benji has four -0.5 cards. What is his total score?**

   $4 \times (-0.5) = -2.0$

   Benji's total score is -2.0 points.

Problem Set Sample Solutions

1. **At lunch time, Benjamin often borrows money from his friends to buy snacks in the school cafeteria. Benjamin borrowed $\$0.75$ from his friend Clyde five days last week to buy ice cream bars. Represent the amount Benjamin borrowed as the product of two rational numbers; then, determine how much Benjamin owed his friend last week.**

   $5\,(-0.75) = -3.75$

   Benjamin owed Clyde $\$3.75$.

2. **Monica regularly records her favorite television show. Each episode of the show requires 3.5% of the total capacity of her video recorder. Her recorder currently has 62% of its total memory free. If Monica records all five episodes this week, how much space will be left on her video recorder?**

   $62 + 5(-3.5) = 62 + (-17.5) = 44.5$

   Monica's recorder will have 44.5% of its total memory free.

For Problems 3–5, find at least two possible sets of values that will work for each problem.

3. **Fill in the blanks with two rational numbers (other than 1 and -1).** $\_\_\_ \times \left(-\frac{1}{2}\right) \times \_\_\_ = -20$

   What must be true about the relationship between the two numbers you chose?

   Answers may vary. Two possible solutions are 10 and 4 or -10 and -4. The two numbers must be factors of 40, and they must both have the same sign.

4. Fill in the blanks with two rational numbers (other than 1 and -1). $-5.6 \times 100 \div 80 \times \_\_\_ \times \_\_\_ = 700$

What must be true about the relationship between the two numbers you chose?

Answers may vary. Two possible solutions are -50 and 2 or 25 and -4. The two numbers must be factors of -100, and they must have opposite signs.

5. Fill in the blanks with two rational numbers. $\_\_\_ \times \_\_\_ = -0.75$

What must be true about the relationship between the two numbers you chose?

Answers may vary. Two possible solutions are -3 and 0.25 or 0.5 and -1.5. The two numbers must be factors of -0.75, and they must have opposite signs.

For Problems 6–8, create word problems that can be represented by each expression, and then represent each product or quotient as a single rational number.

6. $8 \times (-0.25)$

Answers may vary.

Example: Stacey borrowed a quarter from her mother every time she went to the grocery store so that she could buy a gumball from the gumball machine. Over the summer, Stacey went to the grocery store with her mom eight times. What rational number represents the dollar amount change in her mother's money due to the purchase of gumballs?

Answer: -2

7. $-6 \div \left(1\frac{1}{3}\right)$

Answers may vary.

Example: There was a loss of $\$6$ on my investment over one-and-a-third months. Based on this, what was the investment's average dollar amount change per month?

Answer: -4.50

8. $-\frac{1}{2} \times 12$

Answers may vary.

Example: I discarded exactly half of my card-point total in the Integer Game. If my card-point total was 12 before I discarded, which rational number represents the change to my hand's total card-point total?

Answer: -6

Exit Ticket Sample Solutions

1. **Evaluate the expression below using the properties of operations.**

$$18 \div \left(-\frac{2}{3}\right) \times 4 \div (-7) \times (-3) \div \left(\frac{1}{4}\right)$$

$$18 \times \left(-\frac{3}{2}\right) \times 4 \times \left(-\frac{1}{7}\right) \times (-3) \times \left(\frac{4}{1}\right)$$

$$-27 \times 4 \times \left(-\frac{1}{7}\right) \times (-3) \times \left(\frac{4}{1}\right)$$

$$-\frac{1296}{7}$$

Answer: $-185\frac{1}{7}$ *or* $-185.\overline{142857}$

2.

a. **Given the expression below, what will the sign of the product be? Justify your answer.**

$$-4 \times \left(-\frac{8}{9}\right) \times 2.78 \times \left(1\frac{1}{3}\right) \times \left(-\frac{2}{5}\right) \times (-6.2) \times (-0.2873) \times \left(3\frac{1}{11}\right) \times A$$

There are five negative values in the expression. Because the product of two numbers with the same sign yield a positive product, pairs of negative factors have positive products. Given an odd number of negative factors, all but one can be paired into positive products. The remaining negative factor causes the product of the terms without A to be a negative value. If the value of A is negative, then the pair of negative factors forms a positive product. If the value of A is positive, the product of the two factors with opposite signs yields a negative product.

b. **Give a value for A that would result in a positive value for the expression.**

Answers will vary, but the answer must be negative. -2

c. **Give a value for A that would result in a negative value for the expression.**

