Exit Ticket Sample Solutions

1. Write an equivalent expression to $2x + 3 + 5x + 6$ by combining like terms.

   $2x + 3 + 5x + 6$

   $2x + 5x + 3 + 6$

   $7x + 9$

2. Find the sum of $(8a + 2b - 4)$ and $(3b - 5)$.

   $(8a + 2b - 4) + (3b - 5)$

   $8a + 2b + (-4) + 3b + (-5)$

   $8a + 2b + 3b + (-4) + (-5)$

   $8a + (5b) + (-9)$

   $8a + 5b - 9$

3. Write the expression in standard form: $4(2a) + 7(-4b) + (3 \cdot c \cdot 5)$.

   $(4 \cdot 2)a + \big(7 \cdot (-4)\big)b + (3 \cdot 5)c$

   $8a + (-28)b + 15c$

   $8a - 28b + 15c$

Problem Set Sample Solutions

For Problems 1–9, write equivalent expressions by combining like terms. Verify the equivalence of your expression and the given expression by evaluating each for the given values: $a = 2$, $b = 5$, and $c = -3$.

1. $3a + 5a$

   $8a$
   $8(2)$
   16

   $3(2) + 5(2)$
   $6 + 10$
   16

2. $8b - 4b$

   $4b$
   $4(5)$
   20

   $8(5) - 4(5)$
   $40 - 20$
   20

3. $5c + 4c + c$

   $10c$
   $10(-3)$
   -30

   $5(-3) + 4(-3) + (-3)$
   $-15 + (-12) + (-3)$
   $-27 + (-3)$
   -30

4. $3a + 6 + 5a$

   $8a + 6$
   $8(2) + 6$
   $16 + 6$
   22

   $3(2) + 6 + 5(2)$
   $6 + 6 + 10$
   $12 + 10$
   22

5. $8b + 8 - 4b$

   $4b + 8$
   $4(5) + 8$
   $20 + 8$
   28

   $8(5) + 8 - 4(5)$
   $40 + 8 - 20$
   $48 - 20$
   28

6. $5c - 4c + c$

   $2c$
   $2(-3)$
   -6

   $5(-3) - 4(-3) + (-3)$
   $-15 + \big(-4(-3)\big) + (-3)$
   $-15 + (12) + (-3)$
   $-3 + (-3)$
   -6

7. $3a + 6 + 5a - 2$

$8a + 4$
$8(2) + 4$
$16 + 4$
20

$3(2) + 6 + 5(2) - 2$
$6 + 6 + 10 + (-2)$
$12 + 10 + (-2)$
$22 + (-2)$
20

8. $8b + 8 - 4b - 3$

$4b + 5$
$4(5) + 5$
$20 + 5$
25

$8(5) + 8 - 4(5) - 3$
$40 + 8 + \left(-4(5)\right) + (-3)$
$40 + 8 + (-20) + (-3)$
$48 + (-20) + (-3)$
$28 + (-3)$
25

9. $5c - 4c + c - 3c$

$-1c$
$-1(-3)$
3

$5(-3) - 4(-3) + (-3) - 3(-3)$
$-15 + \left(-4(-3)\right) + (-3) + \left(-3(-3)\right)$
$-15 + (12) + (-3) + (9)$
$-3 + (-3) + 9$
$-6 + 9$
3

Use any order, any grouping to write equivalent expressions by combining like terms. Then, verify the equivalence of your expression to the given expression by evaluating for the value(s) given in each problem.

Problem	*Your Expression*	*Given Expression*
10. $3(6a)$; for $a = 3$ $18a$	$18a$ $18(3)$ 54	$3\left(6(3)\right)$ $3(18)$ 54
11. $5d(4)$; for $d = -2$ $20d$	$20d$ $20(-2)$ -40	$5(-2)(4)$ $-10(4)$ -40
12. $(5r)(-2)$; for $r = -3$ $-10r$	$-10r$ $-10(-3)$ 30	$\left(5(-3)\right)(-2)$ $(-15)(-2)$ 30
13. $3b(8) + (-2)(7c)$; for $b = 2$, $c = 3$ $24b - 14c$	$24b - 14c$ $24(2) - 14(3)$ $48 - 42$ 6	$3(2)(8) + (-2)(7(3))$ $6(8) + (-2)(21)$ $48 + (-42)$ 6
14. $-4(3s) + 2(-t)$; for $s = \frac{1}{2}$, $t = -3$ $-12s - 2t$	$-12s - 2t$ $-12\left(\frac{1}{2}\right) - 2(-3)$ $-6 + \left(-2(-3)\right)$ $-6 + (6)$ 0	$-4\left(3\left(\frac{1}{2}\right)\right) + 2\left(-(-3)\right)$ $-4\left(\frac{3}{2}\right) + 2(3)$ $-2(3) + 2(3)$ $-6 + 6$ 0
15. $9(4p) - 2(3q) + p$; for $p = -1$, $q = 4$ $37p - 6q$	$37p - 6q$ $37(-1) - 6(4)$ $-37 + \left(-6(4)\right)$ $-37 + (-24)$ -61	$9\left(4(-1)\right) - 2\left(3(4)\right) + (-1)$ $9(-4) + \left(-2(12)\right) + (-1)$ $-36 + (-24) + (-1)$ $-60 + (-1)$ -61
16. $7(4g) + 3(5h) + 2(-3g)$; for $g = \frac{1}{2}$, $h = \frac{1}{3}$ $28g + 15h + (-6g)$ $22g + 15h$	$22g + 15h$ $22\left(\frac{1}{2}\right) + 15\left(\frac{1}{3}\right)$ $11 + 5$ 16	$7\left(4\left(\frac{1}{2}\right)\right) + 3\left(5\left(\frac{1}{3}\right)\right) + 2\left(-3\left(\frac{1}{2}\right)\right)$ $7(2) + 3\left(\frac{5}{3}\right) + 2\left(-\frac{3}{2}\right)$ $14 + 5 + (-3)$ $19 + (-3)$ 16

The problems below are follow-up questions to Example 1, part (b) from Classwork: Find the sum of $2x + 1$ and $5x$.

17. **Jack got the expression $7x + 1$ and then wrote his answer as $1 + 7x$. Is his answer an equivalent expression? How do you know?**

    Yes; Jack correctly applied any order (the commutative property), changing the order of addition.

18. **Jill also got the expression $7x + 1$ and then wrote her answer as $1x + 7$. Is her expression an equivalent expression? How do you know?**

    No, any order (the commutative property) does not apply to mixing addition and multiplication; therefore, the $7x$ must remain intact as a term.

    $1(4) + 7 = 11$ *and* $7(4) + 1 = 29$*; the expressions do not evaluate to the same value for* $x = 4$.

Problem Set Sample Solutions

1. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating each expression using $x = 5$.

a. $3x + (2 - 4x)$ $-x + 2$ $-5 + 2$ -3 $3(5) + (2 - 4(5))$ $15 + (2 + (-20))$ $15 + (-18)$ -3	b. $3x + (-2 + 4x)$ $7x - 2$ $7(5) - 2$ $35 - 2$ 33 $3(5) + (-2 + 4(5))$ $15 + (-2 + 20)$ $15 + 18$ 33	c. $-3x + (2 + 4x)$ $x + 2$ $5 + 2$ 7 $-3(5) + (2 + 4(5))$ $-15 + (2 + 20)$ $-15 + 22$ 7
d. $3x + (-2 - 4x)$ $-x - 2$ $-5 - 2$ -7 $3(5) + (-2 - 4(5))$ $15 + \left(-2 + (-4(5))\right)$ $15 + (-2 + (-20))$ $15 + (-22)$ -7	e. $3x - (2 + 4x)$ $-x - 2$ $-5 - 2$ -7 $3(5) - (2 + 4(5))$ $15 - (2 + 20)$ $15 - 22$ $15 + (-22)$ -7	f. $3x - (-2 + 4x)$ $-x + 2$ $-5 + 2$ -3 $3(5) - (-2 + 4(5))$ $15 - (-2 + 20)$ $15 - (18)$ $15 + (-18)$ -3
g. $3x - (-2 - 4x)$ $7x + 2$ $7(5) + 2$ $35 + 2$ 37 $3(5) - (-2 - 4(5))$ $15 - \left(-2 + (-4(5))\right)$ $15 - (-2 + (-20))$ $15 - (-22)$ $15 + 22$ 37	h. $3x - (2 - 4x)$ $7x - 2$ $7(5) - 2$ $35 - 2$ 33 $3(5) - (2 - 4(5))$ $15 - \left(2 + (-4(5))\right)$ $15 - (2 + (-20))$ $15 - (-18)$ $15 + 18$ 33	i. $-3x - (-2 - 4x)$ $x + 2$ $5 + 2$ 7 $-3(5) - (-2 - 4(5))$ $-15 - \left(-2 + (-4(5))\right)$ $-15 - (-2 + (-20))$ $-15 - (-22)$ $-15 + 22$ 7

j. In problems (a)–(d) above, what effect does addition have on the terms in parentheses when you removed the parentheses?

By the any grouping property, the terms remained the same with or without the parentheses.

k. In problems (e)–(i), what effect does subtraction have on the terms in parentheses when you removed the parentheses?

The opposite of a sum is the sum of the opposites; each term within the parentheses is changed to its opposite.

2. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating each expression for the given value of the variable.

a. $4y-(3+y);\ y=2$ $3y-3$ $3(2)-3$ $6-3$ 3 $4(2)-(3+2)$ $8-5$ $8+(-5)$ 3	b. $(2b+1)-b;\ b=-4$ $b+1$ $-4+1$ -3 $(2(-4)+1)-(-4)$ $(-8+1)+4$ $(-7)+4$ -3	c. $(6c-4)-(c-3);\ c=-7$ $5c-1$ $5(-7)-1$ $-35-1$ -36 $(6(-7)-4)-(-7-3)$ $(-42-4)-(-10)$ $-42+(-4)+(10)$ $-46+10$ -36
d. $(d+3d)-(-d+2);$ $d=3$ $5d-2$ $5(3)-2$ $15-2$ 13 $\left(3+3(3)\right)-(-3+2)$ $(3+9)-(-1)$ $12+1$ 13	e. $(-5x-4)-(-2-5x);$ $x=3$ -2 $(-5(3)-4)-\left(-2-5(3)\right)$ $(-15-4)-(-2-15)$ $(-19)-(-17)$ $(-19)+17$ -2	f. $11f-(-2f+2);\ f=\frac{1}{2}$ $13f-2$ $13\left(\frac{1}{2}\right)-2$ $\frac{13}{2}-2$ $6\frac{1}{2}-2$ $4\frac{1}{2}$ $11\left(\frac{1}{2}\right)-\left(-2\left(\frac{1}{2}\right)+2\right)$ $\frac{11}{2}-(-1+2)$ $\frac{11}{2}-1$ $\frac{11}{2}+\left(-\frac{2}{2}\right)$ $\frac{9}{2}$ $4\frac{1}{2}$

g. $-5g+(6g-4);\ g=-2$ $g-4$ $-2-4$ -6 $-5(-2)+(6(-2)-4)$ $10+(-12-4)$ $10+\left(-12+(-4)\right)$ $10+(-16)$ -6	h. $(8h-1)-(h+3);$ $h=-3$ $7h-4$ $7(-3)-4$ $-21-4$ -25 $(8(-3)-1)-(-3+3)$ $(-24-1)-(0)$ $(-25)-0$ -25	i. $(7+w)-(w+7);w=-4$ 0 $\left(7+(-4)\right)-(-4+7)$ $3-3$ $3+(-3)$ 0

j. $(2g+9h-5)-(6g-4h+2);\ g=-2$ and $h=5$

$-4g+13h-7$	$(2(-2)+9(5)-5)-(6(-2)-4(5)+2)$
$-4(-2)+13(5)-7$	$(-4+45-5)-\left(-12+\left(-4(5)\right)+2\right)$
$8+65+(-7)$	$(41-5)-(-12+(-20)+2)$
$73+(-7)$	$\left(41+(-5)\right)-(-32+2)$
66	$36-(-30)$
	$36+30$
	66

3. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating both expressions for the given value of the variable.

a. $-3(8x);\ x=\frac{1}{4}$ $-24x$ $-24\left(\frac{1}{4}\right)$ $-\frac{24}{4}$ -6 $-3\left(8\left(\frac{1}{4}\right)\right)$ $-3(2)$ -6	b. $5\cdot k\cdot(-7);\ k=\frac{3}{5}$ $-35k$ $-35\left(\frac{3}{5}\right)$ $-\frac{105}{5}$ -21 $5\left(\frac{3}{5}\right)(-7)$ $3(-7)$ -21	c. $2(-6x)\cdot 2;\ x=\frac{3}{4}$ $-24x$ $-24\left(\frac{3}{4}\right)$ $-\frac{72}{4}$ -18 $2\left(-6\left(\frac{3}{4}\right)\right)\cdot 2$ $2\left(-3\left(\frac{3}{2}\right)\right)\cdot 2$ $2(-3)\left(\frac{3}{2}\right)(2)$ $-6(3)$ -18

d. $-3(8x)+6(4x);\ x=2$ 0 $-3(8(2))+6(4(2))$ $-3(16)+6(8)$ $-48+48$ 0	e. $8(5m)+2(3m);\ m=-2$ $46m$ $46(-2)$ -92 $8(5(-2))+2(3(-2))$ $8(-10)+2(-6)$ $-80+(-12)$ -92	f. $-6(2v)+3a(3);\ v=\frac{1}{3};\ a=\frac{2}{3}$ $-12v+9a$ $-12\left(\frac{1}{3}\right)+9\left(\frac{2}{3}\right)$ $-\frac{12}{3}+\frac{18}{3}$ $-4+6$ 2 $-6\left(2\left(\frac{1}{3}\right)\right)+3\left(\frac{2}{3}\right)(3)$ $-6\left(\frac{2}{3}\right)+2(3)$ $-4+6$ 2

4. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating both expressions for the given value of the variable.

a. $8x\div 2;\ x=-\frac{1}{4}$ $4x$ $4\left(-\frac{1}{4}\right)$ -1 $8\left(-\frac{1}{4}\right)\div 2$ $-2\div 2$ -1	b. $18w\div 6;\ w=6$ $3w$ $3(6)$ 18 $18(6)\div 6$ $108\div 6$ 18	c. $25r\div 5r;\ r=-2$ 5 $25(-2)\div(5(-2))$ $-50\div(-10)$ 5
d. $33y\div 11y;\ y=-2$ 3 $33(-2)\div(11(-2))$ $(-66)\div(-22)$ 3	e. $56k\div 2k;\ k=3$ 28 $56(3)\div(2(3))$ $168\div 6$ 28	f. $24xy\div 6y;\ x=-2; y=3$ $4x$ $4(-2)$ -8 $24(-2)(3)\div(6(3))$ $-48(3)\div 18$ $-144\div 18$ -8

5. For each problem (a)–(g), write an expression in standard form.

a. Find the sum of $-3x$ and $8x$.

$-3x+8x$

$5x$

b. Find the sum of $-7g$ and $4g + 2$.

$-7g + (4g + 2)$

$-3g + 2$

c. Find the difference when $6h$ is subtracted from $2h - 4$.

$(2h - 4) - 6h$

$-4h - 4$

d. Find the difference when $-3n - 7$ is subtracted from $n + 4$.

$(n + 4) - (-3n - 7)$

$4n + 11$

e. Find the result when $13v + 2$ is subtracted from $11 + 5v$.

$(11 + 5v) - (13v + 2)$

$-8v + 9$

f. Find the result when $-18m - 4$ is added to $4m - 14$.

$(4m - 14) + (-18m - 4)$

$-14m - 18$

g. What is the result when $-2x + 9$ is taken away from $-7x + 2$?

$(-7x + 2) - (-2x + 9)$

$-5x - 7$

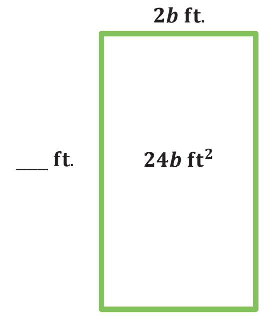
6. Marty and Stewart are stuffing envelopes with index cards. They are putting x index cards in each envelope. When they are finished, Marty has 15 stuffed envelopes and 4 extra index cards, and Stewart has 12 stuffed envelopes and 6 extra index cards. Write an expression in standard form that represents the number of index cards the boys started with. Explain what your expression means.

They inserted the same number of index cards in each envelope, but that number is unknown, x. An expression that represents Marty's index cards is $15x + 4$ because he had 15 envelopes and 4 cards left over. An expression that represents Stewart's index cards is $12x + 6$ because he had 12 envelopes and 6 left over cards. Their total number of cards together would be:

$$15x + 4 + 12x + 6$$
$$15x + 12x + 4 + 6$$
$$27x + 10$$

This means that altogether, they have 27 envelopes with x index cards in each, plus another 10 leftover index cards.

7. **The area of the pictured rectangle below is $24b$ ft^2. Its width is $2b$ ft. Find the height of the rectangle and name any properties used with the appropriate step.**



$24b \div 2b$

$24b \cdot \frac{1}{2b}$ *Multiplying the reciprocal*

$\frac{24b}{2b}$ *Multiplication*

$\frac{24}{2} \cdot \frac{b}{b}$ *Any order, any grouping in multiplication*

$12 \cdot 1$

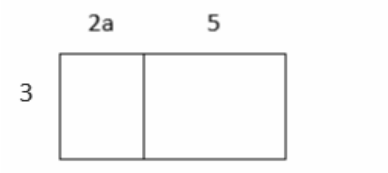
12

The height of the rectangle is 12 ft*.*

Problem Set Sample Solutions

1.

a. **Write two equivalent expressions that represent the rectangular array below.**



$3(2a+5)$ or $6a+15$

b. **Verify informally that the two expressions are equivalent using substitution.**

Let $a=4$.

$3(2a+5)$	$6a+15$
$3(2(4)+5)$	$6(4)+15$
$3(8+5)$	$24+15$
$3(13)$	39
39	

2. **You and your friend made up a basketball shooting game. Every shot made from the free throw line is worth 3 points, and every shot made from the half-court mark is worth 6 points. Write an equation that represents the total number of points, P, if f represents the number of shots made from the free throw line, and h represents the number of shots made from half-court. Explain the equation in words.**

$P=3f+6h$ or $P=3(f+2h)$

The total number of points can be determined by multiplying each free throw shot by 3 and then adding that to the product of each half-court shot multiplied by 6.

The total number of points can also be determined by adding the number of free throw shots to twice the number of half-court shots and then multiplying the sum by three.

3. **Use a rectangular array to write the products in standard form.**

a. $2(x+10)$

$2x+20$

b. $3(4b+12c+11)$

$12b+36c+33$

4. **Use the distributive property to write the products in standard form.**

a. $3(2x - 1)$

$6x - 3$

b. $10(b + 4c)$

$10b + 40c$

c. $9(g - 5h)$

$9g - 45h$

d. $7(4n - 5m - 2)$

$28n - 35m - 14$

e. $a(b + c + 1)$

$ab + ac + a$

f. $(8j - 3l + 9)6$

$48j - 18l + 54$

g. $(40s + 100t) \div 10$

$4s + 10t$

h. $(48p + 24) \div 6$

$8p + 4$

i. $(2b + 12) \div 2$

$b + 6$

j. $(20r - 8) \div 4$

$5r - 2$

k. $(49g - 7) \div 7$

$7g - 1$

l. $(14g + 22h) \div \frac{1}{2}$

$28g + 44h$

5. **Write the expression in standard form by expanding and collecting like terms.**

a. $4(8m - 7n) + 6(3n - 4m)$

$8m - 10n$

b. $9(r - s) + 5(2r - 2s)$

$19r - 19s$

c. $12(1 - 3g) + 8(g + f)$

$-28g + 8f + 12$

Exit Ticket Sample Solutions

1. Write the expression below in standard form.

$3h - 2(1 + 4h)$

$3h + (-2(1 + 4h))$	*Subtraction as adding the inverse*
$3h + (-2 \cdot 1) + (-2h \cdot 4)$	*Distributive property*
$3h + (-2) + (-8h)$	*Apply integer rules*
$-5h - 2$	*Collect like terms*

2. Write the expression below as a product of two factors.

$6m + 8n + 4$

The GCF for the terms is 2. *Therefore, the factors are* $2(3m + 4n + 2)$.

Problem Set Sample Solutions

1. Write each expression as the product of two factors.

a. $1 \cdot 3 + 7 \cdot 3$
$3(1 + 7)$

b. $(1 + 7) + (1 + 7) + (1 + 7)$
$3(1 + 7)$

c. $2 \cdot 1 + (1 + 7) + (7 \cdot 2)$
$3(1 + 7)$

d. $h \cdot 3 + 6 \cdot 3$
$3(h + 6)$

e. $(h + 6) + (h + 6) + (h + 6)$
$3(h + 6)$

f. $2h + (6 + h) + 6 \cdot 2$
$3(h + 6)$

g. $j \cdot 3 + k \cdot 3$
$3(j + k)$

h. $(j + k) + (j + k) + (\mathrm{j} + \mathrm{k})$
$3(j + k)$

i. $2j + (k + j) + 2k$
$3(j + k)$

2. Write each sum as a product of two factors.

a. $6 \cdot 7 + 3 \cdot 7$
$7(6 + 3)$

b. $(8 + 9) + (8 + 9) + (8 + 9)$
$3(8 + 9)$

c. $4 + (12 + 4) + (5 \cdot 4)$
$4(1 + 4 + 5)$

d. $2y \cdot 3 + 4 \cdot 3$
$3(2y + 4)$

e. $(x + 5) + (x + 5)$
$2(x + 5)$

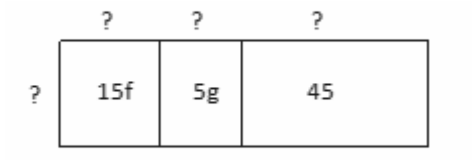
f. $3x + (2 + x) + 5 \cdot 2$
$4(x + 3)$

g. $f \cdot 6 + g \cdot 6$
$6(f + g)$

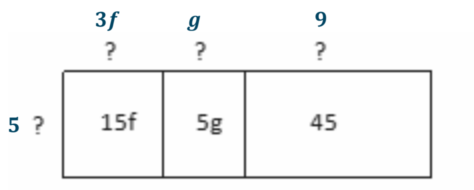
h. $(c + d) + (c + d) + (c + d) + (c + d)$
$4(c + d)$

i. $2r + r + s + 2s$
$3(r + s)$

3. Use the following rectangular array to answer the questions below.



a. Fill in the missing information.



b. Write the sum represented in the rectangular array.

$15f + 5g + 45$

c. Use the missing information from part (a) to write the sum from part (b) as a product of two factors.

$5(3f + g + 9)$

4. Write the sum as a product of two factors.

a. $81w + 48$

$3(27w + 16)$

b. $10 - 25t$

$5(2 - 5t)$

c. $12a + 16b + 8$

$4(3a + 4b + 2)$

5. Xander goes to the movies with his family. Each family member buys a ticket and two boxes of popcorn. If there are five members of his family, let t represent the cost of a ticket and p represent the cost of a box of popcorn. Write two different expressions that represent the total amount his family spent. Explain how each expression describes the situation in a different way.

$5(t + 2p)$

Five people each buy a ticket and two boxes of popcorn, so the cost is five times the quantity of a ticket and two boxes of popcorn.

$5t + 10p$

There are five tickets and 10 boxes of popcorn total. The total cost will be five times the cost of the tickets, plus 10 times the cost of the popcorn.

6. **Write each expression in standard form.**

a. $-3(1 - 8m - 2n)$

$-3\big(1 + (-8m) + (-2n)\big)$

$-3 + 24m + 6n$

b. $5 - 7(-4q + 5)$

$5 + -7(-4q + 5)$

$5 + 28q + (-35)$

$28q - 35 + 5$

$28q - 30$

c. $-(2h - 9) - 4h$

$-\big(2h + (-9)\big) + (-4h)$

$-2h + 9 + (-4h)$

$-6h + 9$

d. $6(-5r - 4) - 2(r - 7s - 3)$

$6(-5r + -4) + -2(r - 7s + -3)$

$-30r + -24 + -2r + 14s + 6$

$-30r + -2r + 14s + -24 + 6$

$-32r + 14s - 18$

7. **Combine like terms to write each expression in standard form.**

a. $(r - s) + (s - r)$

0

b. $(-r + s) + (s - r)$

$-2r + 2s$

c. $(-r - s) - (-s - r)$

0

d. $(r - s) + (s - t) + (t - r)$

0

e. $(r - s) - (s - t) - (t - r)$

$2r - 2s$

Exit Ticket Sample Solutions

1. Find the sum of $5x + 20$ and the opposite of 20. Write an equivalent expression in standard form. Justify each step.

$(5x + 20) + (-20)$	
$5x + (20 + (-20))$	*Associative property of addition*
$5x + 0$	*Additive inverse*
$5x$	*Additive identity property of zero*

2. For $5x + 20$ and the multiplicative inverse of 5, write the product and then write the expression in standard form, if possible. Justify each step.

$(5x + 20)\left(\frac{1}{5}\right)$	
$(5x)\left(\frac{1}{5}\right) + 20\left(\frac{1}{5}\right)$	*Distributive property*
$1x + 4$	*Multiplicative inverses, multiplication*
$x + 4$	*Multiplicative identity property of one*

Problem Set Sample Solutions

1. Fill in the missing parts.

a. The sum of $6c - 5$ and the opposite of $6c$

$(6c - 5) + (-6c)$	
$(6c + (-5)) + (-6c)$	Rewrite subtraction as addition
$6c + (-6c) + (-5)$	*Regrouping/any order (or commutative property of addition)*
$0 + (-5)$	*Additive inverse*
-5	Additive identity property of zero

b. The product of $-2c + 14$ and the multiplicative inverse of -2

$(-2c + 14)\left(-\frac{1}{2}\right)$	
$(-2c)\left(-\frac{1}{2}\right) + (14)\left(-\frac{1}{2}\right)$	*Distributive property*
$1c + (-7)$	Multiplicative inverse, multiplication
$1c - 7$	Adding the additive inverse is the same as subtraction
$c - 7$	*Multiplicative identity property of one*

2. Write the sum, and then rewrite the expression in standard form by removing parentheses and collecting like terms.

a. 6 and $p - 6$

$6 + (p - 6)$

$6 + (-6) + p$

$0 + p$

p

b. $10w + 3$ and -3

$(10w + 3) + (-3)$

$10w + \big(3 + (-3)\big)$

$10w + 0$

$10w$

c. $-x - 11$ and the opposite of -11

$\big(-x + (-11)\big) + 11$

$-x + \big((-11) + (11)\big)$

$-x + 0$

$-x$

d. The opposite of $4x$ and $3 + 4x$

$(-4x) + (3 + 4x)$

$\big((-4x) + 4x\big) + 3$

$0 + 3$

3

e. $2g$ and the opposite of $(1 - 2g)$

$2g + \big(-(1 - 2g)\big)$

$2g + (-1) + 2g$

$2g + 2g + (-1)$

$4g + (-1)$

$4g - 1$

3. Write the product, and then rewrite the expression in standard form by removing parentheses and collecting like terms.

a. $7h - 1$ and the multiplicative inverse of 7

$\big(7h + (-1)\big)\left(\frac{1}{7}\right)$

$\left(\frac{1}{7}\right)(7h) + \left(\frac{1}{7}\right)(-1)$

$h - \frac{1}{7}$

b. The multiplicative inverse of -5 and $10v - 5$

$\left(-\frac{1}{5}\right)(10v - 5)$

$\left(-\frac{1}{5}\right)(10v) + \left(-\frac{1}{5}\right)(-5)$

$-2v + 1$

c. $9 - b$ and the multiplicative inverse of 9

$\left(9 + (-b)\right)\left(\frac{1}{9}\right)$

$\left(\frac{1}{9}\right)(9) + \left(\frac{1}{9}\right)(-b)$

$1 - \frac{1}{9}b$

d. The multiplicative inverse of $\frac{1}{4}$ and $5t - \frac{1}{4}$

$4\left(5t - \frac{1}{4}\right)$

$4(5t) + 4\left(-\frac{1}{4}\right)$

$20t - 1$

e. The multiplicative inverse of $-\frac{1}{10x}$ and $\frac{1}{10x} - \frac{1}{10}$

$(-10x)\left(\frac{1}{10x} - \frac{1}{10}\right)$

$(-10x)\left(\frac{1}{10x}\right) + (-10x)\left(-\frac{1}{10}\right)$

$-1 + x$

4. Write the expressions in standard form.

a. $\frac{1}{4}(4x + 8)$

$\frac{1}{4}(4x) + \frac{1}{4}(8)$

$x + 2$

b. $\frac{1}{6}(r - 6)$

$\frac{1}{6}(r) + \frac{1}{6}(-6)$

$\frac{1}{6}r - 1$

c. $\frac{4}{5}(x + 1)$

$\frac{4}{5}(x) + \frac{4}{5}(1)$

$\frac{4}{5}x + \frac{4}{5}$

d. $\frac{1}{8}(2x+4)$

$\frac{1}{8}(2x)+\frac{1}{8}(4)$

$\frac{1}{4}x+\frac{1}{2}$

e. $\frac{3}{4}(5x-1)$

$\frac{3}{4}(5x)+\frac{3}{4}(-1)$

$\frac{15}{4}x-\frac{3}{4}$

f. $\frac{1}{5}(10x-5)-3$

$\frac{1}{5}(10x)+\frac{1}{5}(-5)+(-3)$

$2x+(-1)+(-3)$

$2x-4$

Exit Ticket Sample Solutions

For the problem $\frac{1}{5}g - \frac{1}{10} - g + 1\frac{3}{10}g - \frac{1}{10}$, Tyson created an equivalent expression using the following steps.

$$\frac{1}{5}g + -1g + 1\frac{3}{10}g + -\frac{1}{10} + -\frac{1}{10}$$
$$-\frac{4}{5}g + 1\frac{1}{10}$$

Is his final expression equivalent to the initial expression? Show how you know. If the two expressions are not equivalent, find Tyson's mistake and correct it.

No, he added the first two terms correctly, but he forgot the third term and added to the other like terms.

If $g = 10$,

$$\frac{1}{5}g + -1g + 1\frac{3}{10}g + -\frac{1}{10} + -\frac{1}{10}$$
$$\frac{1}{5}(10) + -1(10) + 1\frac{3}{10}(10) + -\frac{1}{10} + -\frac{1}{10}$$
$$2 + (-10) + 13 + \left(-\frac{2}{10}\right)$$
$$4\frac{4}{5}$$

$$-\frac{4}{5}g + 1\frac{1}{10}$$
$$-\frac{4}{5}(10) + 1\frac{1}{10}$$
$$-8 + 1\frac{1}{10}$$
$$-6\frac{9}{10}$$

The expressions are not equal.

He should factor out the g and place parentheses around the values using the distributive property in order to make it obvious which rational numbers need to be combined.

$$\frac{1}{5}g + -1g + 1\frac{3}{10}g + -\frac{1}{10} + -\frac{1}{10}$$
$$\left(\frac{1}{5}g + -1g + 1\frac{3}{10}g\right) + \left(-\frac{1}{10} + -\frac{1}{10}\right)$$
$$\left(\frac{1}{5} + -1 + 1\frac{3}{10}\right)g + \left(-\frac{2}{10}\right)$$
$$\left(\frac{2}{10} + \frac{3}{10}\right)g + \left(-\frac{1}{5}\right)$$
$$\frac{1}{2}g - \frac{1}{5}$$

Problem Set Sample Solutions

1. **Write the indicated expressions.**

   a. **$\frac{1}{2}m$ inches in feet**

   $\frac{1}{2}m \times \frac{1}{12} = \frac{1}{24}m$. *It is* $\frac{1}{24}m$ ft.

b. The perimeter of a square with $\frac{2}{3}g$ cm sides

$4 \times \frac{2}{3}g = \frac{8}{3}g$. *The perimeter is* $\frac{8}{3}g$ cm.

c. The number of pounds in 9 oz.

$9 \times \frac{1}{16} = \frac{9}{16}$. *It is* $\frac{9}{16}$ lb.

d. The average speed of a train that travels x miles in $\frac{3}{4}$ hour

$R = \frac{D}{T}$; $\frac{x}{\frac{3}{4}} = \frac{4}{3}x$. *The average speed of the train is* $\frac{4}{3}x$ *miles per hour.*

e. Devin is $1\frac{1}{4}$ years younger than Eli. April is $\frac{1}{5}$ as old as Devin. Jill is 5 years older than April. If Eli is E years old, what is Jill's age in terms of E?

$D = E - 1\frac{1}{4}$, $A = \frac{D}{5}$, $A + 5 = J$, *so* $J = \left(\frac{D}{5}\right) + 5$. $J = \frac{1}{5}\left(E - 1\frac{1}{4}\right) + 5$. $J = \frac{E}{5} + 4\frac{3}{4}$.

2. Rewrite the expressions by collecting like terms.

a. $\frac{1}{2}k - \frac{3}{8}k$

$\frac{4}{8}k - \frac{3}{8}k$

$\frac{1}{8}k$

b. $\frac{2r}{5} + \frac{7r}{15}$

$\frac{6r}{15} + \frac{7r}{15}$

$\frac{13r}{15}$

c. $-\frac{1}{3}a - \frac{1}{2}b - \frac{3}{4} + \frac{1}{2}b - \frac{2}{3}b + \frac{5}{6}a$

$-\frac{1}{3}a + \frac{5}{6}a - \frac{1}{2}b + \frac{1}{2}b - \frac{2}{3}b - \frac{3}{4}$

$-\frac{2}{6}a + \frac{5}{6}a - \frac{2}{3}b - \frac{3}{4}$

$\frac{1}{2}a - \frac{2}{3}b - \frac{3}{4}$

d. $-p + \frac{3}{5}q - \frac{1}{10}q + \frac{1}{9} - \frac{1}{9}p + 2\frac{1}{3}p$

$-p - \frac{1}{9}p + 2\frac{1}{3}p + \frac{3}{5}q - \frac{1}{10}q + \frac{1}{9}$

$-\frac{9}{9}p - \frac{1}{9}p + 2\frac{3}{9}p + \frac{6}{10}q - \frac{1}{10}q + \frac{1}{9}$

$\frac{11}{9}p + \frac{5}{10}q + \frac{1}{9}$

$1\frac{2}{9}p + \frac{1}{2}q + \frac{1}{9}$

e. $\frac{5}{7}y - \frac{y}{14}$

$\frac{10}{14}y - \frac{1}{14}y$

$\frac{9}{14}y$

f. $\frac{3n}{8} - \frac{n}{4} + 2\frac{n}{2}$

$\frac{3n}{8} - \frac{2n}{8} + 2\frac{4n}{8}$

$2\frac{5n}{8}$

3. **Rewrite the expressions by using the distributive property and collecting like terms.**

a. $\frac{4}{5}(15x-5)$

$12x-4$

b. $\frac{4}{5}\left(\frac{1}{4}c-5\right)$

$\frac{1}{5}c-4$

c. $2\frac{4}{5}v-\frac{2}{3}\left(4v+1\frac{1}{6}\right)$

$\frac{2}{15}v-\frac{7}{9}$

d. $8-4\left(\frac{1}{8}r-3\frac{1}{2}\right)$

$-\frac{1}{2}r+22$

e. $\frac{1}{7}(14x+7)-5$

$2x-4$

f. $\frac{1}{5}(5x-15)-2x$

$-x-3$

g. $\frac{1}{4}(p+4)+\frac{3}{5}(p-1)$

$\frac{17}{20}p+\frac{2}{5}$

h. $\frac{7}{8}(w+1)+\frac{5}{6}(w-3)$

$\frac{41}{24}w-\frac{39}{24}$ *or* $\frac{41}{24}w-\frac{13}{8}$

i. $\frac{4}{5}(c-1)-\frac{1}{8}(2c+1)$

$\frac{11}{20}c-\frac{37}{40}$

j. $\frac{2}{3}\left(h+\frac{3}{4}\right)-\frac{1}{3}\left(h+\frac{3}{4}\right)$

$\frac{1}{3}h+\frac{1}{4}$

k. $\frac{2}{3}\left(h+\frac{3}{4}\right)-\frac{2}{3}\left(h-\frac{3}{4}\right)$

1

l. $\frac{2}{3}\left(h+\frac{3}{4}\right)+\frac{2}{3}\left(h-\frac{3}{4}\right)$

$\frac{4}{3}h$

m. $\frac{k}{2}-\frac{4k}{5}-3$

$-\frac{3k}{10}-3$

n. $\frac{3t+2}{7}+\frac{t-4}{14}$

$\frac{1}{2}t$

o. $\frac{9x-4}{10}+\frac{3x+2}{5}$

$\frac{3x}{2}$ *or* $1\frac{1}{2}x$

p. $\frac{3(5g-1)}{4}-\frac{2g+7}{6}$

$3\frac{5}{12}g-1\frac{11}{12}$

q. $-\frac{3d+1}{5}+\frac{d-5}{2}+\frac{7}{10}$

$\frac{-d}{10}-2$

r. $\frac{9w}{6}+\frac{2w-7}{3}-\frac{w-5}{4}$

$\frac{23w-13}{12}$

$\frac{23}{12}w-\frac{13}{12}$

s. $\frac{1+f}{5}-\frac{1+f}{3}+\frac{3-f}{6}$

$\frac{11}{30}-\frac{3}{10}f$

Problem Set Sample Solutions

1. Check whether the given value is a solution to the equation.

   a. $4n - 3 = -2n + 9$ $\quad n = 2$

   $$4(2) - 3 = -2(2) + 9$$
   $$8 - 3 = -4 + 9$$
   $$5 = 5$$

   True

   b. $9m - 19 = 3m + 1$ $\quad m = \frac{10}{3}$

   $$9\left(\frac{10}{3}\right) - 19 = 3\left(\frac{10}{3}\right) + 1$$
   $$\frac{90}{3} - 19 = \frac{30}{3} + 1$$
   $$30 - 19 = 10 + 1$$
   $$11 = 11$$

   True

   c. $3(y + 8) = 2y - 6$ $\quad y = 30$

   $$3(30 + 8) = 2(30) - 6$$
   $$3(38) = 60 - 6$$
   $$114 = 54$$

   False

2. Tell whether each number is a solution to the problem modeled by the following equation.

   Mystery Number: Five more than -8 times a number is 29. What is the number?

   Let the mystery number be represented by n.

   The equation is $5 + (-8)n = 29$.

   a. Is 3 a solution to the equation? Why or why not?

   No, because $5 - 24 \neq 29$.

   b. Is -4 a solution to the equation? Why or why not?

   No, because $5 + 32 \neq 29$.

   c. Is -3 a solution to the equation? Why or why not?

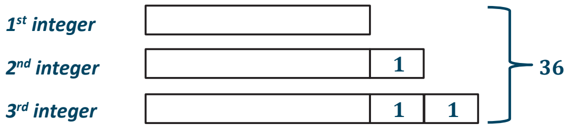
   Yes, because $5 + 24 = 29$.

   d. What is the mystery number?

   -3 *because* 5 *more than* -8 *times* -3 *is* 29.

3. The sum of three consecutive integers is 36.

   a. Find the smallest integer using a tape diagram.



   $36 - 3 = 33$

   $33 \div 3 = 11$

   The smallest integer is 11.

   b. Let n represent the smallest integer. Write an equation that can be used to find the smallest integer.

   Smallest integer: n

   2nd integer: $(n + 1)$

   3rd integer: $(n + 2)$

   Sum of the three consecutive integers: $n + (n + 1) + (n + 2)$

   Equation: $n + (n + 1) + (n + 2) = 36$.

   c. Determine if each value of n below is a solution to the equation in part (b).

$n = 12.5$	*No, it is not an integer and does not make a true equation.*
$n = 12$	*No, it does not make a true equation.*
$n = 11$	*Yes, it makes a true equation.*

4. Andrew is trying to create a number puzzle for his younger sister to solve. He challenges his sister to find the mystery number. "When 4 is subtracted from half of a number, the result is 5." The equation to represent the mystery number is $\frac{1}{2}m - 4 = 5$. Andrew's sister tries to guess the mystery number.

   a. Her first guess is 30. Is she correct? Why or why not?

   No, it does not make a true equation.

$$\frac{1}{2}(30) - 4 = 5$$
$$15 - 4 = 5$$
$$11 = 5$$

   False

   b. Her second guess is 2. Is she correct? Why or why not?