Answers will vary, but the answer must be positive. 3.6

Problem Set Sample Solutions

1. **Evaluate the expression** $-2.2 \times (-2) \div \left(-\frac{1}{4}\right) \times 5$

a. **Using the order of operations only.**

$4.4 \div \left(-\frac{1}{4}\right) \times 5$

-17.6×5

-88

b. **Using the properties and methods used in Lesson 16.**

$-2.2 \times (-2) \times (-4) \times 5$

$-2.2 \times (-2) \times 5 \times (-4)$

$-2.2 \times (-10) \times (-4)$

$22 \times (-4)$

-88

c. **If you were asked to evaluate another expression, which method would you use, (a) or (b), and why?**

Answers will vary; however, most students should have found method (b) to be more efficient.

2. **Evaluate the expressions using the distributive property.**

a. $\left(2\frac{1}{4}\right) \times (-8)$

$2 \times (-8) + \frac{1}{4} \times (-8)$

$-16 + (-2)$

-18

b. $\frac{2}{3}(-7) + \frac{2}{3}(-5)$

$\frac{2}{3}\left(-7 + (-5)\right)$

$\frac{2}{3}(-12)$

-8

3. **Mia evaluated the expression below but got an incorrect answer. Find Mia's error(s), find the correct value of the expression, and explain how Mia could have avoided her error(s).**

$0.38 \times 3 \div \left(-\frac{1}{20}\right) \times 5 \div (-8)$

$0.38 \times 5 \times \left(\frac{1}{20}\right) \times 3 \times (-8)$

$0.38 \times \left(\frac{1}{4}\right) \times 3 \times (-8)$

$0.38 \times \left(\frac{1}{4}\right) \times (-24)$

$0.38 \times (-6)$

-2.28

Mia made two mistakes in the second line; first, she dropped the negative symbol from $-\frac{1}{20}$ *when she changed division to multiplication. The correct term should be* (-20) *because dividing a number is equivalent to multiplying its multiplicative inverse (or reciprocal). Mia's second error occurred when she changed division to multiplication at the end of the expression; she changed only the operation, not the number. The term should be* $\left(-\frac{1}{8}\right)$*. The correct value of the expressions is* $14\frac{1}{4}$*, or* 14.25*.*

Mia could have avoided part of her error if she had determined the sign of the product first. There are two negative values being multiplied, so her answer should have been a positive value.

Problem Set Sample Solutions

1. **A taxi cab in Myrtle Beach charges $\$2$ per mile and $\$1$ for every person. If a taxi cab ride for two people costs $\$12$, how far did the taxi cab travel?**

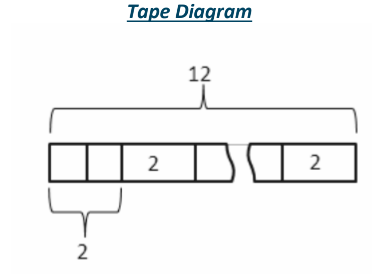
Algebraic Equation & Solution

Number of miles: m

People: 2

$$2m + 2 = 12$$
$$2m + 2 - 2 = 12 - 2$$
$$2m + 0 = 10$$
$$\left(\frac{1}{2}\right)2m = 10\left(\frac{1}{2}\right)$$
$$1m = 5$$
$$m = 5$$

Tape Diagram



$12 - 2 = 10$ $\qquad$ $10 \div 2 = 5$

The taxi cab traveled 5 miles.

2. **Heather works as a waitress at her family's restaurant. She works 2 hours every morning during the breakfast shift and returns to work each evening for the dinner shift. In the last four days, she worked 28 hours. If Heather works the same number of hours every evening, how many hours did she work during each dinner shift?**

Algebraic Equation & Solution

Number of morning hours: 2

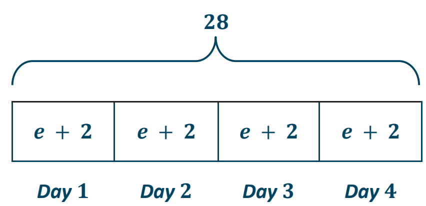
Number of evening hours: e

$$4(e + 2) = 28$$
$$4e + 8 - 8 = 28 - 8$$
$$4e + 0 = 20$$
$$\left(\frac{1}{4}\right)4e = 20\left(\frac{1}{4}\right)$$
$$1e = 5$$
$$e = 5$$

OR

$$\left(\frac{1}{4}\right)4(e + 2) = 28\left(\frac{1}{4}\right)$$
$$e + 2 = 7$$
$$e + 2 - 2 = 7 - 2$$
$$e = 5$$

Tape Diagram



$4e$ $\qquad$ $28 - 8 = 20$

$4(2) = 8$ $\qquad$ $20 \div 4 = 5$

Heather worked 5 hours in the evening.

3. Jillian exercises 5 times a week. She runs 3 miles each morning and bikes in the evening. If she exercises a total of 30 miles for the week, how many miles does she bike each evening?

Algebraic Equation & Solution

Run: 3 mi.

Bikes: b mi.

$$5(b+3)=30$$
$$5b+15-15=30-15$$
$$5b+0=15$$
$$\left(\frac{1}{5}\right)5b=15\left(\frac{1}{5}\right)$$
$$1b=3$$
$$b=3$$

OR

$$\left(\frac{1}{5}\right)(b+3)=30\left(\frac{1}{5}\right)$$
$$b+3=6$$
$$b+3-3=6-3$$
$$b=3$$

Tape Diagram

30

$b+3$	$b+3$	$b+3$	$b+3$	$b+3$
Day 1	*Day* 2	*Day* 3	*Day* 4	*Day* 5

$5(3)=15$ $\quad 30-15=15$

$15\div 5=3$

Jillian bikes 3 *miles every evening.*

4. Marc eats an egg sandwich for breakfast and a big burger for lunch every day. The egg sandwich has 250 calories. If Marc has 5,250 calories for breakfast and lunch for the week in total, how many calories are in one big burger?

Algebraic Equation & Solution

Egg Sandwich: 250 cal.

Hamburger: m cal

$$7(m+250)=5,250$$
$$7m+1,750-1750=5250-1750$$
$$7m+0=3,500$$
$$\left(\frac{1}{7}\right)7m=3,500\left(\frac{1}{7}\right)$$
$$1m=500$$
$$m=500$$

OR

$$\left(\frac{1}{7}\right)7(m+250)=\left(\frac{1}{7}\right)5,250$$
$$m+250=750$$
$$m+250-250=750-250$$
$$m=500$$

Tape Diagram

5,250

$m+250$	$m+250$	$m+250$	$m+250$	$m+250$	$m+250$	$m+250$
Day 1	*Day* 2	*Day* 3	*Day* 4	*Day* 5	*Day* 6	*Day* 7

$7(250)=1,750$

$5,250-1,750=3,500$

$3,500\div 7=500$

Each hamburger has 500 *calories.*

5. **Jackie won tickets playing the bowling game at the local arcade. The first time, she won 60 tickets. The second time, she won a bonus, which was 4 times the number of tickets of the original second prize. Altogether she won 200 tickets. How many tickets was the original second prize?**

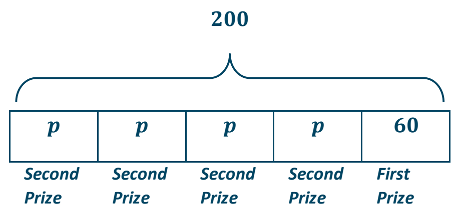
Algebraic Equation & Solution

First Prize: 60 tickets

Second Prize: p tickets

$$4p + 60 = 200$$
$$4p + 60 - 60 = 200 - 60$$
$$4p + 0 = 140$$
$$\left(\frac{1}{4}\right)4p = 140\left(\frac{1}{4}\right)$$
$$1p = 35$$
$$p = 35$$

Tape Diagram



$4p$ $\quad$ $200 - 60 = 140$

$140 \div 4 = 35$

The original second prize was 35 tickets.

Exit Ticket Sample Solutions

Bradley and Louie are roommates at college. At the beginning of the semester, they each paid a security deposit of A dollars. When they move out, their landlord will deduct from this deposit any expenses (B) for excessive wear and tear and refund the remaining amount. Bradley and Louie will share the expenses equally.