   No, it does not make a true equation.

$$\frac{1}{2}(2) - 4 = 5$$
$$1 - 4 = 5$$
$$-3 = 5$$

   False

c. Her final guess is $4\frac{1}{2}$. Is she correct? Why or why not?

No, it does not make a true equation.

$$\frac{1}{2}\left(4\frac{1}{2}\right) - 4 = 5$$
$$2\frac{1}{4} - 4 = 5$$
$$-1\frac{3}{4} = 5$$

False

Exit Ticket Sample Solutions

Mrs. Canale's class is selling frozen pizzas to earn money for a field trip. For every pizza sold, the class makes $\$5.35$. They have already earned $\$182.90$, but they need $\$750$. How many more pizzas must they sell to earn $\$750$? Solve this problem first by using an arithmetic approach, then by using an algebraic approach. Compare the calculations you made using each approach.

Arithmetic Approach:

Amount of money needed: $750 - 182.90 = 567.10$

Number of pizzas needed: $567.10 \div 5.35 = 106$

If the class wants to earn a total of $\$750$*, then they must sell* 106 *more pizzas.*

Algebraic Approach:

Let x *represent the number of additional pizzas they need to sell.*

$$\begin{aligned} 5.35x + 182.90 &= 750 \\ 5.35x + 182.90 - 182.90 &= 750 - 182.90 \\ 5.35x + 0 &= 567.10 \\ \left(\frac{1}{5.35}\right)(5.35x) &= \left(\frac{1}{5.35}\right)(567.10) \\ x &= 106 \end{aligned}$$

OR

$$\begin{aligned} 5.35x + 182.90 &= 750 \\ 100(5.35x + 182.90) &= 100(750) \\ 535x + 18290 &= 75000 \\ 535x + 18290 - 18290 &= 75000 - 18290 \\ \left(\frac{1}{535}\right)(535x) &= \left(\frac{1}{535}\right)(56710) \\ x &= 106 \end{aligned}$$

If the class wants to earn $\$750$*, then they must sell* 106 *more pizzas.*

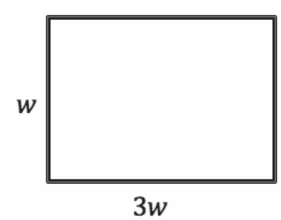
Both approaches subtract 182.90 *from* 750 *to get* 567.10*. Dividing by* 5.35 *is the same as multiplying by* $\frac{1}{5.35}$*. Both result in* 106 *more pizzas that the class needs to sell.*

Problem Set Sample Solutions

Write and solve an equation for each problem.

1. **The perimeter of a rectangle is 30 inches. If its length is three times its width, find the dimensions.**

   The width of the rectangle: w *inches*
   The length of the rectangle: $3w$ *inches*

   $\text{Perimeter} = 2(\text{length} + \text{width})$

   $$\begin{aligned} 2(w + 3w) &= 30 \\ 2(4w) &= 30 \\ 8w &= 30 \\ \left(\frac{1}{8}\right)(8w) &= \left(\frac{1}{8}\right)(30) \\ w &= 3\frac{3}{4} \end{aligned}$$

   OR

   $$\begin{aligned} 2(w + 3w) &= 30 \\ (w + 3w) &= 15 \\ 4w &= 15 \\ w &= 3\frac{3}{4} \end{aligned}$$

   The width is $3\frac{3}{4}$ *inches.*

   The length is $(3)\left(3\frac{3}{4}\text{ in.}\right) = (3)\left(\frac{15}{4}\text{ in.}\right) = 11\frac{1}{4}\text{ in.}$

2. A cell phone company has a basic monthly plan of $\$40$ plus $\$0.45$ for any minutes used over 700. Before receiving his statement, John saw he was charged a total of $\$48.10$. Write and solve an equation to determine how many minutes he must have used during the month. Write an equation without decimals.

The number of minutes over 700: m minutes

$$40 + 0.45m = 48.10$$
$$0.45m + 40 - 40 = 48.10 - 40$$
$$0.45m = 8.10$$
$$\left(\frac{1}{0.45}\right)(0.45m) = 8.10\left(\frac{1}{0.45}\right)$$
$$m = 18$$

$$4000 + 45m = 4810$$
$$45m + 4000 - 4000 = 4810 - 4000$$
$$45m = 810$$
$$\left(\frac{1}{45}\right)(45m) = 810\left(\frac{1}{45}\right)$$
$$m = 18$$

John used 18 minutes over 700 for the month. He used a total of 718 minutes.

3. A volleyball coach plans her daily practices to include 10 minutes of stretching, $\frac{2}{3}$ of the entire practice scrimmaging, and the remaining practice time working on drills of specific skills. On Wednesday, the coach planned 100 minutes of stretching and scrimmaging. How long, in hours, is the entire practice?

The duration of the entire practice: x hours

$$\frac{2}{3}x + \frac{10}{60} = \frac{100}{60}$$
$$\frac{2}{3}x + \frac{1}{6} = \frac{5}{3}$$
$$\frac{2}{3}x + \frac{1}{6} - \frac{1}{6} = \frac{5}{3} - \frac{1}{6}$$
$$\frac{2}{3}x = \frac{9}{6}$$
$$\left(\frac{3}{2}\right)\left(\frac{2}{3}x\right) = \frac{3}{2}\left(\frac{9}{6}\right)$$
$$x = \frac{27}{12} = 2\frac{1}{4}$$

The entire practice is a length of $2\frac{1}{4}$ hours, or 2.25 hours.

4. The sum of two consecutive even numbers is 54. Find the numbers.

First consecutive even integer: x

Second consecutive even integer: $x + 2$

$$x + (x + 2) = 54$$
$$2x + 2 = 54$$
$$2x + 2 - 2 = 54 - 2$$
$$2x + 0 = 52$$
$$\left(\frac{1}{2}\right)(2x) = \left(\frac{1}{2}\right)(52)$$
$$x = 26$$

The consecutive even integers are 26 and 28.

5. **Justin has $\$7.50$ more than Eva, and Emma has $\$12$ less than Justin. Together, they have a total of $\$63.00$. How much money does each person have?**

The amount of money Eva has: x dollars

The amount of money Justin has: $(x+7.50)$ dollars

The amount of money Emma has: $\big((x+7.50)-12\big)$ dollars, or $(x-4.50)$ dollars

$$\begin{aligned} x+(x+7.50)+(x-4.50) &= 63 \\ 3x+3 &= 63 \\ 3x+3-3 &= 63-3 \\ 3x+0 &= 60 \\ \left(\frac{1}{3}\right)3x &= \left(\frac{1}{3}\right)60 \\ x &= 20 \end{aligned}$$

If the total amount of money all three people have is $\$63$, then Eva has $\$20$, Justin has $\$27.50$, and Emma has $\$15.50$.

6. **Barry's mountain bike weighs 6 pounds more than Andy's. If their bikes weigh 42 pounds altogether, how much does Barry's bike weigh? Identify the if-then moves in your solution.**

If we let a represent the weight in pounds of Andy's bike, then $a+6$ represents the weight in pounds of Barry's bike.

$$\begin{aligned} a+(a+6) &= 42 \\ (a+a)+6 &= 42 \\ 2a+6 &= 42 \\ 2a+6-6 &= 42-6 && \text{If } 2a+6=42\text{, then } 2a+6-6=42-6. \\ 2a+0 &= 36 \\ 2a &= 36 \\ \frac{1}{2}\cdot 2a &= \frac{1}{2}\cdot 36 && \text{If } 2a=36\text{, then } \frac{1}{2}\cdot 2a=\frac{1}{2}\cdot 36. \\ 1\cdot a &= 18 \\ a &= 18 \end{aligned}$$

Barry's Bike: $a+6$

$(18)+6=24$

Barry's bike weighs 24 pounds.

7. **Trevor and Marissa together have 26 T-shirts to sell. If Marissa has 6 fewer T-shirts than Trevor, find how many T-shirts Trevor has. Identify the if-then moves in your solution.**

Let t represent the number of T-shirts that Trevor has, and let $t-6$ represent the number of T-shirts that Marissa has.

$$t+(t-6)=26$$
$$(t+t)+(-6)=26$$
$$2t+(-6)=26$$
$$2t+(-6)+6=26+6$$ *If-then move: Addition property of equality*
$$2t+0=32$$
$$2t=32$$
$$\frac{1}{2}\cdot 2t=\frac{1}{2}\cdot 32$$ *If-then move: Multiplication property of equality*
$$1\cdot t=16$$
$$t=16$$

Trevor has 16 T-shirts to sell, and Marissa has 10 T-shirts to sell.

8. **A number is $\frac{1}{7}$ of another number. The difference of the numbers is 18. (Assume that you are subtracting the smaller number from the larger number.) Find the numbers.**

If we let n represent a number, then $\frac{1}{7}n$ represents the other number.

$$n-\left(\frac{1}{7}n\right)=18$$
$$\frac{7}{7}n-\frac{1}{7}n=18$$
$$\frac{6}{7}n=18$$
$$\frac{7}{6}\cdot\frac{6}{7}n=\frac{7}{6}\cdot 18$$
$$1n=7\cdot 3$$
$$n=21$$

The numbers are 21 and 3.

9. A number is 6 greater than $\frac{1}{2}$ another number. If the sum of the numbers is 21, find the numbers.

If we let n represent a number, then $\frac{1}{2}n + 6$ represents the first number.

$$n + \left(\frac{1}{2}n + 6\right) = 21$$
$$\left(n + \frac{1}{2}n\right) + 6 = 21$$
$$\left(\frac{2}{2}n + \frac{1}{2}n\right) + 6 = 21$$
$$\frac{3}{2}n + 6 = 21$$
$$\frac{3}{2}n + 6 - 6 = 21 - 6$$
$$\frac{3}{2}n + 0 = 15$$
$$\frac{3}{2}n = 15$$
$$\frac{2}{3} \cdot \frac{3}{2}n = \frac{2}{3} \cdot 15$$
$$1n = 2 \cdot 5$$
$$n = 10$$

Since the numbers sum to 21, they are 10 and 11.

10. Kevin is currently twice as old as his brother. If Kevin was 8 years old 2 years ago, how old is Kevin's brother now?

If we let b represent Kevin's brother's age in years, then Kevin's age in years is $2b$.

$$2b - 2 = 8$$
$$2b - 2 + 2 = 8 + 2$$
$$2b = 10$$
$$\left(\frac{1}{2}\right)(2b) = \left(\frac{1}{2}\right)(10)$$
$$b = 5$$

Kevin's brother is currently 5 years old.

11. The sum of two consecutive odd numbers is 156. What are the numbers?

If we let n represent one odd number, then $n + 2$ represents the next consecutive odd number.

$$n + (n + 2) = 156$$
$$2n + 2 - 2 = 156 - 2$$
$$2n = 154$$
$$\left(\frac{1}{2}\right)(2n) = \left(\frac{1}{2}\right)(154)$$
$$n = 77$$

The two numbers are 77 and 79.

12. If n represents an odd integer, write expressions in terms of n that represent the next three consecutive odd integers. If the four consecutive odd integers have a sum of 56, find the numbers.

If we let n represent an odd integer, then $n+2$, $n+4$, and $n+6$ represent the next three consecutive odd integers.

$$\begin{aligned} n+(n+2)+(n+4)+(n+6) &= 56 \\ 4n+12 &= 56 \\ 4n+12-12 &= 56-12 \\ 4n &= 44 \\ n &= 11 \end{aligned}$$

The numbers are 11, 13, 15, and 17.

13. The cost of admission to a history museum is $\$3.25$ per person over the age of 3; kids 3 and under get in for free. If the total cost of admission for the Warrick family, including their two 6-month old twins, is $\$19.50$, find how many family members are over 3 years old.

If we let w represent the number of Warrick family members, then $w-2$ represents the number of family members over the age of 3 years.

$$\begin{aligned} 3.25(w-2) &= 19.5 \\ 3.25w-6.5 &= 19.5 \\ 3.25w-6.5+6.5 &= 19.5+6.5 \\ 3.25w &= 26 \\ w &= 8 \\ w-2 &= 6 \end{aligned}$$

There are 6 members of the Warrick family over the age of 3 years.

14. Six times the sum of three consecutive odd integers is -18. Find the integers.

If we let n represent the first odd integer, then $n+2$ and $n+4$ represent the next two consecutive odd integers.

$$\begin{aligned} 6\big(n+(n+2)+(n+4)\big) &= -18 \\ 6(3n+6) &= -18 \\ 18n+36 &= -18 \\ 18n+36-36 &= -18-36 \\ 18n &= -54 \\ n &= -3 \end{aligned}$$

$$n+2=-1$$

$$n+4=1$$

The integers are -3, -1, and 1.

15. I am thinking of a number. If you multiply my number by 4, add -4 to the product, and then take $\frac{1}{3}$ of the sum, the result is -6. Find my number.

Let n represent the given number.

$$\frac{1}{3}(4n + (-4)) = -6$$
$$\frac{4}{3}n - \frac{4}{3} = -6$$
$$\frac{4}{3}n - \frac{4}{3} + \frac{4}{3} = -6 + \frac{4}{3}$$
$$\frac{4}{3}n = \frac{-14}{3}$$
$$n = -3\frac{1}{2}$$

16. A vending machine has twice as many quarters in it as dollar bills. If the quarters and dollar bills have a combined value of $\$96.00$, how many quarters are in the machine?

If we let d represent the number of dollar bills in the machine, then $2d$ represents the number of quarters in the machine.

$$2d \cdot \left(\frac{1}{4}\right) + 1d \cdot (1) = 96$$
$$\frac{1}{2}d + 1d = 96$$
$$1\frac{1}{2}d = 96$$
$$\frac{3}{2}d = 96$$
$$\frac{2}{3}\left(\frac{3}{2}d\right) = \frac{2}{3}(96)$$
$$d = 64$$
$$2d = 128$$

There are 128 quarters in the machine.

Problem Set Sample Solutions

1. A company buys a digital scanner for $\$12,000$. The value of the scanner is $12,000\left(1-\frac{n}{5}\right)$ after n years. The company has budgeted to replace the scanner when the trade-in value is $\$2,400$. After how many years should the company plan to replace the machine in order to receive this trade-in value?

$$12,000\left(1-\frac{n}{5}\right)=2,400$$
$$12,000-2,400n=2,400$$
$$-2,400n+12,000-12,000=2,400-12,000$$
$$-2,400n=-9,600$$
$$n=4$$

They will replace the scanner after 4 years.

2. Michael is 17 years older than John. In 4 years, the sum of their ages will be 49. Find Michael's present age.

x represents Michael's age now in years.

	Now	*4 years later*
Michael	x	$x+4$
John	$x-17$	$(x-17)+4$

$$x+4+x-17+4=49$$
$$x+4+x-13=49$$
$$2x-9=49$$
$$2x-9+9=49+9$$
$$2x=58$$
$$\left(\frac{1}{2}\right)(2x)=\left(\frac{1}{2}\right)(58)$$
$$x=29$$

Michael's present age is 29 years old.

3. Brady rode his bike 70 miles in 4 hours. He rode at an average speed of 17 mph for t hours and at an average rate of speed of 22 mph for the rest of the time. How long did Brady ride at the slower speed? Use the variable t to represent the time, in hours, Brady rode at 17 mph.

	Rate (mph)	*Time (hours)*	*Distance (miles)*
Brady speed 1	17	t	$17t$
Brady speed 2	22	$4-t$	$22(4-t)$

Total distance

The total distance he rode: $17t+22(4-t)$

The total distance equals 70 miles:

$$17t+22(4-t)=70$$
$$17t+88-22t=70$$
$$-5t+88=70$$
$$-5t+88-88=70-88$$
$$-5t=-18$$
$$t=3.6$$

Brady rode at 17 mph for 3.6 hours.

4. **Caitlan went to the store to buy school clothes. She had a store credit from a previous return in the amount of $\$39.58$. If she bought 4 of the same style shirt in different colors and spent a total of $\$52.22$ after the store credit was taken off her total, what was the price of each shirt she bought? Write and solve an equation with integer coefficients.**

t: the price of one shirt

$$4t - 39.58 = 52.22$$
$$4t - 39.58 + 39.58 = 52.22 + 39.58$$
$$4t + 0 = 91.80$$
$$\left(\frac{1}{4}\right)(4t) = \left(\frac{1}{4}\right)(91.80)$$
$$t = 22.95$$

The price of one shirt was $\$22.95$.

5. **A young boy is growing at a rate of 3.5 cm per month. He is currently 90 cm tall. At that rate, in how many months will the boy grow to a height of 132 cm?**

Let m represent the number of months.

$$3.5m + 90 = 132$$
$$3.5m + 90 - 90 = 132 - 90$$
$$3.5m = 42$$
$$\left(\frac{1}{3.5}\right)(3.5m) = \left(\frac{1}{3.5}\right)(42)$$
$$m = 12$$

The boy will grow to be 132 cm tall 12 months from now.

6. **The sum of a number, $\frac{1}{6}$ of that number, $2\frac{1}{2}$ of that number, and 7 is $12\frac{1}{2}$. Find the number.**

Let n represent the given number.

$$n + \frac{1}{6}n + \left(2\frac{1}{2}\right)n + 7 = 12\frac{1}{2}$$
$$n\left(1 + \frac{1}{6} + \frac{5}{2}\right) + 7 = 12\frac{1}{2}$$
$$n\left(\frac{6}{6} + \frac{1}{6} + \frac{15}{6}\right) + 7 = 12\frac{1}{2}$$
$$n\left(\frac{22}{6}\right) + 7 = 12\frac{1}{2}$$
$$\frac{11}{3}n + 7 - 7 = 12\frac{1}{2} - 7$$
$$\frac{11}{3}n + 0 = 5\frac{1}{2}$$
$$\frac{11}{3}n = 5\frac{1}{2}$$
$$\frac{3}{11}\cdot\frac{11}{3}n = \frac{3}{11}\cdot\frac{11}{2}$$
$$1n = \frac{3}{2}$$
$$n = 1\frac{1}{2}$$

The number is $1\frac{1}{2}$.

7. **The sum of two numbers is 33 and their difference is 2. Find the numbers.**

Let x represent the first number, then $33 - x$ represents the other number since their sum is 33.

$$\begin{aligned} x - (33 - x) &= 2 \\ x + \left(-(33 - x)\right) &= 2 \\ x + (-33) + x &= 2 \\ 2x + (-33) &= 2 \\ 2x + (-33) + 33 &= 2 + 33 \\ 2x + 0 &= 35 \\ 2x &= 35 \\ \frac{1}{2} \cdot 2x &= \frac{1}{2} \cdot 35 \\ 1x &= \frac{35}{2} \\ x &= 17\frac{1}{2} \end{aligned}$$

$33 - x = 33 - \left(17\frac{1}{2}\right) = 15\frac{1}{2}$

$\left\{17\frac{1}{2}, 15\frac{1}{2}\right\}$

8. **Aiden refills three token machines in an arcade. He puts twice the number of tokens in machine A as in machine B, and in machine C, he puts $\frac{3}{4}$ of what he put in machine A. The three machines took a total of $18,324$ tokens. How many did each machine take?**

Let A represent the number of tokens in machine A. Then $\frac{1}{2}A$ represents the number of tokens in machine B, and $\frac{3}{4}A$ represents the number of tokens in machine C.

$$\begin{aligned} A + \frac{1}{2}A + \frac{3}{4}A &= 18,324 \\ \frac{9}{4}A &= 18,324 \\ A &= 8,144 \end{aligned}$$

Machine A took $8,144$ tokens, machine B took $4,072$ tokens, and machine C took $6,108$ tokens.

9. **Paulie ordered 250 pens and 250 pencils to sell for a theatre club fundraiser. The pens cost 11 cents more than the pencils. If Paulie's total order costs $\$42.50$, find the cost of each pen and pencil.**

Let l represent the cost of a pencil in dollars. Then, the cost of a pen in dollars is $l + 0.11$.

$$\begin{aligned} 250(l + l + 0.11) &= 42.5 \\ 250(2l + 0.11) &= 42.5 \\ 500l + 27.5 &= 42.5 \\ 500l + 27.5 + (-27.5) &= 42.5 + (-27.5) \\ 500l + 0 &= 15 \\ 500l &= 15 \\ \frac{500l}{500} &= \frac{15}{500} \\ l &= 0.03 \end{aligned}$$

A pencil costs $\$0.03$, and a pen costs $\$0.14$.