- **Write an expression that describes the amount each roommate will receive from the landlord when the lease expires.**
- **Evaluate the expression using the following information: Each roommate paid a $\$125$ deposit, and the landlord deducted $\$50$ total for damages.**

Deposit each person paid: A

Total damages: B

Each roommate receives: $A - \frac{B}{2}$

$A = 125$, $B = 50$

$A - \frac{B}{2}$

$125 - \frac{50}{2}$

$125 - 25$

100

Problem Set Sample Solutions

1. **Sally is paid a fixed amount of money to walk her neighbor's dog every day after school. When she is paid each month, she puts aside $\$20$ to spend and saves the remaining amount. Write an expression that represents the amount Sally will save in 6 months if she earns m dollars each month. If Sally is paid $\$65$ each month, how much will she save in 6 months?**

   m *= monthly pay*

   $6(m - 20)$

   $6m - 120$

   For $m = 65$

$6(m - 20)$	*or*	$6(m - 20)$
$6(65 - 20)$		$6(65 - 20)$
$6(45)$		$390 - 120$
270		270

   Sally will save $\$270$ *in* 6 *months.*

2. A football team scored 3 touchdowns, 3 extra points, and 4 field goals.

   a. Write an expression to represent the total points the football team scored.

   t = number of points for a touchdown.

   e = number of points for the extra point.

   f = number of points for a field goal.

   $3t + 3e + 4f$

   b. Write another expression that is equivalent to the one written above.

   Answers may vary. Sample response: $3t + 3e + 2f + 2f$

   c. If each touchdown is worth 6 points, each extra point is 1 point, and each field goal is 3 points, how many total points did the team score?

   $3t + 3e + 4f$

   $3(6) + 3(1) + 4(3)$

   $18 + 3 + 12$

   33

3. Write three other expressions that are equivalent to $8x - 12$.

   Answers may vary.

   $4(2x - 3)$

   $6x + 2x - 12$

   $8(x - 1) - 4$

   $-12 + 8x$

4. Profit is defined as earnings less expenses (earnings $-$ expenses). At the local hot-air balloon festival, the Ma & Pops Ice Cream Truck sells ice cream pops, which cost them $\$0.75$ each but are sold for $\$2$ each. They also paid $\$50$ to the festival's organizers for a vendor permit. The table below shows the earnings, expenses, and profit earned when 50, 75, and 100 ice cream pops were sold at the festival.

Number of Pops Sold	Earnings	Expenses	Profit
50	$50(2) = 100$	$50(0.75) + 50$ $37.5 + 50 = 87.5$	$100 - 87.5 = 12.50$
75	$75(2) = 150$	$75(0.75) + 50$ $56.25 + 50 = 106.25$	$150 - 106.25 = 43.75$
100	$100(2) = 200$	$100(0.75) + 50$ $75 + 50 = 125$	$200 - 125 = 75$

   a. Write an expression that represents the profit (in dollars) Ma & Pops earned by selling ice cream pops at the festival.

   p represents the number of pops sold.

   $2p - 0.75p - 50$

b. **Write an equivalent expression.**

$\mathbf{1.25p - 50}$

c. **How much of a profit did Ma & Pops Ice Cream Truck make if it sold 20 ice cream pops? What does this mean? Explain why this might be the case.**

$\mathbf{1.25p - 50}$

$\mathbf{1.25(20) - 50}$

$\mathbf{25 - 50}$

$\mathbf{-25}$

They did not make any money; they lost $\$25$. A possible reason is it could have been cold or rainy and people were not buying ice cream.

d. **How much of a profit did Ma & Pops Ice Cream Truck make if it sold 75 ice cream pops? What does this mean? Explain why this might be the case.**

$\mathbf{1.25p - 50}$

$\mathbf{1.25(75) - 50}$

$\mathbf{93.75 - 50}$

$\mathbf{43.75}$

They made a profit of $\$43.75$. Possible reasons are the weather could have been warmer and people bought the ice cream, or people just like to eat ice cream no matter what the weather is.

Exit Ticket Sample Solutions

1. Write three equivalent expressions that can be used to find the final price of an item costing g dollars that is on sale for 15% off and charged 7% sales tax.

   $(x - 0.15x) + 0.07(x - 0.15x)$ $1.07(x - 0.15x)$ $1.07(0.85x)$ *or* $0.85(1.07)x$

2. Using all of the expressions, determine the final price for an item that costs $\$75$. If necessary, round to the nearest penny.

   $x = 75$

$(x - 0.15x) + 0.07(x - 0.15x)$	$1.07(x - 0.15x)$	$1.07(0.85x)$ *or* $0.85(1.07)x$
$(75 - 0.15(75)) + 0.07(75 - 0.15(75))$	$1.07(75 - 0.15(75))$	$1.07(0.85(75))$
$63.75 + 0.07(63.75)$	$1.07(63.75)$	$1.07(63.75)$
$63.75 + 4.46$	68.21	68.21
68.21		

   The final price of an item that costs $\$75$ *is* $\$68.21$*.*

3. If each expression yields the same final sale price, is there anything to be gained by using one over the other?

   Using the final two expressions makes the problem shorter and offers fewer areas to make errors. However, all three expressions are correct.