10. A family left their house in two cars at the same time. One car traveled an average of 7 miles per hour faster than the other. When the first car arrived at the destination after $5\frac{1}{2}$ hours of driving, both cars had driven a total of 599.5 miles. If the second car continues at the same average speed, how much time, to the nearest minute, will it take before the second car arrives?

Let r represent the speed in miles per hour of the faster car, then $r-7$ represents the speed in miles per hour of the slower car.

$$5\frac{1}{2}(r)+5\frac{1}{2}(r-7)=599.5$$
$$5\frac{1}{2}(r+r-7)=599.5$$
$$5\frac{1}{2}(2r-7)=599.5$$
$$\frac{11}{2}(2r-7)=599.5$$
$$\frac{2}{11}\cdot\frac{11}{2}(2r-7)=\frac{2}{11}\cdot 599.5$$
$$1\cdot(2r-7)=\frac{1199}{11}$$
$$2r-7=109$$
$$2r-7+7=109+7$$
$$2r+0=116$$
$$2r=116$$
$$\frac{1}{2}\cdot 2r=\frac{1}{2}\cdot 116$$
$$1r=58$$
$$r=58$$

The average speed of the faster car is 58 miles per hour, so the average speed of the slower car is 51 miles per hour.

$$\text{distance}=\text{rate}\cdot\text{time}$$
$$d=51\cdot 5\frac{1}{2}$$
$$d=51\cdot\frac{11}{2}$$
$$d=280.5$$

The slower car traveled 280.5 miles in $5\frac{1}{2}$ hours.

$$d=58\cdot 5\frac{1}{2}$$
$$d=58\cdot\frac{11}{2}$$
$$d=319$$

OR

$$599.5-280.5=319$$

The faster car traveled 319 miles in $5\frac{1}{2}$ hours.

The slower car traveled 280.5 miles in $5\frac{1}{2}$ hours. The remainder of their trip is 38.5 miles because $319-280.5=38.5$.

$$\text{distance}=\text{rate}\cdot\text{time}$$
$$38.5=51\,(t)$$
$$\frac{1}{51}(38.5)=\frac{1}{51}(51)(t)$$
$$\frac{38.5}{51}=1t$$
$$\frac{77}{102}=t$$

This time is in hours. To convert to minutes, multiply by 60 minutes per hour.

$$\frac{77}{102}\cdot 60=\frac{77}{51}\cdot 30=\frac{2310}{51}\approx 45$$

The slower car will arrive approximately 45 minutes after the first.

11. **Emily counts the triangles and parallelograms in an art piece and determines that altogether, there are 42 triangles and parallelograms. If there are 150 total sides, how many triangles and parallelograms are there?**

If t represents the number of triangles that Emily counted, then $42 - t$ represents the number of parallelograms that she counted.

$$3t + 4(42 - t) = 150$$
$$3t + 4\big(42 + (-t)\big) = 150$$
$$3t + 4(42) + 4(-t) = 150$$
$$3t + 168 + (-4t) = 150$$
$$3t + (-4t) + 168 = 150$$
$$-t + 168 = 150$$
$$-t + 168 - 168 = 150 - 168$$
$$-t + 0 = -18$$
$$-t = -18$$
$$-1 \cdot (-t) = -1 \cdot (-18)$$
$$1t = 18$$
$$t = 18$$

There are 18 triangles and 24 parallelograms.

Note to the Teacher: Problems 12 and 13 are more difficult and may not be suitable to assign to all students to solve independently.

12. **Stefan is three years younger than his sister Katie. The sum of Stefan's age 3 years ago and $\frac{2}{3}$ of Katie's age at that time is 12. How old is Katie now?**

If s represents Stefan's age in years, then $s + 3$ represents Katie's current age, $s - 3$ represents Stefan's age 3 years ago, and s also represents Katie's age 3 years ago.

$$(s - 3) + \left(\frac{2}{3}\right)s = 12$$
$$s + (-3) + \frac{2}{3}s = 12$$
$$s + \frac{2}{3}s + (-3) = 12$$
$$\frac{3}{3}s + \frac{2}{3}s + (-3) = 12$$
$$\frac{5}{3}s + (-3) = 12$$
$$\frac{5}{3}s + (-3) + 3 = 12 + 3$$
$$\frac{5}{3}s + 0 = 15$$
$$\frac{5}{3}s = 15$$
$$\frac{3}{5} \cdot \frac{5}{3}s = \frac{3}{5} \cdot 15$$
$$1s = 3 \cdot 3$$
$$s = 9$$

Stefan's current age is 9 years, so Katie is currently 12 years old.

13. Lucas bought a certain weight of oats for his horse at a unit price of $\$0.20$ per pound. The total cost of the oats left him with $\$1$. He wanted to buy the same weight of enriched oats instead, but at $\$0.30$ per pound, he would have been $\$2$ short of the total amount due. How much money did Lucas have to buy oats?

The difference in the costs is $\$3.00$ for the same weight in feed.

Let w represent the weight in pounds of feed.

$$0.3w - 0.2w = 3$$
$$0.1w = 3$$
$$\frac{1}{10}w = 3$$
$$10 \cdot \frac{1}{10}w = 10 \cdot 3$$
$$1w = 30$$
$$w = 30$$

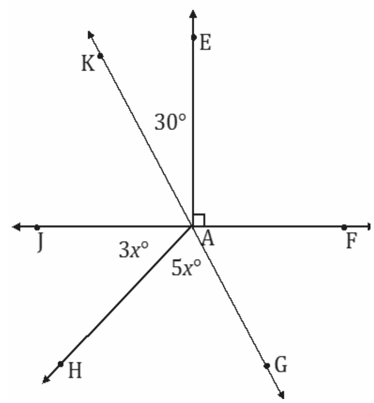
Lucas bought 30 pounds of oats.

$\text{Cost} = \text{unit price} \times \text{weight}$
$\text{Cost} = (\$0.20 \text{ per pound}) \cdot (30 \text{ pounds})$
$\text{Cost} = \$6.00$

Lucas paid $\$6$ for 30 pounds of oats. Lucas had $\$1$ left after his purchase, so he started with $\$7$.

Exit Ticket Sample Solutions

In a complete sentence, describe the relevant angle relationships in the following diagram. That is, describe the angle relationships you could use to determine the value of x.



$\angle KAE$ and $\angle EAF$ are adjacent angles whose measurements are equal to $\angle KAF$; $\angle KAF$ and $\angle JAG$ are vertical angles and are of equal measurement.

Use the angle relationships described above to write an equation to solve for x. Then, determine the measurements of $\angle JAH$ and $\angle HAG$.

$$5x + 3x = 90 + 30$$
$$8x = 120$$
$$\left(\frac{1}{8}\right)(8x) = \left(\frac{1}{8}\right)(120)$$
$$x = 15$$

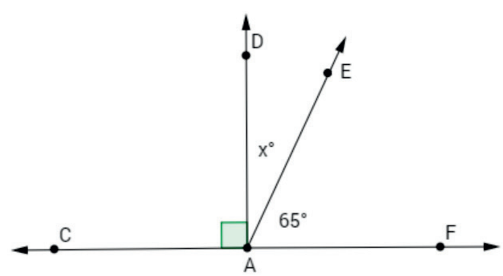
$m\angle JAH = 3(15°) = 45°$

$m\angle HAG = 5(15°) = 75°$

Problem Set Sample Solutions

For each question, use angle relationships to write an equation in order to solve for each variable. Determine the indicated angles. You can check your answers by measuring each angle with a protractor.

1. **In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurement of $\angle DAE$.**

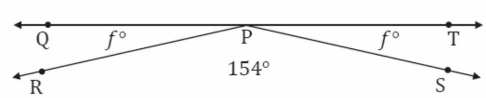


One possible response: $\angle CAD$, $\angle DAE$, and $\angle FAE$ are angles on a line and their measures sum to $180°$.

$$90 + x + 65 = 180$$
$$x + 155 = 180$$
$$x + 155 - 155 = 180 - 155$$
$$x = 25$$

$m\angle DAE = 25°$

2. **In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurement of $\angle QPR$.**



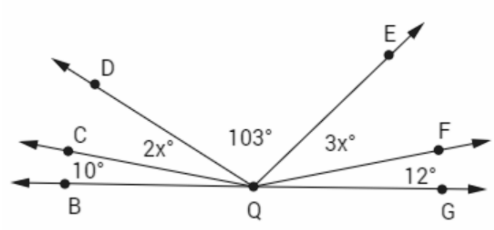
$\angle QPR$, $\angle RPS$, and $\angle SPT$ are angles on a line and their measures sum to $180°$.

$$f + 154 + f = 180$$
$$2f + 154 = 180$$
$$2f + 154 - 154 = 180 - 154$$
$$2f = 26$$
$$\left(\frac{1}{2}\right)2f = \left(\frac{1}{2}\right)26$$
$$f = 13$$

$m\angle QPR = 13°$

3. **In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurements of $\angle CQD$ and $\angle EQF$.**

$\angle BQC$, $\angle CQD$, $\angle DQE$, $\angle EQF$, and $\angle FQG$ are angles on a line and their measures sum to $180°$.



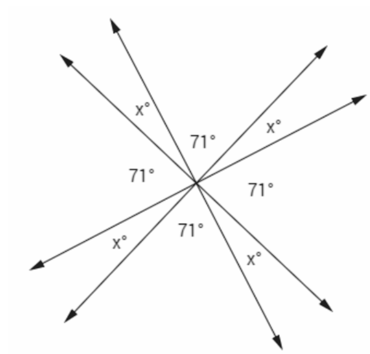
$$10 + 2x + 103 + 3x + 12 = 180$$
$$5x + 125 = 180$$
$$5x + 125 - 125 = 180 - 125$$
$$5x = 55$$
$$\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)55$$
$$x = 11$$

$$m\angle CQD = 2(11°) = 22°$$

$$m\angle EQF = 3(11°) = 33°$$

4. **In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measure of x.**

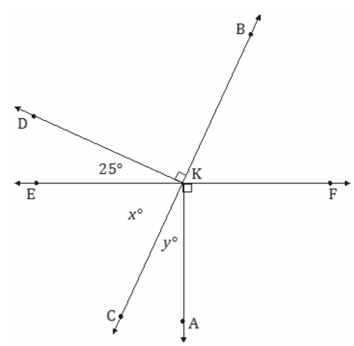
All of the angles in the diagram are angles at a point and their measures sum to $360°$.



$$4(x + 71) = 360$$
$$4x + 284 = 360$$
$$4x + 284 - 284 = 360 - 284$$
$$4x = 76$$
$$\left(\frac{1}{4}\right)4x = \left(\frac{1}{4}\right)76$$
$$x = 19$$

5. **In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of x and y.**

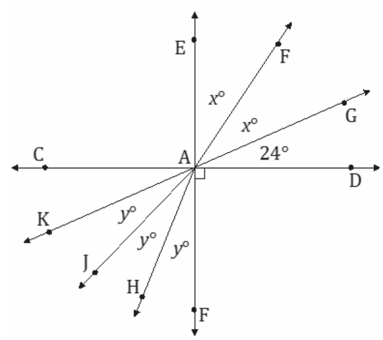
$\angle CKE$, $\angle EKD$, and $\angle DKB$ are angles on a line and their measures sum to $180°$. Since $\angle FKA$ and $\angle AKE$ form a straight angle and the measurement of $\angle FKA$ is $90°$, $\angle AKE$ is $90°$, making $\angle CKE$ and $\angle AKC$ form a right angle and their measures have a sum of $90°$.



$$x + 25 + 90 = 180$$
$$x + 115 = 180$$
$$x + 115 - 115 = 180 - 115$$
$$x = 65$$

$$(65) + y = 90$$
$$65 - 65 + y = 90 - 65$$
$$y = 25$$

6. **In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of x and y.**

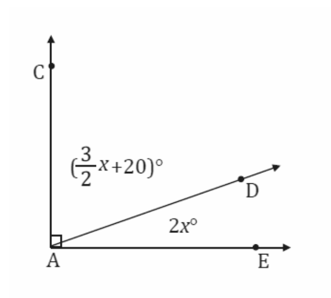


$\angle EAG$ and $\angle FAK$ are vertical angles and are of equal measurement. $\angle EAG$ and $\angle GAD$ form a right angle and their measures have a sum of $90°$.

$$2x + 24 = 90$$
$$2x + 24 - 24 = 90 - 24$$
$$2x = 66$$
$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right)66$$
$$x = 33$$

$$3y = 66$$
$$\left(\frac{1}{3}\right)3y = \left(\frac{1}{3}\right)66$$
$$y = 22$$

7. **In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of $\angle CAD$ and $\angle DAE$.**



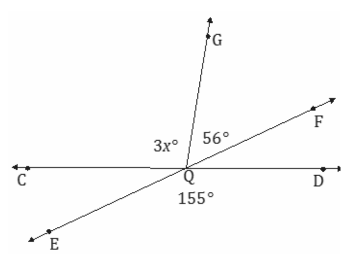
$\angle CAD$ and $\angle DAE$ form a right angle and their measures have a sum of $90°$.

$$\left(\frac{3}{2}x + 20\right) + 2x = 90$$
$$\frac{7}{2}x + 20 = 90$$
$$\frac{7}{2}x + 20 - 20 = 90 - 20$$
$$\frac{7}{2}x = 70$$
$$\left(\frac{2}{7}\right)\frac{7}{2}x = 70\left(\frac{2}{7}\right)$$
$$x = 20$$

$$m\angle CAD = \frac{3}{2}(20°) + 20° = 50°$$

$$m\angle DAE = 2(20°) = 40°$$

8. **In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measure of $\angle CQG$.**



$\angle DQE$ and $\angle CQF$ are vertical angles and are of equal measurement. $\angle CQG$ and $\angle GQF$ are adjacent angles and their measures sum to the measure of $\angle CQF$.

$$3x + 56 = 155$$
$$3x + 56 - 56 = 155 - 56$$
$$3x = 99$$
$$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)99$$
$$x = 33$$
$$m\angle CQG = 3(33°) = 99°$$

9. The ratio of the measures of a pair of adjacent angles on a line is $4 : 5$.

a. Find the measures of the two angles.

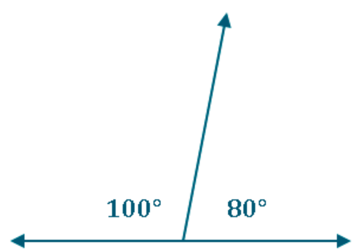
$\angle 1 = 4x, \angle 2 = 5x$

$$4x + 5x = 180$$
$$9x = 180$$
$$\left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)180$$
$$x = 20$$

$\angle 1 = 4(20°) = 80°$

$\angle 2 = 5(20°) = 100°$

b. Draw a diagram to scale of these adjacent angles. Indicate the measurements of each angle.



10. The ratio of the measures of three adjacent angles on a line is $3 : 4 : 5$.

a. Find the measures of the three angles.

$\angle 1 = 3x, \angle 2 = 4x, \angle 3 = 5x$

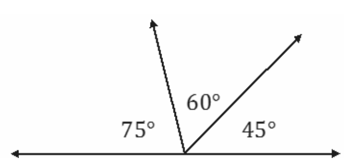
$$3x + 4x + 5x = 180$$
$$12x = 180$$
$$\left(\frac{1}{12}\right)12x = \left(\frac{1}{12}\right)180$$
$$x = 15$$

$\angle 1 = 3(15°) = 45°$

$\angle 2 = 4(15°) = 60°$

$\angle 3 = 5(15°) = 75°$

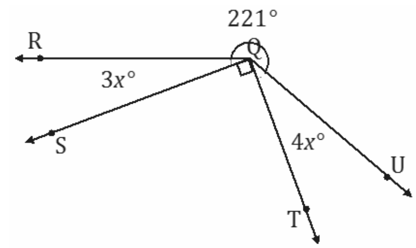
b. Draw a diagram to scale of these adjacent angles. Indicate the measurements of each angle.



Exit Ticket Sample Solutions

Write an equation for the angle relationship shown in the figure and solve for x. Find the measures of $\angle RQS$ and $\angle TQU$.

$$3x + 90 + 4x + 221 = 360$$
$$7x + 311 = 360$$
$$7x + 311 - 311 = 360 - 311$$
$$7x = 49$$
$$\left(\frac{1}{7}\right)7x = \left(\frac{1}{7}\right)49$$
$$x = 7$$



$m\angle RQS = 3(7°) = 21°$

$m\angle TQU = 4(7°) = 28°$

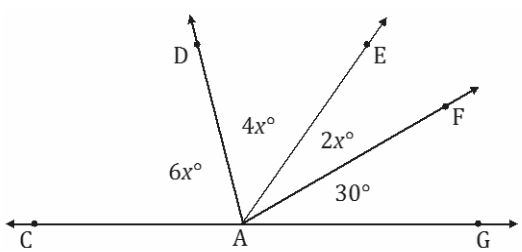
Problem Set Sample Solutions

In a complete sentence, describe the angle relationships in each diagram. Write an equation for the angle relationship(s) shown in the figure, and solve for the indicated unknown angle. You can check your answers by measuring each angle with a protractor.

1. **Find the measures of $\angle EAF$, $\angle DAE$, and $\angle CAD$.**

$\angle GAF$, $\angle EAF$, $\angle DAE$, and $\angle CAD$ are angles on a line and their measures have a sum of $180°$.

$$6x + 4x + 2x + 30 = 180$$
$$12x + 30 = 180$$
$$12x + 30 - 30 = 180 - 30$$
$$12x = 150$$
$$x = 12.5$$



$m\angle EAF = 2(12.5°) = 25°$

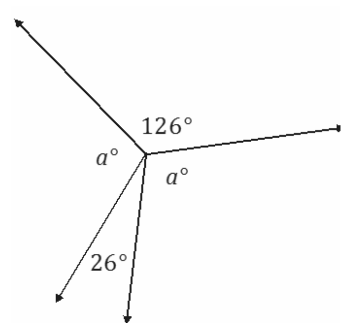
$m\angle DAE = 4(12.5°) = 50°$

$m\angle CAD = 6(12.5°) = 75°$

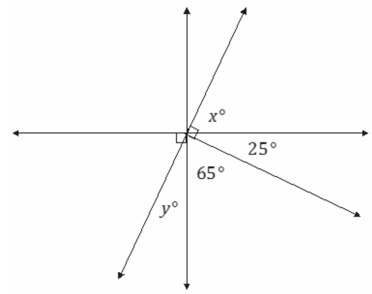
2. **Find the measure of a.**

Angles $a°$, $26°$, $a°$, and $126°$ are angles at a point and have a sum of $360°$.

$$a + 126 + a + 26 = 360$$
$$2a + 152 = 360$$
$$2a + 152 - 152 = 360 - 152$$
$$2a = 208$$
$$\left(\frac{1}{2}\right)2a = \left(\frac{1}{2}\right)208$$
$$a = 104$$



3. Find the measures of x and y.

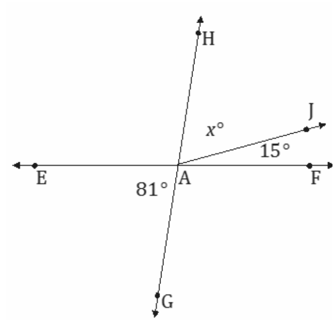


Angles $y°$ and $65°$ and angles $25°$ and $x°$ have a sum of $90°$.

$$x + 25 = 90$$
$$x + 25 - 25 = 90 - 25$$
$$x = 65$$

$$65 + y = 90$$
$$65 + y = 90$$
$$65 - 65 + y = 90 - 65$$
$$y = 25$$

4. Find the measure of $\angle HAJ$.

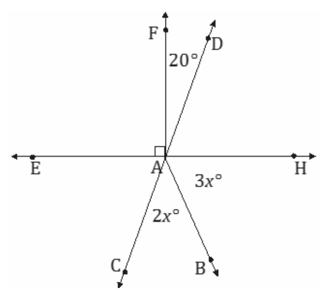


Adjacent angles $x°$ and $15°$ together are vertically opposite from and are equal to angle $81°$.

$$x + 15 = 81$$
$$x + 15 - 15 = 81 - 15$$
$$x = 66$$

$$m\angle HAJ = 66°$$

5. Find the measures of $\angle HAB$ and $\angle CAB$.

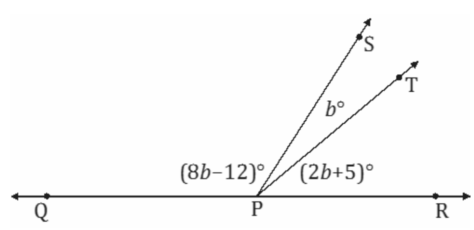


The measures of adjacent angles $\angle CAB$ and $\angle HAB$ have a sum of the measure of $\angle CAH$, which is vertically opposite from and equal to the measurement of $\angle DAE$.

$$2x + 3x + 70 = 180$$
$$5x = 110$$
$$\left(\frac{1}{5}\right)5x = \left(\frac{1}{5}\right)110$$
$$x = 22$$

$$m\angle HAB = 3(22°) = 66°$$
$$m\angle CAB = 2(22°) = 44°$$

6. The measure of $\angle SPT$ is $b°$. The measure of $\angle TPR$ is five more than two times $\angle SPT$. The measure of $\angle QPS$ is twelve less than eight times the measure of $\angle SPT$. Find the measures of $\angle SPT$, $\angle TPR$, and $\angle QPS$.



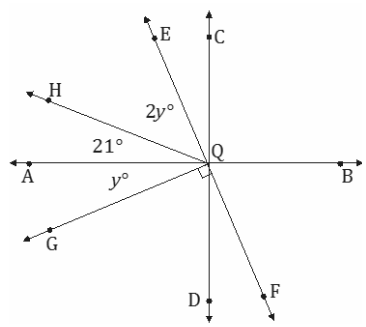
$\angle QPS$, $\angle SPT$, and $\angle TPR$ are angles on a line and their measures have a sum of $180°$.

$$(8b - 12) + b + (2b + 5) = 180$$
$$11b - 7 = 180$$
$$11b - 7 + 7 = 180 + 7$$
$$11b = 187$$
$$\left(\frac{1}{11}\right)11b = \left(\frac{1}{11}\right)187$$
$$b = 17$$

$$m\angle SPT = (17°) = 17°$$
$$m\angle TPR = 2(17°) + 5° = 39°$$
$$m\angle QPS = 8(17°) - 12° = 124°$$

7. **Find the measures of $\angle HQE$ and $\angle AQG$.**

$\angle AQG$, $\angle AQH$, and $\angle HQE$ are adjacent angles whose measures have a sum of $90°$.



$$2y + 21 + y = 90$$
$$3y + 21 = 90$$
$$3y + 21 - 21 = 90 - 21$$
$$3y = 69$$
$$\left(\frac{1}{3}\right)3y = \left(\frac{1}{3}\right)69$$
$$y = 23$$

$m\angle HQE = 2(23°) = 46°$

$m\angle AQG = (23°) = 23°$

8. **The measures of three angles at a point are in the ratio of $2 : 3 : 5$. Find the measures of the angles.**

$\angle A = 2x$, $\angle B = 3x$, $\angle C = 5x$

$$2x + 3x + 5x = 360$$
$$10x = 360$$
$$\left(\frac{1}{10}\right)10x = \left(\frac{1}{10}\right)360$$
$$x = 36$$

$\angle A = 2(36°) = 72°$

$\angle B = 3(36°) = 108°$

$\angle C = 5(36°) = 180°$

9. **The sum of the measures of two adjacent angles is $72°$. The ratio of the smaller angle to the larger angle is $1 : 3$. Find the measures of each angle.**

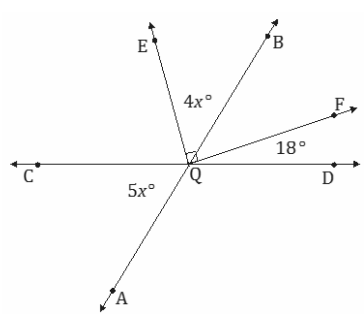
$\angle A = x$, $\angle B = 3x$

$$x + 3x = 72$$
$$4x = 72$$
$$\left(\frac{1}{4}\right)(4x) = \left(\frac{1}{4}\right)(72)$$
$$x = 18$$

$\angle A = (18°) = 18°$

$\angle B = 3(18°) = 54°$

10. **Find the measures of $\angle CQA$ and $\angle EQB$.**



$$4x + 5x = 108$$
$$9x = 108$$
$$\left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)108$$
$$x = 12$$

$m\angle CQA = 5(12°) = 60°$

$m\angle EQB = 4(12°) = 48°$

e. Subtract -3 from both sides.

The inequality symbol is preserved.

$$2 > -4$$
$$2 - (-3) > -4 - (-3)$$
$$5 > -1$$

Problem Set Sample Solutions

1. For each problem, use the properties of inequalities to write a true inequality statement.
The two integers are -2 and -5.

a. Write a true inequality statement.

$-5 < -2$

b. Subtract -2 from each side of the inequality. Write a true inequality statement.

$-7 < -4$

c. Multiply each number by -3. Write a true inequality statement.

$15 > 6$

2. On a recent vacation to the Caribbean, Kay and Tony wanted to explore the ocean elements. One day they went in a submarine 150 feet below sea level. The second day they went scuba diving 75 feet below sea level.

a. Write an inequality comparing the submarine's elevation and the scuba diving elevation.

$-150 < -75$

b. If they only were able to go one-fifth of the capable elevations, write a new inequality to show the elevations they actually achieved.

$-30 < -15$

c. Was the inequality symbol preserved or reversed? Explain.

The inequality symbol was preserved because the number that was multiplied to both sides was NOT negative.

3. If a is a negative integer, then which of the number sentences below is true? If the number sentence is not true, give a reason.

a. $5 + a < 5$

True

b. $5 + a > 5$

False because adding a negative number to 5 will decrease 5, which will not be greater than 5.

c. $5 - a > 5$

True

d. $5 - a < 5$

False because subtracting a negative number is adding a positive number to 5, which will be larger than 5.

e. $5a < 5$

True

f. $5a > 5$

False because a negative number multiplied by a positive number is negative, which will be less than 5.

g. $5 + a > a$

True

h. $5 + a < a$

False because adding 5 to a negative number is greater than the negative number itself.

i. $5 - a > a$

True

j. $5 - a < a$

False because subtracting a negative number is the same as adding a positive number, which is greater than the negative number itself.

k. $5a > a$

False because a negative number multiplied by a 5 is negative and will be 5 times smaller than a.

l. $5a < a$

True

Exit Ticket Sample Solutions

Shaggy earned $\$7.55$ per hour plus an additional $\$100$ in tips waiting tables on Saturday. He earned at least $\$160$ in all. Write an inequality and find the minimum number of hours, to the nearest hour, that Shaggy worked on Saturday.

Let h represent the number of hours worked.

$$7.55h + 100 \geq 160$$
$$7.55h + 100 - 100 \geq 160 - 100$$
$$7.55h \geq 60$$
$$\left(\frac{1}{7.55}\right)(7.55h) \geq \left(\frac{1}{7.55}\right)(60)$$
$$h \geq 7.9$$

If Shaggy earned at least $\$160$, he would have worked at least 8 hours.

Note: The solution shown above is rounded to the nearest tenth. The overall solution, though, is rounded to the nearest hour since that is what the question asks for.

Problem Set Sample Solutions

1. **Match each problem to the inequality that models it. One choice will be used twice.**

c	The sum of three times a number and -4 is greater than 17.	a.	$3x + -4 \geq 17$
b	The sum of three times a number and -4 is less than 17.	b.	$3x + -4 < 17$
d	The sum of three times a number and -4 is at most 17.	c.	$3x + -4 > 17$
d	The sum of three times a number and -4 is no more than 17.	d.	$3x + -4 \leq 17$
a	The sum of three times a number and -4 is at least 17.		

2. **If x represents a positive integer, find the solutions to the following inequalities.**

a. $x < 7$

$x < 7$ *or* $1, 2, 3, 4, 5, 6$

b. $x - 15 < 20$

$x < 35$

c. $x + 3 \leq 15$

$x \leq 12$

d. $-x > 2$

There are no positive integer solutions.

e. $10 - x > 2$

$x < 8$

f. $-x \geq 2$

There are no positive integer solutions.

g. $\frac{x}{3} < 2$

$x < 6$

h. $-\frac{x}{3} > 2$

There are no positive integer solutions.

i. $3 - \frac{x}{4} > 2$

$x < 4$

3. Recall that the symbol $\neq$ means *not equal to*. If x represents a positive integer, state whether each of the following statements is always true, sometimes true, or false.

a. $x > 0$

Always true

b. $x < 0$

False

c. $x > -5$

Always true

d. $x > 1$

Sometimes true

e. $x \geq 1$

Always true

f. $x \neq 0$

Always true

g. $x \neq -1$

Always true

h. $x \neq 5$

Sometimes true

4. Twice the smaller of two consecutive integers increased by the larger integer is at least 25.

Model the problem with an inequality, and determine which of the given values 7, 8, and/or 9 are solutions. Then, find the smallest number that will make the inequality true.

$2x + x + 1 \geq 25$

For $x = 7$:

$$\begin{aligned} 2x + x + 1 &\geq 25 \\ 2(7) + 7 + 1 &\geq 25 \\ 14 + 7 + 1 &\geq 25 \\ 22 &\geq 25 \end{aligned}$$

False

For $x = 8$:

$$\begin{aligned} 2x + x + 1 &\geq 25 \\ 2(8) + 8 + 1 &\geq 25 \\ 16 + 8 + 1 &\geq 25 \\ 25 &\geq 25 \end{aligned}$$

True

For $x = 9$:

$$\begin{aligned} 2x + x + 1 &\geq 25 \\ 2(9) + 9 + 1 &\geq 25 \\ 18 + 9 + 1 &\geq 25 \\ 28 &\geq 25 \end{aligned}$$

True

The smallest integer would be 8.

5.

a. The length of a rectangular fenced enclosure is 12 feet more than the width. If Farmer Dan has 100 feet of fencing, write an inequality to find the dimensions of the rectangle with the largest perimeter that can be created using 100 feet of fencing.

Let w *represent the width of the fenced enclosure.*

$w + 12$: *length of the fenced enclosure*

$$\begin{aligned} w + w + w + 12 + w + 12 &\leq 100 \\ 4w + 24 &\leq 100 \end{aligned}$$

b. **What are the dimensions of the rectangle with the largest perimeter? What is the area enclosed by this rectangle?**

$$4w + 24 \leq 100$$
$$4w + 24 - 24 \leq 100 - 24$$
$$4w + 0 \leq 76$$
$$\left(\frac{1}{4}\right)(4w) \leq \left(\frac{1}{4}\right)(76)$$
$$w \leq 19$$

Maximum width is 19 feet.

Maximum length is 31 feet.

Maximum area: $A = lw$

$A = (19)(31)$

$A = 589$

The area is 589 ft^2.

6. **At most, Kyle can spend $\$50$ on sandwiches and chips for a picnic. He already bought chips for $\$6$ and will buy sandwiches that cost $\$4.50$ each. Write and solve an inequality to show how many sandwiches he can buy. Show your work, and interpret your solution.**

Let s represent the number of sandwiches.

$$4.50s + 6 \leq 50$$
$$4.50s + 6 - 6 \leq 50 - 6$$
$$4.50s \leq 44$$
$$\left(\frac{1}{4.50}\right)(4.50s) \leq \left(\frac{1}{4.50}\right)(44)$$
$$s \leq 9\frac{7}{9}$$

At most, Kyle can buy 9 sandwiches with $\$50$.

Exit Ticket Sample Solutions

Games at the carnival cost $\$3$ each. The prizes awarded to winners cost $\$145.65$. How many games must be played to make at least $\$50$?

Let g represent the number of games played.

$$3g - 145.65 \geq 50$$
$$3g - 145.65 + 145.65 \geq 50 + 145.65$$
$$3g + 0 \geq 195.65$$
$$\left(\frac{1}{3}\right)(3g) \geq \left(\frac{1}{3}\right)(195.65)$$
$$g \geq 65.217$$

There must be at least 66 games played to make at least $\$50$.

Problem Set Sample Solutions

1. As a salesperson, Jonathan is paid $\$50$ per week plus 3% of the total amount he sells. This week, he wants to earn at least $\$100$. Write an inequality with integer coefficients for the total sales needed to earn at least $\$100$, and describe what the solution represents.

   Let the variable p represent the purchase amount.

$$50 + \frac{3}{100}p \geq 100$$
$$\frac{3}{100}p + 50 \geq 100$$
$$(100)\left(\frac{3}{100}p\right) + 100(50) \geq 100(100)$$
$$3p + 5000 \geq 10000$$
$$3p + 5000 - 5000 \geq 10000 - 5000$$
$$3p + 0 \geq 5000$$
$$\left(\frac{1}{3}\right)(3p) \geq \left(\frac{1}{3}\right)(5000)$$
$$p \geq 1666\frac{2}{3}$$

   Jonathan must sell $\$1,666.67$ in total purchases.

2. Systolic blood pressure is the higher number in a blood pressure reading. It is measured as the heart muscle contracts. Heather was with her grandfather when he had his blood pressure checked. The nurse told him that the upper limit of his systolic blood pressure is equal to half his age increased by 110.

   a. a is the age in years, and p is the systolic blood pressure in millimeters of mercury (mmHg). Write an inequality to represent this situation.

   $$p \leq \frac{1}{2}a + 110$$

b. Heather's grandfather is 76 years old. What is *normal* for his systolic blood pressure?

$p \le \frac{1}{2}a + 110$, where $a = 76$.

$$p \le \frac{1}{2}(76) + 110$$
$$p \le 38 + 110$$
$$p \le 148$$

A systolic blood pressure for his age is normal if it is at most 148 $mmHG$.

3. Traci collects donations for a dance marathon. One group of sponsors will donate a total of $\$6$ for each hour she dances. Another group of sponsors will donate $\$75$ no matter how long she dances. What number of hours, to the nearest minute, should Traci dance if she wants to raise at least $\$1,000$?

Let the variable h represent the number of hours Traci dances.

$$6h + 75 \ge 1000$$
$$6h + 75 - 75 \ge 1000 - 75$$
$$6h + 0 \ge 925$$
$$\left(\frac{1}{6}\right)(6h) \ge \left(\frac{1}{6}\right)(925)$$
$$h \ge 154\frac{1}{6}$$

Traci would have to dance at least 154 hours and 10 minutes.

4. Jack's age is three years more than twice the age of his younger brother, Jimmy. If the sum of their ages is at most 18, find the greatest age that Jimmy could be.

Let the variable j represent Jimmy's age in years.

Then, the expression $3 + 2j$ represents Jack's age in years.

$$j + 3 + 2j \le 18$$
$$3j + 3 \le 18$$
$$3j + 3 - 3 \le 18 - 3$$
$$3j \le 15$$
$$\left(\frac{1}{3}\right)(3j) \le \left(\frac{1}{3}\right)(15)$$
$$j \le 5$$

Jimmy's age is 5 years or less.

5. Brenda has $\$500$ in her bank account. Every week she withdraws $\$40$ for miscellaneous expenses. How many weeks can she withdraw the money if she wants to maintain a balance of a least $\$200$?

Let the variable w represent the number of weeks.

$$500 - 40w \ge 200$$
$$500 - 500 - 40w \ge 200 - 500$$
$$-40w \ge -300$$
$$\left(-\frac{1}{40}\right)(-40w) \le \left(-\frac{1}{40}\right)(-300)$$
$$w \le 7.5$$

$\$40$ can be withdrawn from the account for seven weeks if she wants to maintain a balance of at least $\$200$.

6. A scooter travels 10 miles per hour faster than an electric bicycle. The scooter traveled for 3 hours, and the bicycle traveled for $5\frac{1}{2}$ hours. Altogether, the scooter and bicycle traveled no more than 285 miles. Find the maximum speed of each.

	Speed	*Time*	*Distance*
Scooter	$x+10$	3	$3(x+10)$
Bicycle	x	$5\frac{1}{2}$	$5\frac{1}{2}x$

$$3(x+10)+5\frac{1}{2}x \leq 285$$
$$3x+30+5\frac{1}{2}x \leq 285$$
$$8\frac{1}{2}x+30 \leq 285$$
$$8\frac{1}{2}x+30-30 \leq 285-30$$
$$8\frac{1}{2}x \leq 255$$
$$\frac{17}{2}x \leq 255$$
$$\left(\frac{2}{17}\right)\left(\frac{17}{2}x\right) \leq (255)\left(\frac{2}{17}\right)$$
$$x \leq 30$$

The maximum speed the bicycle traveled was 30 miles per hour, and the maximum speed the scooter traveled was 40 miles per hour.

Problem Set Sample Solutions

1. Ben has agreed to play fewer video games and spend more time studying. He has agreed to play less than 10 hours of video games each week. On Monday through Thursday, he plays video games for a total of $5\frac{1}{2}$ hours. For the remaining 3 days, he plays video games for the same amount of time each day. Find t, the amount of time he plays video games for each of the 3 days. Graph your solution.

   Let t represent the time in hours spent playing video games.

$$3t + 5\frac{1}{2} < 10$$
$$3t + 5\frac{1}{2} - 5\frac{1}{2} < 10 - 5\frac{1}{2}$$
$$3t + 0 < 4\frac{1}{2}$$
$$\left(\frac{1}{3}\right)(3t) < \left(\frac{1}{3}\right)\left(4\frac{1}{2}\right)$$
$$t < 1.5$$

   Graph:

   Ben plays less than 1.5 hours of video games each of the three days.

2. Gary's contract states that he must work more than 20 hours per week. The graph below represents the number of hours he can work in a week.

   a. Write an algebraic inequality that represents the number of hours, h, Gary can work in a week.

      $h > 20$

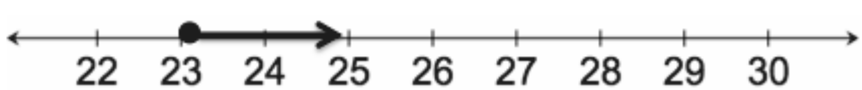
   b. Gary is paid $\$15.50$ per hour in addition to a weekly salary of $\$50$. This week he wants to earn more than $\$400$. Write an inequality to represent this situation.

      $15.50h + 50 > 400$

   c. Solve and graph the solution from part (b). Round your answer to the nearest hour.

$$15.50h + 50 - 50 > 400 - 50$$
$$15.50h > 350$$
$$\left(\frac{1}{15.50}\right)(15.50h) > 350\left(\frac{1}{15.50}\right)$$
$$h > 22.58$$

      Gary has to work 23 or more hours to earn more than $\$400$.



3. Sally's bank account has $\$650$ in it. Every week, Sally withdraws $\$50$ to pay for her dog sitter. What is the maximum number of weeks that Sally can withdraw the money so there is at least $\$75$ remaining in the account? Write and solve an inequality to find the solution, and graph the solution on a number line.

Let w represent the number of weeks Sally can withdraw the money.

$$650 - 50w \geq 75$$
$$650 - 50w - 650 \geq 75 - 650$$
$$-50w \geq -575$$
$$\left(\frac{1}{-50}\right)(-50w) \geq \left(\frac{1}{-50}\right)(-575)$$
$$w \leq 11.5$$

The maximum number of weeks Sally can withdraw the weekly dog sitter fee is 11 weeks.

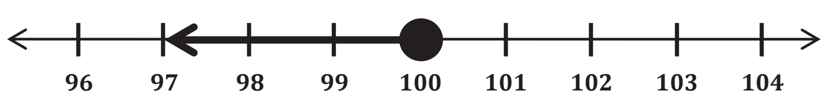
4. On a cruise ship, there are two options for an Internet connection. The first option is a fee of $\$5$ plus an additional $\$0.25$ per minute. The second option costs $\$50$ for an unlimited number of minutes. For how many minutes, m, is the first option cheaper than the second option? Graph the solution.

Let m represent the number of minutes of Internet connection.

$$5 + 0.25m < 50$$
$$5 + 0.25m - 5 < 50 - 5$$
$$0.25m + 0 < 45$$
$$\left(\frac{1}{0.25}\right)(0.25m) < \left(\frac{1}{0.25}\right)(45)$$
$$m < 180$$

If there are less than 180 minutes, or 3 hours, used on the Internet, then the first option would be cheaper. If 180 minutes or more are planned, then the second option is more economical.

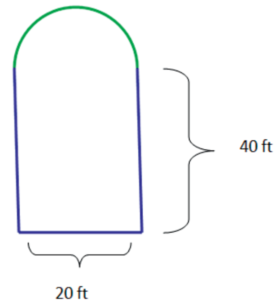
5. The length of a rectangle is 100 centimeters, and its perimeter is greater than 400 centimeters. Henry writes an inequality and graphs the solution below to find the width of the rectangle. Is he correct? If yes, write and solve the inequality to represent the problem and graph. If no, explain the error(s) Henry made.