4. Describe the benefits, special characteristics, and properties of each expression.

   The second and third expressions collect like terms. The third expression can be written either way using the commutative property of multiplication. The first and second expressions find the discount price first, whereas the third expression is written in terms of percent paid.

Problem Set Sample Solutions

Solve the following problems. If necessary, round to the nearest penny.

1. A family of 12 went to the local Italian restaurant for dinner. Every family member ordered a drink and meal, 3 ordered an appetizer, and 6 people ordered cake for dessert.

   a. Write an expression that can be used to figure out the cost of the bill. Include the definitions for the variables the server used.

      $d =$ *drink*

      $m =$ *meal*

      $a =$ *appetizer*

      $c =$ *cake*

      $12d + 12m + 3a + 6c$

   b. The waitress wrote on her ordering pad the following expression: $3(4d + 4m + a + 2c)$.

      Was she correct? Explain why or why not.

      Yes, she was correct because her expression is equivalent to the expression from part (a). If the distributive property is applied, the expressions would be exact.

c. What is the cost of the bill if a drink costs $\$3$, a meal costs $\$20$, an appetizer costs $\$5.50$, and a slice of cake costs $\$3.75$?

$12d + 12m + 3a + 6c$

$12(3) + 12(20) + 3(5.50) + 6(3.75)$

$36 + 240 + 16.50 + 22.50$

315

The cost of the bill is $\$315$.

d. Suppose the family had a 10% discount coupon for the entire check and then left an 18% tip. What is the total?

$\left(315 - 315(0.10)\right) + 0.18\left(315 - 315(0.10)\right)$

$1.18\left(315 - 315(0.10)\right)$

$1.18\left(315(0.90)\right)$

334.53

After the discount and tip, the new total is $\$334.53$.

2. Sally designs web pages for customers. She charges $\$135.50$ per web page; however, she must pay a monthly rental fee of $\$650$ for her office. Write an expression to determine her take-home pay after expenses. If Sally designed 5 web pages last month, what was her take-home pay after expenses?

w = *number of webpages Sally designs*

$135.50w - 650$

$135.50(5) - 650$

27.50

After expenses, Sally's take-home pay is $\$27.50$.

3. While shopping, Megan and her friend Rylie find a pair of boots on sale for 25% off the original price. Megan calculates the final cost of the boots by first deducting the 25% and then adding the 6% sales tax. Rylie thinks Megan will pay less if she pays the 6% sales tax first and then takes the 25% discount.

a. Write an expression to represent each girl's scenario if the original price of the boots was x dollars.

Megan	*Rylie*
$(x - 0.25x) + 0.06(x - 0.25x)$	$(x + 0.06x) - 0.25(x + 0.06x)$
$1.06(x - 0.25x)$	$0.75(x + 0.06x)$
$1.06(0.75x)$	$0.75(1.06x)$

b. Evaluate each expression if the boots originally cost $\$200$.

Megan	*Rylie*
$1.06(0.75x)$	$0.75(1.06x)$
$1.06\left(0.75(200)\right)$	$0.75\left(1.06(200)\right)$
159	159

Using both Megan's and Rylie's methods would show that the boots would cost $\$159$.

c. **Who was right? Explain how you know.**

Neither girl was right. They both pay the same amount.

d. **Explain how both girls' expressions are equivalent.**

Two expressions are equivalent if they yield the same number for every substitution of numbers for the variables in each expression. Since multiplication is commutative, the order of the multiplication can be reversed, and the result will remain the same.

Exit Ticket Sample Solutions

1. **Using the incomplete register below, work forward and backward to determine the beginning and ending balances after the series of transactions listed.**

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	367.11
1/31/12	Paycheck		350.55	717.66
2/1/12	Gillian's Chocolate Factory (Candy)	32.40		685.26
2/4/12	Main Street Jeweler's	425.30		259.96
2/14/12	Saratoga Steakhouse	125.31		134.65

2. **Write an expression to represent the balance after the paycheck was deposited on 1/31/12. Let x represent the beginning balance.**

   $x + 350.55$

3. **Write a numerical expression to represent the balance after the transaction for Main Street Jeweler's was made.**

   $685.26 - 425.30$

Problem Set Sample Solutions

1. **You are planning a fundraiser for your student council. The fundraiser is a Glow in the Dark Dance. Solve each entry below, and complete the transaction log to determine the ending balance in the student account.**

   a. **The cost of admission to the dance is $\$7$ per person, and all tickets were sold on November 1. Write an expression to represent the total amount of money collected for admission. Evaluate the expression if 250 people attended the dance.**

   p represents the number of people attending dance.