Henry's graph is incorrect. The inequality should be $2(100) + 2w > 400$. When you solve the inequality, you get $w > 100$. The circle on 100 on the number line is correct; however, the circle should be an open circle since the perimeter is not equal to 400. Also, the arrow should be pointing in the opposite direction because the perimeter is greater than 400, which means the width is greater than 100. The given graph indicates an inequality of less than or equal to.

Exit Ticket Sample Solutions

Brianna's parents built a swimming pool in the backyard. Brianna says that the distance around the pool is 120 feet.



1. Is she correct? Explain why or why not.

   Brianna is incorrect. The distance around the pool is 131.4 ft. She found the distance around the rectangle only and did not include the distance around the semicircular part of the pool.

2. Explain how Brianna would determine the distance around the pool so that her parents would know how many feet of stone to buy for the edging around the pool.

   In order to find the distance around the pool, Brianna must first find the circumference of the semicircle, which is $C = \frac{1}{2} \cdot \pi \cdot 20$ ft., or 10π ft., or about 31.4 ft. The sum of the three other sides is $20\text{ ft.} + 40\text{ ft.} + 40\text{ ft.} = 100\text{ ft.}$; the perimeter is $100\text{ ft.} + 31.4\text{ ft.} = 131.4\text{ ft.}$

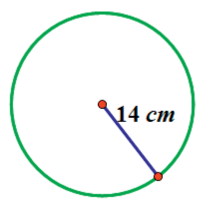
3. Explain the relationship between the circumference of the semicircular part of the pool and the width of the pool.

   The relationship between the circumference of the semicircular part and the width of the pool is the same as half of π because this is half the circumference of the entire circle.

Problem Set Sample Solutions

Students should work in cooperative groups to complete the tasks for this exercise.

1. Find the circumference.



   a. Give an exact answer in terms of π.

   $C = 2\pi r$
   $C = 2\pi \cdot 14\text{ cm}$
   $C = 28\pi\text{ cm}$

   b. Use $\pi \approx \frac{22}{7}$, and express your answer as a fraction in lowest terms.

   $C \approx 2 \cdot \frac{22}{7} \cdot 14\text{ cm}$
   $C \approx 88\text{ cm}$

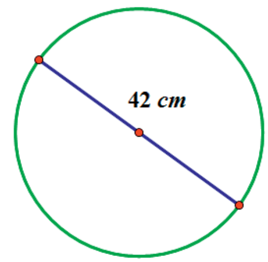
   c. Use the π button on your calculator, and express your answer to the nearest hundredth.

   $C = 2 \cdot \pi \cdot 14\text{ cm}$

   $C \approx 87.96\text{ cm}$

2. Find the circumference.

   a. Give an exact answer in terms of π.



$d = 42 \text{ cm}$

$C = \pi d$

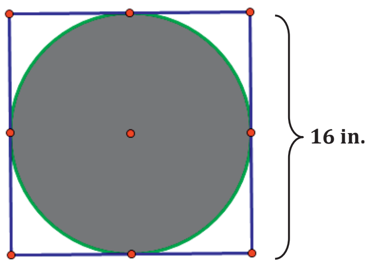
$C = 42\pi \text{ cm}$

   b. Use $\pi \approx \frac{22}{7}$, and express your answer as a fraction in lowest terms.

$C \approx 42 \text{ cm} \cdot \frac{22}{7}$

$C \approx 132 \text{ cm}$

3. The figure shows a circle within a square. Find the circumference of the circle. Let $\pi \approx 3.14$.



The diameter of the circle is the same as the length of the side of the square.

$C = \pi d$

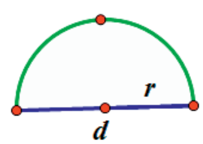
$C = \pi \cdot 16 \text{ in.}$

$C \approx 3.14 \cdot 16 \text{ in.}$

$C \approx 50.24 \text{ in.}$

4. Consider the diagram of a semicircle shown.

   a. Explain in words how to determine the perimeter of a semicircle.



The perimeter is the sum of the length of the diameter and half of the circumference of a circle with the same diameter.

   b. Using d to represent the diameter of the circle, write an algebraic equation that will result in the perimeter of a semicircle.

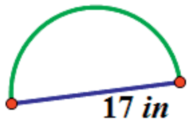
$P = d + \frac{1}{2}\pi d$

   c. Write another algebraic equation to represent the perimeter of a semicircle using r to represent the radius of a semicircle.

$P = 2r + \frac{1}{2}\pi \cdot 2r$

$P = 2r + \pi r$

5. Find the perimeter of the semicircle. Let $\pi \approx 3.14$.



$P = d + \frac{1}{2}\pi d$

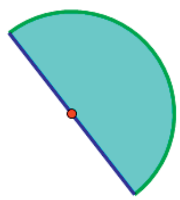
$P \approx 17 \text{ in.} + \frac{1}{2} \cdot 3.14 \cdot 17 \text{ in.}$

$P \approx 17 \text{ in.} + 26.69 \text{ in.}$

$P \approx 43.69 \text{ in.}$

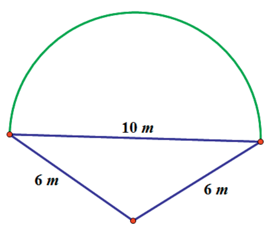
6. Ken's landscape gardening business makes odd-shaped lawns that include semicircles. Find the length of the edging material needed to border the two lawn designs. Use 3.14 for π.

a. The radius of this flowerbed is 2.5 m.



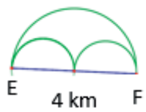
A semicircle has half of the circumference of a circle. If the circumference of the semicircle is $C = \frac{1}{2}(\pi \cdot 2 \cdot 2.5 \text{ m})$, then the circumference approximates 7.85 m. The length of the edging material must include the circumference and the diameter; $7.85 \text{ m} + 5 \text{ m} = 12.85 \text{ m}$. Ken needs 12.85 meters of edging to complete his design.

b. The diameter of the semicircular section is 10 m, and the lengths of the two sides are 6 m.



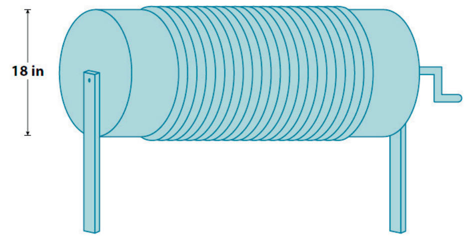
The circumference of the semicircular part has half of the circumference of a circle. The circumference of the semicircle is $C = \frac{1}{2}\pi \cdot 10 \text{ m}$, which is approximately 15.7 m. The length of the edging material must include the circumference of the semicircle and the perimeter of two sides of the triangle; $15.7 \text{ m} + 6 \text{ m} + 6 \text{ m} = 27.7 \text{ m}$. Ken needs 27.7 meters of edging to complete his design.

7. Mary and Margaret are looking at a map of a running path in a local park. Which is the shorter path from E to F, along the two semicircles or along the larger semicircle? If one path is shorter, how much shorter is it? Let $\pi \approx 3.14$.



A semicircle has half of the circumference of a circle. The circumference of the large semicircle is $C = \frac{1}{2}\pi \cdot 4 \text{ km}$, or 6.28 km. The diameter of the two smaller semicircles is 2 km. The total circumference would be the same as the circumference for a whole circle with the same diameter. If $C = \pi \cdot 2 \text{ km}$, then $C = 6.28$ km. The distance around the larger semicircle is the same as the distance around both of the semicircles. So, both paths are equal in distance.

8. Alex the electrician needs 34 yards of electrical wire to complete a job. He has a coil of wiring in his workshop. The coiled wire is 18 inches in diameter and is made up of 21 circles of wire. Will this coil be enough to complete the job? Let $\pi \approx 3.14$.



The circumference of the coil of wire is $C = \pi \cdot 18 \text{ in.}$, or approximately 56.52 in. If there are 21 circles of wire, then the number of circles times the circumference will yield the total number of inches of wire in the coil. If $56.52 \text{ in.} \cdot 21 \approx 1186.92 \text{ in.}$, then $\frac{1186.92 \text{ in.}}{36 \text{ in.}} \approx 32.97 \text{ yd.}$ ($1 \text{ yd.} = 3 \text{ ft.} = 36 \text{ in.}$ When converting inches to yards, you must divide the total inches by the number of inches in a yard, which is 36 inches.) Alex will not have enough wire for his job in this coil of wire.

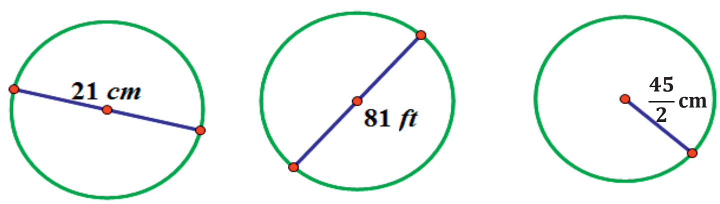
Exit Ticket Sample Solutions

Complete each statement using the words or algebraic expressions listed in the word bank below.

1. The length of the *height* of the rectangular region approximates the length of the *radius* of the circle.
2. The *base* of the rectangle approximates the length of one-half of the circumference of the circle.
3. The circumference of the circle is $2\pi r$.
4. The *diameter* of the *circle* is $2r$.
5. The ratio of the circumference to the diameter is π.
6. Area (circle) $=$ Area of (*rectangle*) $= \frac{1}{2} \cdot \text{circumference} \cdot r = \frac{1}{2}(2\pi r) \cdot r = \pi \cdot r \cdot r = \pi r^2$.

Problem Set Sample Solutions

1. The following circles are not drawn to scale. Find the area of each circle. (Use $\frac{22}{7}$ as an approximation for π.)



$346.5\ \text{cm}^2$ $\quad$ $5{,}155.1\ \text{ft}^2$ $\quad$ $1{,}591.1\ \text{cm}^2$

2. A circle has a diameter of 20 inches.
   a. Find the exact area, and find an approximate area using $\pi \approx 3.14$.

   If the diameter is 20 in., *then the radius is* 10 in. *If* $A = \pi r^2$, *then* $A = \pi \cdot (10\ \text{in.})^2$ *or* $100\pi\ \text{in}^2$.
   $A \approx (100 \cdot 3.14)\ \text{in}^2 \approx 314\ \text{in}^2$.

   b. What is the circumference of the circle using $\pi \approx 3.14$?

   If the diameter is 20 in., *then the circumference is* $C = \pi d$ *or* $C \approx 3.14 \cdot 20\ \text{in.} \approx 62.8\ \text{in.}$

3. A circle has a diameter of 11 inches.
   a. Find the exact area and an approximate area using $\pi \approx 3.14$.

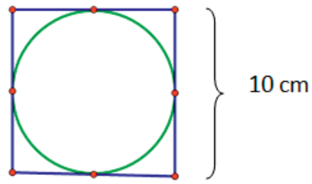
   If the diameter is 11 in., *then the radius is* $\frac{11}{2}$ in. *If* $A = \pi r^2$, *then* $A = \pi \cdot \left(\frac{11}{2}\ \text{in.}\right)^2$ *or* $\frac{121}{4}\pi\ \text{in}^2$.

   $$A \approx \left(\frac{121}{4} \cdot 3.14\right)\text{in}^2 \approx 94.985\ \text{in}^2$$

   b. What is the circumference of the circle using $\pi \approx 3.14$?

   If the diameter is 11 *inches, then the circumference is* $C = \pi d$ *or* $C \approx 3.14 \cdot 11\ \text{in.} \approx 34.54\ \text{in.}$

4. **Using the figure below, find the area of the circle.**



In this circle, the diameter is the same as the length of the side of the square. The diameter is 10 cm; so, the radius is 5 cm. $A = \pi r^2$, so $A = \pi(5 \text{ cm})^2 = 25\pi \text{ cm}^2$.

5. **A path bounds a circular lawn at a park. If the inner edge of the path is 132 ft. around, approximate the amount of area of the lawn inside the circular path. Use $\pi \approx \frac{22}{7}$.**

The length of the path is the same as the circumference. Find the radius from the circumference; then, find the area.

$$C = 2\pi r$$
$$132 \text{ ft.} \approx 2 \cdot \frac{22}{7} \cdot r$$
$$132 \text{ ft.} \approx \frac{44}{7} r$$
$$\frac{7}{44} \cdot 132 \text{ ft.} \approx \frac{7}{44} \cdot \frac{44}{7} r$$
$$21 \text{ ft.} \approx r$$

$$A \approx \frac{22}{7} \cdot (21 \text{ ft.})^2$$
$$A \approx 1386 \text{ ft}^2$$

6. **The area of a circle is $36\pi \text{ cm}^2$. Find its circumference.**

Find the radius from the area of the circle; then, use it to find the circumference.

$$A = \pi r^2$$
$$36\pi \text{ cm}^2 = \pi r^2$$
$$\frac{1}{\pi} \cdot 36\pi \text{ cm}^2 = \frac{1}{\pi} \cdot \pi r^2$$
$$36 \text{ cm}^2 = r^2$$
$$6 \text{ cm} = r$$

$$C = 2\pi r$$
$$C = 2\pi \cdot 6 \text{ cm}$$
$$C = 12\pi \text{ cm}$$

7. **Find the ratio of the area of two circles with radii 3 cm and 4 cm.**

The area of the circle with radius 3 cm is $9\pi \text{ cm}^2$. The area of the circle with the radius 4 cm is $16\pi \text{ cm}^2$. The ratio of the area of the two circles is $9\pi : 16\pi$ or $9 : 16$.

8. **If one circle has a diameter of 10 cm and a second circle has a diameter of 20 cm, what is the ratio of the area of the larger circle to the area of the smaller circle?**

The area of the circle with the diameter of 10 cm has a radius of 5 cm. The area of the circle with the diameter of 10 cm is $\pi \cdot (5 \text{ cm})^2$, or $25\pi \text{ cm}^2$. The area of the circle with the diameter of 20 cm has a radius of 10 cm. The area of the circle with the diameter of 20 cm is $\pi \cdot (10 \text{ cm})^2$ or $100\pi \text{ cm}^2$. The ratio of the diameters is 20 to 10 or $2:1$, while the ratio of the areas is 100π to 25π or $4:1$.

9. **Describe a rectangle whose perimeter is 132 ft. and whose area is less than 1 ft^2. Is it possible to find a circle whose circumference is 132 ft. and whose area is less than 1 ft^2? If not, provide an example or write a sentence explaining why no such circle exists.**

A rectangle that has a perimeter of 132 ft. can have a length of 65.995 ft. and a width of 0.005 ft. The area of such a rectangle is 0.329975 ft^2, which is less than 1 ft^2. No, because a circle that has a circumference of 132 ft. has a radius of approximately 21 ft.

$A = \pi r^2 = \pi(21)^2 = 1387.96 \neq 1$

10. **If the diameter of a circle is double the diameter of a second circle, what is the ratio of the area of the first circle to the area of the second?**

If I choose a diameter of 24 cm for the first circle, then the diameter of the second circle is 12 cm. The first circle has a radius of 12 cm and an area of $144\pi \text{ cm}^2$. The second circle has a radius of 6 cm and an area of $36\pi \text{ cm}^2$. The ratio of the area of the first circle to the second is 144π to 36π, which is a 4 to 1 ratio. The ratio of the diameters is 2, while the ratio of the areas is the square of 2, or 4.

Exit Ticket Sample Solutions

1. Ken's landscape gardening business creates odd-shaped lawns that include semicircles. Find the area of this semicircular section of the lawn in this design. Use $\frac{22}{7}$ for π.

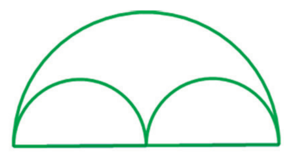
If the diameter is 5 m, then the radius is $\frac{5}{2}$ m. Using the formula for area of a semicircle,

$A = \frac{1}{2}\pi r^2$, $A \approx \frac{1}{2} \cdot \frac{22}{7} \cdot \left(\frac{5}{2} \text{ m}\right)^2$. *Using the order of operations,*

$A \approx \frac{1}{2} \cdot \frac{22}{7} \cdot \frac{25}{4} \text{ m}^2 \approx \frac{550}{56} \text{ m}^2 \approx 9.8 \text{ m}^2$.

2. In the figure below, Ken's company has placed sprinkler heads at the center of the two small semicircles. The radius of the sprinklers is 5 ft. If the area in the larger semicircular area is the shape of the entire lawn, how much of the lawn will not be watered? Give your answer in terms of π and to the nearest tenth. Explain your thinking.

The area not covered by the sprinklers would be the area between the larger semicircle and the two smaller ones. The area for the two semicircles is the same as the area of one circle with the same radius of 5 ft. The area not covered by the sprinklers can be found by subtracting the area of the two smaller semicircles from the area of the large semicircle.



$$\text{Area Not Covered} = \text{Area of large semicircle} - \text{Area of two smaller semicircles}$$

$$A = \frac{1}{2}\pi \cdot (10 \text{ ft})^2 - \left(2 \cdot \left(\frac{1}{2}(\pi \cdot (5 \text{ ft})^2)\right)\right)$$

$$A = \frac{1}{2}\pi \cdot 100 \text{ ft}^2 - \pi \cdot 25 \text{ ft}^2$$

$$A = 50\pi \text{ ft}^2 - 25\pi \text{ ft}^2 = 25\pi \text{ ft}^2$$

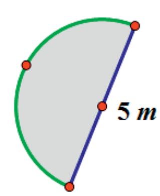
Let $\pi \approx 3.14$

$A \approx 78.5 \text{ ft}^2$

The sprinklers will not cover $25\pi \text{ ft}^2$ or 78.5 ft^2 of the lawn.

Problem Set Sample Solutions

1. Mark created a flower bed that is semicircular in shape. The diameter of the flower bed is 5 m.



   a. What is the perimeter of the flower bed? (Approximate π to be 3.14.)

   The perimeter of this flower bed is the sum of the diameter and one-half the circumference of a circle with the same diameter.

   $P = \text{diameter} + \frac{1}{2}\pi \cdot \text{diameter}$

   $P \approx 5 \text{ m} + \frac{1}{2} \cdot 3.14 \cdot 5 \text{ m}$

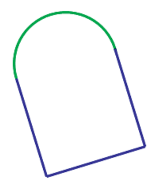
   $P \approx 12.85 \text{ m}$

b. What is the area of the flower bed? (Approximate π to be 3.14.)

$$A = \frac{1}{2}\pi\,(2.5\text{ m})^2$$
$$A = \frac{1}{2}\pi\,(6.25\text{ m}^2)$$
$$A \approx 0.5 \cdot 3.14 \cdot 6.25\text{ m}^2$$
$$A \approx 9.8\text{ m}^2$$

2. A landscape designer wants to include a semicircular patio at the end of a square sandbox. She knows that the area of the semicircular patio is 25.12 cm^2.

a. Draw a picture to represent this situation.



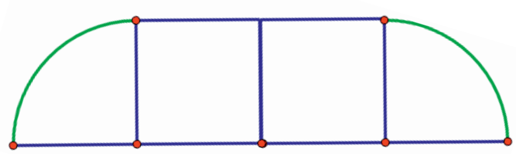
b. What is the length of the side of the square?

If the area of the patio is 25.12 cm^2, then we can find the radius by solving the equation $A = \frac{1}{2}\pi r^2$ and substituting the information that we know. If we approximate π to be 3.14 and solve for the radius, r, then

$$25.12\text{ cm}^2 \approx \frac{1}{2}\pi r^2$$
$$\frac{2}{1}\cdot 25.12\text{ cm}^2 \approx \frac{2}{1}\cdot\frac{1}{2}\pi r^2$$
$$50.24\text{ cm}^2 \approx 3.14r^2$$
$$\frac{1}{3.14}\cdot 50.24\text{ cm}^2 \approx \frac{1}{3.14}\cdot 3.14r^2$$
$$16\text{ cm}^2 \approx r^2$$
$$4\text{ cm} \approx r$$

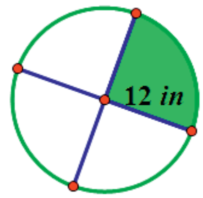
The length of the diameter is 8 cm; therefore, the length of the side of the square is 8 cm.