   $7p$

   $7(250) = 1750$

   If 250 people attended the dance, $\$1,750$ would be collected.

   b. **The following expenses were necessary for the dance, and checks were written to each company.**

   - **DJ for the dance—*Music Madness DJ* costs $\$200$ and paid for on November 3.**
   - **Glow sticks from *Glow World, Inc*. for the first 100 entrants. Cost of glow sticks were $\$0.75$ each plus 8% sales tax and bought on November 4.**

   Complete the transaction log below based on this information

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	$1,243.56$
November 1	*Dance Admission*		$1,750.00$	$2,993.56$
November 3	*Music Madness DJ*	200.00		$2,793.56$
November 4	*Glow Sticks from Glow World, Inc.*	81.00		$2,712.56$

c. **Write a numerical expression to determine the cost of the glow sticks.**

$100\left(\frac{8}{100}(0.75)+0.75\right)$

All 100 *glow sticks cost* $\$81$*.*

Analyze the results.

d. **Write an algebraic expression to represent the profit earned from the fundraiser. (Profit is the amount of money collected in admissions minus all expenses.)**

$7p-200-81$

$7p\ +\ (-200)\ +\ (-81)$

$7p\ +\ (-281)$ *or* $7p-281$

e. **Evaluate the expression to determine the profit if 250 people attended the dance. Use the variable p to represent the number of people attending the dance (from part (a)).**

$7p\ +\ (-281)$

$7(250)\ +\ (-281)$

$1,750\ +\ (-281)$

$1,469$

The profit is $\$1,469$*.*

f. **Using the transaction log above, what was the amount of the profit earned?**

$2,712.56-1,243.56=1469$ *The profit is* $\$1,469$*.*

2. **The register below shows a series of transactions made to an investment account. Vinnie and Anthony both completed the register in hopes of finding the beginning balance. As you can see, they do not get the same answer. Who was correct? What mistake did the other person make? What was the monthly gain or loss?**

Original Register

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	
3/1/11	Broker's Fee	250.00		
3/10/11	Loan Withdrawal	895.22		
3/15/11	Refund – Misc. Fee		50.00	
3/31/11	Investment Results		$2,012.22$	$18,917.00$

Vinnie's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	$18,000.00$
3/1/11	Broker's Fee	250.00		$17,750.00$
3/10/11	Loan Withdrawal	895.22		$16,854.78$
3/15/11	Refund – Misc. Fee		50.00	$16,904.78$
3/31/11	Investment Results		$2,012.22$	$18,917.00$

Anthony's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	$19,834.00$
3/1/11	Broker's Fee	250.00		$20,084.00$
3/10/11	Loan Withdrawal	895.22		$20,979.22$
3/15/11	Refund – Misc. Fee		50.00	$20,929.22$
3/31/11	Investment Results		$2,012.22$	$18,917.00$

The correct register is Vinnie's.

Anthony made the mistake of using the operations for moving forward. He added the deposits and subtracted the payments, but since he was working backward in the problem, he needed to do just the opposite.

The monthly gain was $\$917$. This was a gain because the ending balance was greater than the beginning balance, and the amount of the gain was calculated by $18,917 - 18,000 = 917$.

Exit Ticket Sample Solutions

Compare the two expressions. **Expression 1:** $6 + 7 + -5$

Expression 2: $-5 + 10 + 3$

1. **Are the two expressions equivalent? How do you know?**

   Yes, the expressions are equivalent because Expression 1 is equal to 8 and Expression 2 is equal to 8, as well. When two expressions evaluate to the same number, they are equivalent.

2. **Subtract -5 from each expression. Write the new numerical expression, and write a conclusion as an if–then statement.**

   Expression 1: $6 + 7 \pm 5 - (-5)$ *Expression 2:* $-5 + 10 + 3 - (-5)$

   13 13

   If $6 + 7 + -5 = -5 + 10 + 3$, *then* $6 + 7 + -5 - (-5) = -5 + 10 + 3 - (-5)$.

   If Expression 1= Expression 2, then (Expression 1$-(-5)$) = (Expression 2$-(-5)$).

3. **Add 4 to each expression. Write the new numerical expression, and write a conclusion as an if–then statement.**

   Expression 1: $6 + 7 \pm 5 + 4$ *Expression 2:* $-5 + 10 + 3 + 4$

   12 12

   If $6 + 7 + -5 = -5 + 10 + 3$, *then* $6 + 7 + -5 + 4 = -5 + 10 + 3 + 4$.

   If Expression 1 = Expression 2, then (Expression 1 $+ 4$) = (Expression 2 $+ 4$).

4. **Divide each expression by -2. Write the new numerical expression, and write a conclusion as an if–then statement.**

   Expression 1: $(6 + 7 + -5) \div -2$ *Expression 2:* $(-5 + 10 + 3) \div -2$

   $8 \div -2$ $8 \div -2$

   -4 -4

   If $6 + 7 + -5 = -5 + 10 + 3$, *then* $(6 + 7 + -5) \div -2 = (-5 + 10 + 3) \div -2$

   If Expression 1= Expression 2, then (Expression 1 $\div -2$) = (Expression 2 $\div -2$).

Problem Set Sample Solutions

This Problem Set provides students with additional practice evaluating numerical expressions and applying different moves while seeing the effect on number sentences.

1. **Evaluate the following numerical expressions.**

   a. $2 + (-3) + 7 = 6$

   b. $-4 - 1 = -5$

   c. $-\frac{5}{2} \times 2 = -5$

   d. $-10 \div 2 + 3 = -2$

   e. $\left(\frac{1}{2}\right)(8) + 2 = 6$

   f. $3 + (-4) - 1 = -2$

2. Which expressions from Exercise 1 are equal?

Expressions (a) and (e) are equivalent.

Expressions (b) and (c) are equivalent.

Expressions (d) and (f) are equivalent.

3. If two of the equivalent expressions from Exercise 1 are divided by 3, write an if–then statement using the properties of equality.

If $2+(-3)+7=\left(\frac{1}{2}\right)(8)+2$, *then* $(2+(-3)+7)\div 3=\left(\left(\frac{1}{2}\right)(8)+2\right)\div 3$.

4. Write an if–then statement if -3 is multiplied by the following equation: $-1-3=-4$.

If $-1-3=-4$, *then* $-3(-1-3)=-3(-4)$

5. Simplify the expression.

$5+6-5+4+7-3+6-3$

$=17$

Using the expression, write an equation.

$5+6-5+4+7-3+6-3=17$

Rewrite the equation if 5 is added to both expressions.

$5+6-5+4+7-3+6-3+5=17+5$

Write an if–then statement using the properties of equality.

If $5+6-5+4+7-3+6-3=17$, *then* $5+6-5+4+7-3+6-3+5=17+5$

Exit Ticket Sample Solutions

Andrew's math teacher entered the seventh-grade students in a math competition. There was an enrollment fee of $\$30$ and also an $\$11$ charge for each packet of 10 tests. The total cost was $\$151$. How many tests were purchased?

Set up an equation to model this situation, solve it using if-then statements, and justify the reasons for each step in your solution.

Let p represent the number of test packets.

Enrollment fee + cost of test $= 151$

If: $30 + 11p = 151$

Then: $30 - 30 + 11p = 151 - 30$ — *Subtraction property of equality for the additive inverse of* 30

If: $0 + 11p = 121$

Then: $11p = 121$ — *Additive identity*

If: $11p = 121$

Then: $\frac{1}{11}(11p) = \frac{1}{11}(121)$ — *Multiplication property of equality using the multiplicative inverse of* 11

If: $1p = 11$

Then: $p = 11$ — *Multiplicative identity*

Andrew's math teacher bought 11 *packets of tests. There were* 10 *tests in each packet, and* $10 \times 11 = 110$.

So, there were 110 *tests purchased.*

Problem Set Sample Solutions

For Exercises 1–4, solve each equation algebraically using if-then statements to justify your steps.

1. $\frac{2}{3}x - 4 = 20$

   If: $\frac{2}{3}x - 4 = 20$

   Then: $\frac{2}{3}x - 4 + 4 = 20 + 4$ — *Addition property of equality using the additive inverse of* -4

   If: $\frac{2}{3}x + 0 = 24$

   Then: $\frac{2}{3}x = 24$ — *Additive identity*

   If: $\frac{2}{3}x = 24$

   Then: $\left(\frac{3}{2}\right)\frac{2}{3}x = \left(\frac{3}{2}\right)24$ — *Multiplication property of equality using the multiplicative inverse of* $\frac{2}{3}$