3. A window manufacturer designed a set of windows for the top of a two-story wall. If the window is comprised of 2 squares and 2 quarter circles on each end, and if the length of the span of windows across the bottom is 12 feet, approximately how much glass will be needed to complete the set of windows?



The area of the windows is the sum of the areas of the two quarter circles and the two squares that make up the bank of windows. If the span of windows is 12 feet across the bottom, then each window is 3 feet wide on the bottom. The radius of the quarter circles is 3 feet, so the area for one quarter circle window is $A = \frac{1}{4}\pi\cdot(3\text{ ft})^2$, or $A \approx 7.065\text{ ft}^2$. The area of one square window is $A = (3\text{ ft})^2$, or 9 ft^2. The total area is $A = 2(\text{area of quarter circle}) + 2(\text{area of square})$, or $A \approx (2\cdot 7.065\text{ ft}^2) + (2\cdot 9\text{ ft}^2) \approx 32.13\text{ ft}^2$.

4. Find the area of the shaded region. (Approximate π to be $\frac{22}{7}$.)



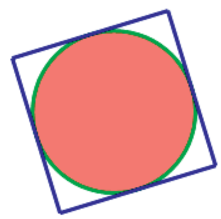
$A = \frac{1}{4}\pi(12 \text{ in})^2$

$A = \frac{1}{4}\pi \cdot 144 \text{ in}^2$

$A \approx \frac{1}{4} \cdot \frac{22}{7} \cdot 144 \text{ in}^2$

$A \approx \frac{792}{7} \text{in}^2$ *or* 113.1 in^2

5. The figure below shows a circle inside of a square. If the radius of the circle is 8 cm, find the following and explain your solution.



a. The circumference of the circle

$C = 2\pi \cdot 8 \text{ cm}$

$C = 16\pi \text{ cm}$

b. The area of the circle

$A = \pi \cdot (8 \text{ cm})^2$

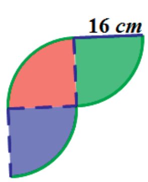
$A = 64\,\pi \text{ cm}^2$

c. The area of the square

$A = 16 \text{ cm} \cdot 16 \text{ cm}$

$A = 256 \text{ cm}^2$

6. Michael wants to create a tile pattern out of three quarter circles for his kitchen backsplash. He will repeat the three quarter circles throughout the pattern. Find the area of the tile pattern that Michael will use. Approximate π as 3.14.



There are three quarter circles in the tile design. The area of one quarter circle multiplied by 3 *will result in the total area.*

$A = \frac{1}{4}\pi \cdot (16 \text{ cm})^2$

$A \approx \frac{1}{4} \cdot 3.14 \cdot 256 \text{ cm}^2$

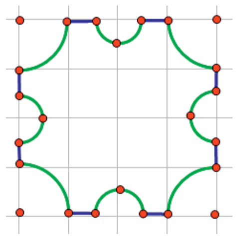
$A \approx 200.96 \text{ cm}^2$

$A \approx 3 \cdot 200.96 \text{ cm}^2$

$A \approx 602.88 \text{ cm}^2$

The area of the tile pattern is approximately 602.88 cm^2.

7. A machine shop has a square metal plate with sides that measure 4 cm each. A machinist must cut four semicircles with a radius of $\frac{1}{2}$ cm and four quarter circles with a radius of 1 cm from its sides and corners. What is the area of the plate formed? Use $\frac{22}{7}$ to approximate π.



The area of the metal plate is determined by subtracting the four quarter circles (corners) and the four half-circles (on each side) from the area of the square. Area of the square: $A = (4\text{ cm})^2 = 16\text{ cm}^2$.

The area of four quarter circles is the same as the area of a circle with a radius of 1 cm*:* $A \approx \frac{22}{7}(1\text{ cm})^2 \approx \frac{22}{7}\text{ cm}^2$.

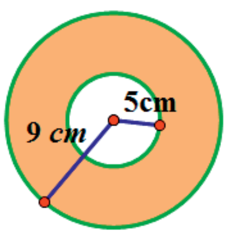
The area of the four semicircles with radius $\frac{1}{2}$ cm *is*

$$A \approx 4 \cdot \frac{1}{2} \cdot \frac{22}{7} \cdot \left(\frac{1}{2}\text{ cm}\right)^2$$
$$A \approx 4 \cdot \frac{1}{2} \cdot \frac{22}{7} \cdot \frac{1}{4}\text{ cm}^2 \approx \frac{11}{7}\text{ cm}^2.$$

The area of the metal plate is

$$A \approx 16\text{ cm}^2 - \frac{22}{7}\text{ cm}^2 - \frac{11}{7}\text{ cm}^2 \approx \frac{79}{7}\text{ cm}^2$$

8. A graphic artist is designing a company logo with two concentric circles (two circles that share the same center but have different radii). The artist needs to know the area of the shaded band between the two concentric circles. Explain to the artist how he would go about finding the area of the shaded region.



The artist should find the areas of both the larger and smaller circles. Then, the artist should subtract the area of the smaller circle from the area of the larger circle to find the area between the two circles. The area of the larger circle is

$A = \pi \cdot (9\text{ cm})^2$ *or* $81\pi\text{ cm}^2$.

The area of the smaller circle is

$A = \pi(5\text{ cm})^2$ *or* $25\pi\text{ cm}^2$.

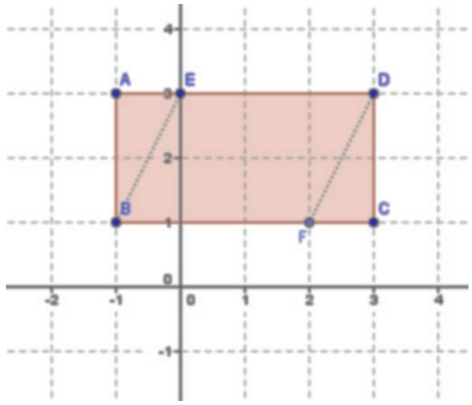
The area of the region between the circles is $81\pi\text{ cm}^2 - 25\pi\text{ cm}^2 = 56\pi\text{ cm}^2$. *If we approximate* π *to be* 3.14, *then* $A \approx 175.84\text{ cm}^2$.

9. Create your own shape made up of rectangles, squares, circles, or semicircles, and determine the area and perimeter.

Student answers may vary.

Exit Ticket Sample Solutions

The figure $ABCD$ is a rectangle. $AB = 2$ units, $AD = 4$ units, and $AE = FC = 1$ unit.



1. Find the area of rectangle $ABCD$.

   Area $= 4$ units $\times$ 2 units $= 8$ sq. units

2. Find the area of triangle ABE.

   Area $= \frac{1}{2} \times 1$ unit $\times$ 2 units $= 1$ sq. unit

3. Find the area of triangle DCF.

   Area $= \frac{1}{2} \times 1$ unit $\times$ 2 units $= 1$ sq. unit

4. Find the area of the parallelogram $BEDF$ two different ways.

   Area $=$ Area of $ABCD$ – Area of ABE – Area of DCF

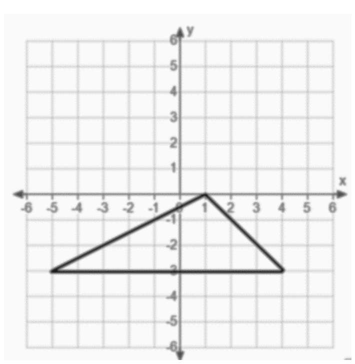
   $= (8 - 1 - 1)$ sq. units $= 6$ sq. units

   Area $=$ base $\times$ height

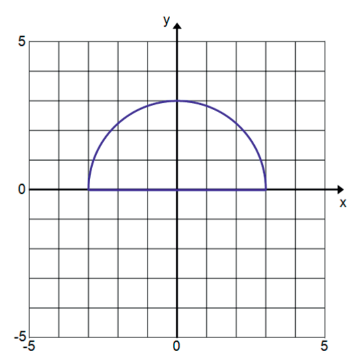
   $= 3$ units $\times$ 2 units $= 6$ sq. units

Problem Set Sample Solutions

Find the area of each figure.

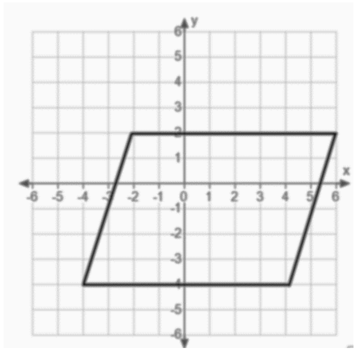
1. 

   Area $= 13.5$ sq. units

2. 

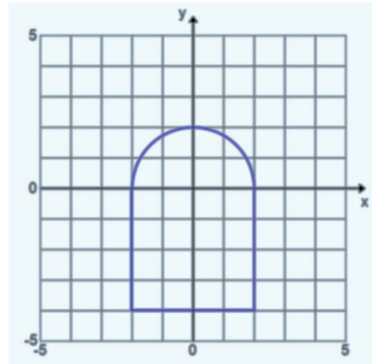
   Area $= 4.5\pi$ sq. units ≈ 14.13 sq. units

3.



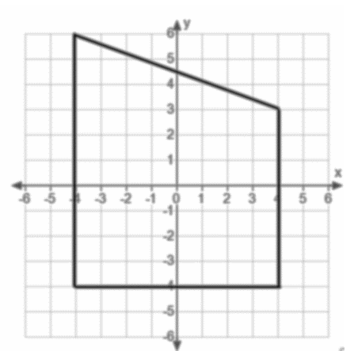
Area $= 48$ sq. units

4.



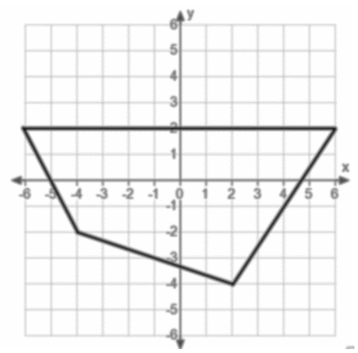
Area $= (2\pi + 16)$ sq. units ≈ 22.28 sq. units

5.



Area $= 68$ sq. units

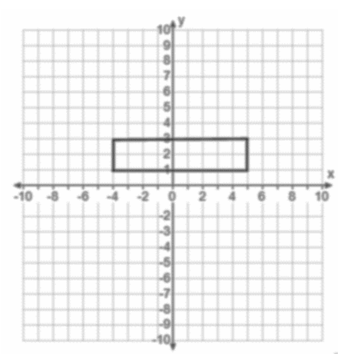
6.



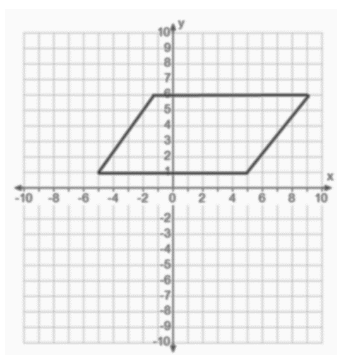
Area $= 46$ sq. units

For Problems 7–9, draw a figure in the coordinate plane that matches each description.

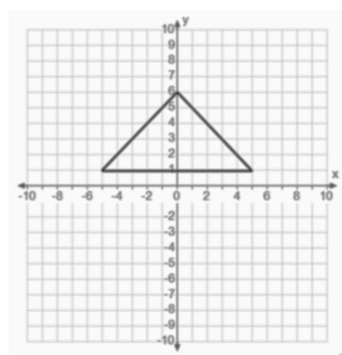
7. A rectangle with an area of 18 sq. units



8. A parallelogram with an area of 50 sq. units

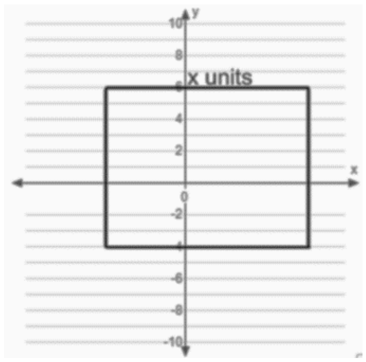


9. A triangle with an area of 25 sq. units



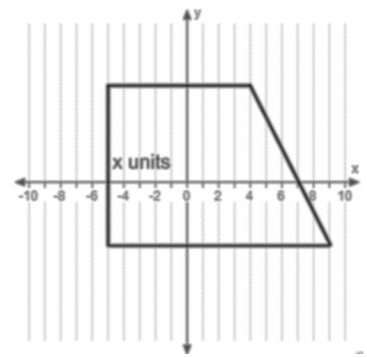
Find the unknown value labled as x on each figure.

10. The rectangle has an area of 80 sq. units.



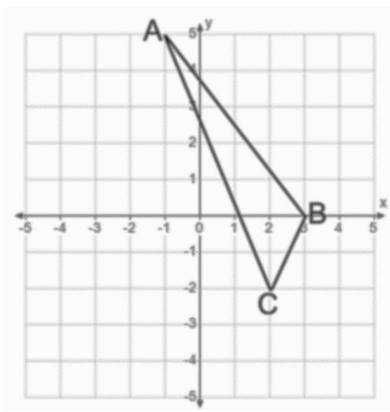
$x = 8$

11. The trapezoid has an area of 115 sq. units.



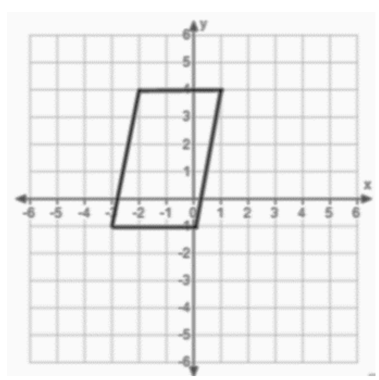
$x = 10$

12. Find the area of triangle ABC.



Area $= 6.5$ sq. units

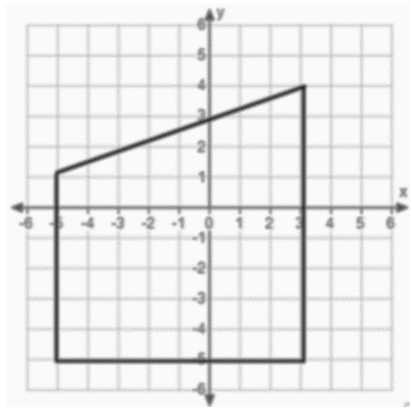
13. Find the area of the quadrilateral using two different methods. Describe the methods used, and explain why they result in the same area.



Area $= 15$ sq. units

*One method is by drawing a rectangle around the figure. The area of the parallelogram is equal to the area of the rectangle minus the area of the two triangles. A second method is to use the area formula for a parallelogram (*Area $=$ base $\times$ height*).*

14. **Find the area of the quadrilateral using two different methods. What are the advantages or disadvantages of each method?**

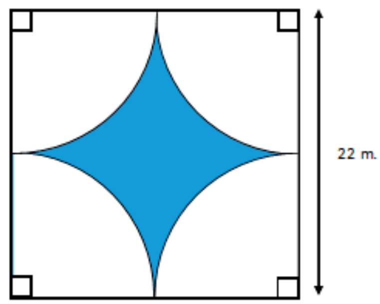


Area $= 60$ sq. units

One method is to use the area formula for a trapezoid, $A = \frac{1}{2}(\text{base } 1 + \text{base } 2) \times \text{height}$. ***The second method is to split the figure into a rectangle and a triangle. The second method requires more calculations. The first method requires first recognizing the figure as a trapezoid and recalling the formula for the area of a trapezoid.***

Exit Ticket Sample Solutions

The unshaded regions are quarter circles. Approximate the area of the shaded region. Use $\pi \approx 3.14$.



Area of the square − area of the 4 quarter circles = area of the shaded region

$(22\text{ m} \cdot 22\text{ m}) - ((11\text{ m})^2 \cdot 3.14)$

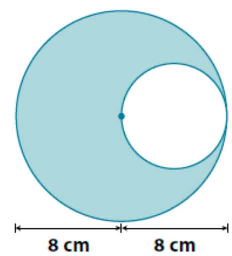
$484\text{ m}^2 - 379.94\text{ m}^2$

104.06 m^2

The area of the shaded region is approximately 104.06 m^2.

Problem Set Sample Solutions

1. Find the area of the shaded region. Use 3.14 for π.



Area of large circle– area of small circle

$$(\pi \times (8\text{ cm})^2) - (\pi \times (4\text{ cm})^2)$$

$$(3.14)(64\text{ cm}^2) - (3.14)(16\text{ cm}^2)$$

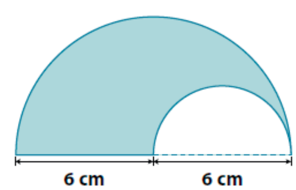
$$200.96\text{ cm}^2 - 50.24\text{ cm}^2$$

$$150.72\text{ cm}^2$$

The area of the region is approximately 150.72 cm^2.

2. The figure shows two semicircles. Find the area of the shaded region. Use 3.14 for π.

Area of large semicircle region − area of small semicircle region = area of the shaded region



$$\left(\frac{1}{2}\right)(\pi \times (6\text{ cm})^2) - \left(\frac{1}{2}\right)(\pi \times (3\text{ cm})^2)$$

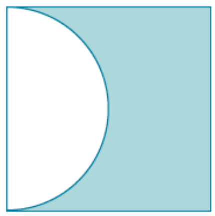
$$\left(\frac{1}{2}\right)(3.14)(36\text{ cm}^2) - \left(\frac{1}{2}\right)(3.14)(9\text{ cm}^2)$$

$$56.52\text{ cm}^2 - 14.13\text{ cm}^2$$

$$42.39\text{ cm}^2$$

The area is approximately 42.39 cm^2.

3. The figure shows a semicircle and a square. Find the area of the shaded region. Use 3.14 for π.



Area of the square − area of the semicircle

$$(24 \text{ cm} \times 24 \text{ cm}) - \left(\frac{1}{2}\right)\left(\pi \times (12 \text{ cm})^2\right)$$

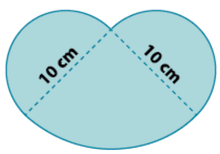
$$576 \text{ cm}^2 - \left(\frac{1}{2}\right)(3.14 \times 144 \text{ cm}^2)$$

$$576 \text{ cm}^2 - 226.08 \text{ cm}^2$$

$$349.92 \text{ cm}^2$$

The area is approximately 349.92 cm^2.

4. The figure shows two semicircles and a quarter of a circle. Find the area of the shaded region. Use 3.14 for π.



Area of two semicircles + area of quarter of the larger circle

$$2\left(\frac{1}{2}\right)\left(\pi \times (5 \text{ cm})^2\right) + \left(\frac{1}{4}\right)\left(\pi \times (10 \text{ cm})^2\right)$$

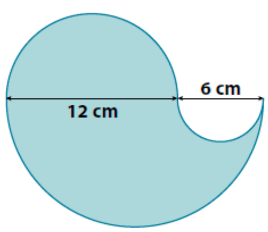
$$(3.14)(25 \text{ cm}^2) + (3.14)(25 \text{ cm}^2)$$

$$78.5 \text{ cm}^2 + 78.5 \text{ cm}^2$$

$$157 \text{ cm}^2$$

The area is approximately 157 cm^2.

5. Jillian is making a paper flower motif for an art project. The flower she is making has four petals; each petal is formed by three semicircles as shown below. What is the area of the paper flower? Provide your answer in terms of π.



Area of medium semicircle + (area of larger semicircle − area of small semicircle)

$$\left(\frac{1}{2}\right)\left(\pi \times (6 \text{ cm})^2\right) + \left(\left(\frac{1}{2}\right)\left(\pi \times (9 \text{ cm})^2\right) - \left(\frac{1}{2}\right)\left(\pi \times (3 \text{ cm})^2\right)\right)$$

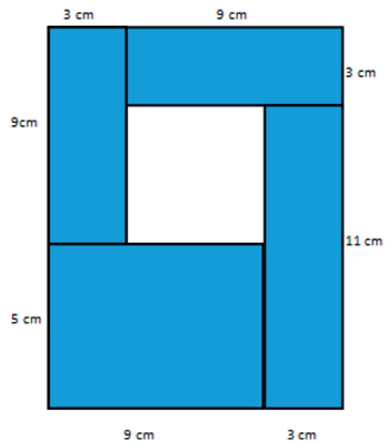
$$18\pi \text{ cm}^2 + 40.5\pi \text{ cm}^2 - 4.5\pi \text{ cm}^2 = 54\pi \text{ cm}^2$$

$$54\pi \text{ cm}^2 \times 4$$

$$216\pi \text{ cm}^2$$

The area is $216\pi \text{ cm}^2$.