   If: $1x = 36$

   Then: $x = 36$ — *Multiplicative identity*

2. $4 = \frac{-1+x}{2}$

If: $4 = \frac{-1+x}{2}$

Then: $2\,(4) = 2\left(\frac{-1+x}{2}\right)$ — *Multiplication property of equality using the multiplicative inverse of* $\frac{1}{2}$

If: $8 = 1\,(-1+x)$

Then: $8 = -1+x$ — *Multiplicative identity*

If: $8 = -1+x$

Then: $8-(-1) = -1-(-1)+x$ — *Subtraction property of equality for the additive inverse of* -1

If: $9 = 0+x$

Then: $9 = x$ — *Additive identity*

3. $12(x+9) = -108$

If: $12(x+9) = -108$

Then: $\left(\frac{1}{12}\right)12(x+9) = \left(\frac{1}{12}\right)(-108)$ — *Multiplication property of equality using the multiplicative inverse of* 12

If: $1\,(x+9) = -9$

Then: $x+9 = -9$ — *Multiplicative identity*

If: $x+9 = -9$

Then: $x+9-9 = -9-9$ — *Subtraction property of equality for the additive inverse of* 9

If: $x+0 = -18$

Then: $x = -18$ — *Additive identity*

4. $5x + 14 = -7$

If: $5x + 14 = -7$

Then: $5x + 14 - 14 = -7-14$ — *Subtraction property of equality for the additive inverse of* 14

If: $5x + 0 = -21$

Then: $5x = -21$ — *Additive identity*

If: $5x = -21$

Then: $\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)(-21)$ — *Multiplication property of equality using the multiplicative inverse of* 5

If: $1x = -4.2$

Then: $x = -4.2$ — *Multiplicative identity*

For Exercises 5–7, write an equation to represent each word problem. Solve the equation showing the steps, and then state the value of the variable in the context of the situation.

5. **A plumber has a very long piece of pipe that is used to run city water parallel to a major roadway. The pipe is cut into two sections. One section of pipe is 12 ft. shorter than the other. If $\frac{3}{4}$ of the length of the shorter pipe is 120 ft., how long is the longer piece of the pipe?**

   Let x represent the longer piece of pipe.

   If: $\frac{3}{4}(x-12)=120$

   Then: $\frac{4}{3}\left(\frac{3}{4}\right)(x-12)=\left(\frac{4}{3}\right)120$ — *Multiplication property of equality using the multiplicative inverse of* $\frac{3}{4}$

   If: $1(x-12)=160$

   Then: $x-12=160$ — *Multiplicative identity*

   If: $x-12=160$

   Then: $x-12+12=160+12$ — *Addition property of equality for the additive inverse of* -12

   If: $x+0=172$

   Then: $x=172$ — *Additive identity*

   The longer piece of pipe is 172 ft.

6. **Bob's monthly phone bill is made up of a $\$10$ fee plus $\$0.05$ per minute. Bob's phone bill for July was $\$22$. Write an equation to model the situation using m to represent the number of minutes. Solve the equation to determine the number of phone minutes Bob used in July.**

   Let m represent the number of phone minutes Bob used.

   If: $10+0.05m=22$

   Then: $10-10+0.05m=22-10$ — *Subtraction property of equality for the additive inverse of* 10

   If: $0+0.05m=12$

   Then: $0.05m=12$ — *Additive identity*

   If: $0.05m=12$

   Then: $\left(\frac{1}{0.05}\right)0.05m=\left(\frac{1}{0.05}\right)12$ — *Multiplication property of equality using the multiplicative inverse of* 0.05

   If: $1m=240$

   Then: $m=240$ — *Multiplicative identity*

   Bob used 240 phone minutes in July.

7. **Kym switched cell phone plans. She signed up for a new plan that will save her $\$3.50$ per month compared to her old cell phone plan. The cost of the new phone plan for an entire year is $\$294$. How much did Kym pay per month under her old phone plan?**

Let n represent the amount Kym paid per month for her old cell phone plan.

If: $294 = 12(n - 3.50)$

Then: $\left(\frac{1}{12}\right)(294) = \left(\frac{1}{12}\right)12(n - 3.50)$ — *Multiplication property of equality using the multiplicative inverse of 12*

If: $24.5 = 1\,(n - 3.50)$

Then: $24.5 = n - 3.50$ — *Multiplicative identity*

If: $24.5 = n - 3.50$

Then: $24.5 + 3.50 = n - 3.50 + 3.50$ — *Addition property of equality for the additive inverse of -3.50*

If: $28 = n + 0$

Then: $28 = n$ — *Additive identity*

Kym paid $\$28$ per month for her old cell phone plan.