6. The figure is formed by five rectangles. Find the area of the unshaded rectangular region.



Area of the whole rectangle − area of the sum of the shaded rectangles = area of the unshaded rectangular region

$$(12\text{ cm} \times 14\text{ cm}) - \big(2(3\text{ cm} \times 9\text{ cm}) + (11\text{ cm} \times 3\text{ cm}) + (5\text{ cm} \times 9\text{ cm})\big)$$

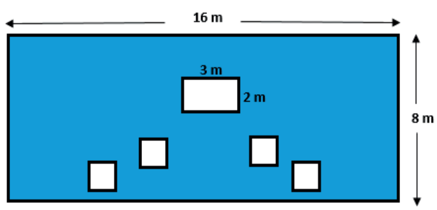
$$168\text{ cm}^2 - (54\text{ cm}^2 + 33\text{ cm}^2 + 45\text{ cm}^2)$$

$$168\text{ cm}^2 - 132\text{ cm}^2$$

$$36\text{ cm}^2$$

The area is 36 cm^2.

7. The smaller squares in the shaded region each have side lengths of 1.5 m. Find the area of the shaded region.



Area of the 16 m *by* 8 m *rectangle − the sum of the area of the smaller unshaded rectangles = area of the shaded region*

$$(16\text{ m} \times 8\text{ m}) - \big((3\text{ m} \times 2\text{ m}) + (4(1.5\text{ m} \times 1.5\text{ m}))\big)$$

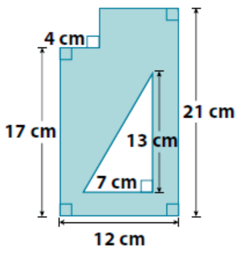
$$128\text{ m}^2 - (6\text{ m}^2 + 4(2.25\text{ m}^2))$$

$$128\text{ m}^2 - 15\text{ m}^2$$

$$113\text{ m}^2$$

The area is 113 m^2.

8. Find the area of the shaded region.



Area of the sum of the rectangles − area of the right triangle = area of shaded region

$$\big((17\text{ cm} \times 4\text{ cm}) + (21\text{ cm} \times 8\text{ cm})\big) - \left(\left(\frac{1}{2}\right)(13\text{ cm} \times 7\text{ cm})\right)$$

$$(68\text{ cm}^2 + 168\text{ cm}^2) - \left(\frac{1}{2}\right)(91\text{ cm}^2)$$

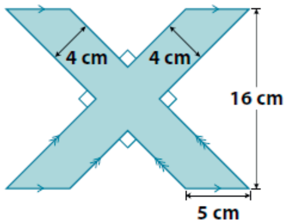
$$236\text{ cm}^2 - 45.5\text{ cm}^2$$

$$190.5\text{ cm}^2$$

The area is 190.5 cm^2.

9.

a. Find the area of the shaded region.



Area of the two parallelograms − area of square in the center = area of the shaded region

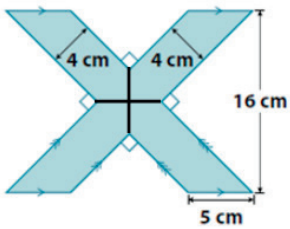
$$2(5\text{ cm} \times 16\text{ cm}) - (4\text{ cm} \times 4\text{ cm})$$

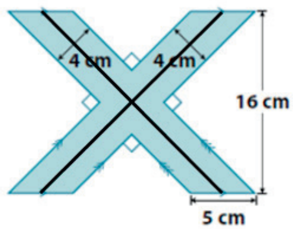
$$160\text{ cm}^2 - 16\text{ cm}^2$$

$$144\text{ cm}^2$$

The area is 144 cm^2.

b. Draw two ways the figure above can be divided in four equal parts.



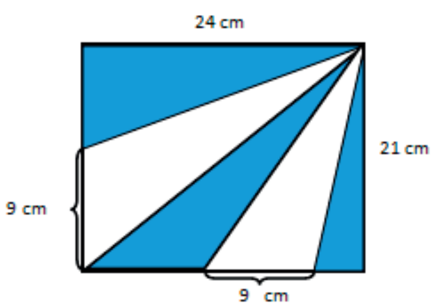


c. What is the area of one of the parts in (b)?

$144\text{ cm}^2 \div 4 = 36\text{ cm}^2$

The area of one of the parts in (b) is 36 cm^2.

10. The figure is a rectangle made out of triangles. Find the area of the shaded region.



Area of the rectangle − area of the unshaded triangles = area of the shaded region

$$(24\text{ cm} \times 21\text{ cm}) - \left(\left(\frac{1}{2}\right)(9\text{ cm} \times 21\text{ cm}) + \left(\frac{1}{2}\right)(9\text{ cm} \times 24\text{ cm})\right)$$

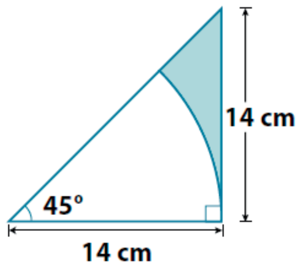
$$504\text{ cm}^2 - (94.5\text{ cm}^2 + 108\text{ cm}^2)$$

$$504\text{ cm}^2 - 202.5\text{ cm}^2$$

$$301.5\text{ cm}^2$$

The area is 301.5 cm^2.

11. **The figure consists of a right triangle and an eighth of a circle. Find the area of the shaded region. Use $\frac{22}{7}$ for π.**



Area of right triangle – area of eighth of the circle= area of shaded region

$$\left(\frac{1}{2}\right)(14\text{ cm} \times 14\text{ cm}) - \left(\frac{1}{8}\right)(\pi \times 14\text{ cm} \times 14\text{ cm})$$

$$\left(\frac{1}{2}\right)(196\text{ cm}^2) - \left(\frac{1}{8}\right)\left(\frac{22}{7}\right)(2\text{ cm} \times 7\text{ cm} \times 2\text{ cm} \times 7\text{ cm})$$

$$98\text{ cm}^2 - 77\text{ cm}^2$$

$$21\text{ cm}^2$$

The area is approximately 21 cm^2.

Exit Ticket Sample Solutions

Find the surface area of the right trapezoidal prism. Show all necessary work.

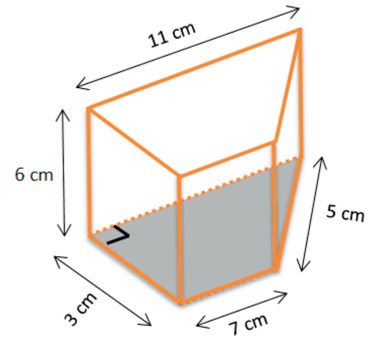
$SA = LA + 2B$

$LA = P \cdot h$

$LA = (3 \text{ cm } + 7 \text{ cm } + 5 \text{ cm } + 11 \text{ cm }) \cdot 6 \text{ cm}$

$LA = 26 \text{ cm} \cdot 6 \text{ cm}$

$LA = 156 \text{ cm}^2$



Each base consists of a 3 cm *by* 7 cm *rectangle and right triangle with a base of* 3 cm *and a height of* 4 cm. *Therefore, the area of each base:*

$B = A_r + A_t$

$B = lw + \frac{1}{2}bh$

$B = (7 \text{ cm } \cdot 3 \text{ cm}) + \left(\frac{1}{2} \cdot 3 \text{ cm} \cdot 4 \text{ cm}\right)$

$B = 21 \text{ cm}^2 + 6 \text{ cm}^2$

$B = 27 \text{ cm}^2$

$SA = LA + 2B$

$SA = 156 \text{ cm}^2 + 2(27 \text{ cm}^2)$

$SA = 156 \text{ cm}^2 + 54 \text{ cm}^2$

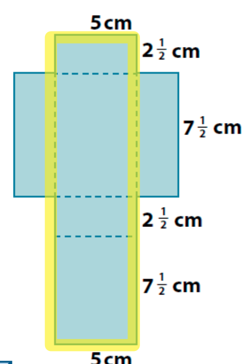
$SA = 210 \text{ cm}^2$

The surface of the right trapezoidal prism is 210 cm^2.

Problem Set Sample Solutions

1. **For each of the following nets, highlight the perimeter of the lateral area, draw the solid represented by the net, indicate the type of solid, and then find the solid's surface area.**

   a. *Right rectangular prism*



$SA = LA + 2B$

$LA = P \cdot h$

$LA = \left(2\frac{1}{2} \text{ cm} + 7\frac{1}{2} \text{ cm} + 2\frac{1}{2} \text{ cm} + 7\frac{1}{2} \text{ cm}\right) \cdot 5 \text{ cm}$

$LA = 20 \text{ cm} \cdot 5 \text{ cm}$

$LA = 100 \text{ cm}^2$

$B = lw$

$B = 2\frac{1}{2} \text{ cm} \cdot 7\frac{1}{2} \text{ cm}$

$B = \frac{5}{2} \text{cm} \cdot \frac{15}{2} \text{cm}$

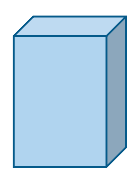
$B = \frac{75}{4} \text{ cm}^2$

$SA = 100 \text{ cm}^2 + 2\left(\frac{75}{4} \text{ cm}^2\right)$

$SA = 100 \text{ cm}^2 + 37.5 \text{ cm}^2$

$SA = 137.5 \text{ cm}^2$

The surface area of the right rectangular prism is 137.5 cm^2



(3-Dimensional Form)

b. ***Right triangular prism***

$SA = LA + 2B$

$LA = P \cdot h$

$LA = (10 \text{ in.} + 8 \text{ in.} + 10 \text{ in.}) \cdot 12 \text{ in.}$

$LA = 28 \text{ in.} \cdot 12 \text{ in.}$

$LA = 336 \text{ in}^2$

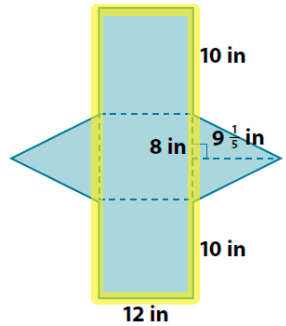
$B = \frac{1}{2}bh$

$B = \frac{1}{2}(8 \text{ in.})\left(9\frac{1}{5} \text{ in.}\right)$

$B = 4 \text{ in.}\left(9\frac{1}{5} \text{ in.}\right)$

$B = \left(36 + \frac{4}{5}\right) \text{in}^2$

$B = 36\frac{4}{5} \text{ in}^2$



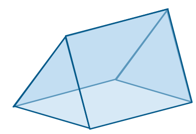
$SA = 336 \text{ in}^2 + 2\left(36\frac{4}{5} \text{ in}^2\right)$

$SA = 336 \text{ in}^2 + \left(72 + \frac{8}{5}\right) \text{in}^2$

$SA = 408 \text{ in}^2 + 1\frac{3}{5} \text{in}^2$

$SA = 409\frac{3}{5} \text{ in}^2$

The surface area of the right triangular prism is $409\frac{3}{5}$ in^2.


(3-Dimensional Form)

2. Given a cube with edges that are $\frac{3}{4}$ inch long:

a. Find the surface area of the cube.

$SA = 6s^2$

$SA = 6\left(\frac{3}{4} \text{ in.}\right)^2$

$SA = 6\left(\frac{3}{4} \text{ in.}\right) \cdot \left(\frac{3}{4} \text{ in.}\right)$

$SA = 6\left(\frac{9}{16} \text{ in}^2\right)$

$SA = \frac{27}{8} \text{ in}^2$ or $3\frac{3}{8} \text{ in}^2$

b. Joshua makes a scale drawing of the cube using a scale factor of 4. Find the surface area of the cube that Joshua drew.

$\frac{3}{4} \text{ in.} \cdot 4 = 3 \text{ in.}$; *The edge lengths of Joshua's drawing would be* 3 *inches.*

$SA = 6(3 \text{ in.})^2$

$SA = 6(9 \text{ in}^2)$

$SA = 54 \text{ in}^2$

c. **What is the ratio of the surface area of the scale drawing to the surface area of the actual cube, and how does the value of the ratio compare to the scale factor?**

$54 \div 3\frac{3}{8}$

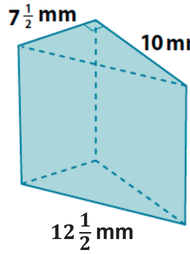
$54 \div \frac{27}{8}$

$54 \cdot \frac{8}{27}$

$2 \cdot 8 = 16$. *The ratios of the surface area of the scale drawing to the surface area of the actual cube is* $16:1$. *The value of the ratio is* 16. *The scale factor of the drawing is* 4, *and the value of the ratio of the surface area of the drawing to the surface area of the actual cube is* 4^2 *or* 16.

3. **Find the surface area of each of the following right prisms using the formula $SA = LA + 2B$.**

a.



$SA = LA + 2B$

$LA = P \cdot h$

$LA = \left(12\frac{1}{2}\text{ mm} + 10\text{ mm} + 7\frac{1}{2}\text{ mm}\right) \cdot 15\text{ mm}$

$LA = 30\text{ mm} \cdot 15\text{ mm}$

$LA = 450\text{ mm}^2$

$B = \frac{1}{2}bh$

$B = \frac{1}{2} \cdot \left(7\frac{1}{2}\text{ mm}\right) \cdot (10\text{ mm})$

$B = \frac{1}{2} \cdot (70 + 5)\text{ mm}^2$

$B = \frac{1}{2} \cdot 75\text{ mm}^2$

$B = \frac{75}{2}\text{ mm}^2$

$SA = 450\text{ mm}^2 + 2\left(\frac{75}{2}\text{ mm}^2\right)$

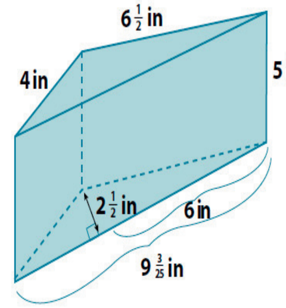
$SA = 450\text{ mm}^2 + 75\text{ mm}^2$

$SA = 525\text{ mm}^2$

The surface area of the prism is 525 mm^2.

b.

$SA = LA + 2B$



$LA = P \cdot h$

$LA = \left(9\frac{3}{25}\text{ in.} + 6\frac{1}{2}\text{ in.} + 4\text{ in.}\right) \cdot 5\text{ in}$

$LA = \left(\frac{228}{25}\text{ in.} + \frac{13}{2}\text{ in.} + 4\text{ in.}\right) \cdot 5\text{ in}$

$LA = \left(\frac{456}{50}\text{ in.} + \frac{325}{50}\text{ in.} + \frac{200}{50}\text{ in.}\right) \cdot 5\text{ in.}$

$LA = \left(\frac{981}{50}\text{ in.}\right) \cdot 5\text{ in.}$

$LA = \frac{49{,}050}{50}\text{ in}^2$

$LA = 98\frac{1}{10}\text{ in}^2$

$B = \frac{1}{2}bh$

$B = \frac{1}{2} \cdot 9\frac{3}{25}\text{ in.} \cdot 2\frac{1}{2}\text{ in.}$

$B = \frac{1}{2} \cdot \frac{228}{25}\text{ in.} \cdot \frac{5}{2}\text{ in.}$

$B = \frac{1{,}140}{100}\text{ in}^2$

$B = 11\frac{2}{5}\text{ in}^2$

$2B = 2 \cdot 11\frac{2}{5}\text{ in}^2$

$2B = 22\frac{4}{5}\text{ in}^2$

$SA = LA + 2B$

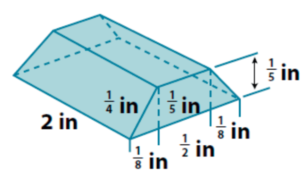
$SA = 98\frac{1}{10}\text{ in}^2 + 22\frac{4}{5}\text{ in}^2$

$SA = 120\frac{9}{10}\text{ in}^2$

The surface area of the prism is $120\frac{9}{10}\text{ in}^2$.

c.

$SA = LA + 2B$



$LA = P \cdot h$

$LA = \left(\frac{1}{8}\text{ in.} + \frac{1}{2}\text{ in.} + \frac{1}{8}\text{ in.} + \frac{1}{4}\text{ in.} + \frac{1}{2}\text{ in.} + \frac{1}{4}\text{ in.}\right) \cdot 2\text{ in.}$

$LA = \left(1\frac{3}{4}\text{ in.}\right) \cdot 2\text{ in.}$

$LA = 2\text{ in}^2 + 1\frac{1}{2}\text{ in}^2$

$LA = 3\frac{1}{2}\text{ in}^2$

$B = A_{rectangle} + 2A_{triangle}$

$B = \left(\frac{1}{2}\text{ in.} \cdot \frac{1}{5}\text{ in.}\right) + 2 \cdot \frac{1}{2}\left(\frac{1}{8}\text{ in.} \cdot \frac{1}{5}\text{ in.}\right)$

$B = \left(\frac{1}{10}\text{ in}^2\right) + \left(\frac{1}{40}\text{ in}^2\right)$

$B = \frac{1}{10}\text{ in}^2 + \frac{1}{40}\text{ in}^2$

$B = \frac{4}{40}\text{ in}^2 + \frac{1}{40}\text{ in}^2$

$B = \frac{5}{40}\text{ in}^2$

$B = \frac{1}{8}\text{ in}^2$

$SA = 3\frac{1}{2}\ in^2 + 2\left(\frac{1}{8}\ in^2\right)$

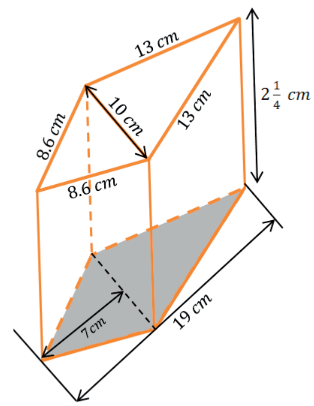
$SA = 3\frac{1}{2}\ in^2 + \frac{1}{4}\ in^2$

$SA = 3\frac{2}{4}\ in^2 + \frac{1}{4}\ in^2$

$SA = 3\frac{3}{4}\ in^2$

The surface area of the prism is $3\frac{3}{4}\text{ in}^2$.

d.



$SA = LA + 2B$

$LA = P \cdot h$

$LA = (13 \text{ cm} + 13 \text{ cm} + 8.6 \text{ cm} + 8.6 \text{ cm}) \cdot 2\frac{1}{4} \text{ cm}$

$LA = (26 \text{ cm} + 17.2 \text{ cm}) \cdot 2\frac{1}{4} \text{ cm}$

$LA = (43.2)\text{cm} \cdot 2\frac{1}{4} \text{cm}$

$LA = (86.4 \text{ cm}^2 + 10.8 \text{ cm}^2)$

$LA = 97.2 \text{ cm}^2$

$SA = LA + 2B$ $\quad$ $B = \frac{1}{2}(10 \text{ cm} \cdot 7 \text{ cm}) + \frac{1}{2}(12 \text{ cm} \cdot 10 \text{ cm})$

$SA = 97.2\ cm^2 + 2(95 \text{ cm}^2)$ $\quad$ $B = \frac{1}{2}(70 \text{ cm}^2 + 120 \text{ cm}^2)$

$SA = 97.2 \text{ cm}^2 + 190 \text{ cm}^2$ $\quad$ $B = \frac{1}{2}(190 \text{ cm}^2)$

$SA = 287.2 \text{ cm}^2$ $\quad$ $B = 95 \text{ cm}^2$

The surface area of the prism is 287.2 cm^2.

4. A cube has a volume of 64 m^3. What is the cube's surface area?

A cube's length, width, and height must be equal. $64 = 4 \cdot 4 \cdot 4 = 4^3$*, so the length, width, and height of the cube are all* 4 m.

$SA = 6s^2$

$SA = 6(4 \text{ m})^2$

$SA = 6(16 \text{ m}^2)$

$SA = 96 \text{ m}^2$

5. The height of a right rectangular prism is $4\frac{1}{2}$ ft. The length and width of the prism's base are 2 ft. and $1\frac{1}{2}$ ft. Use the formula $SA = LA + 2B$ to find the surface area of the right rectangular prism.

$SA = LA + 2B$

$LA = P \cdot h$ $\quad$ $B = lw$

$LA = \left(2 \text{ ft.} + 2 \text{ ft.} + 1\frac{1}{2} \text{ ft.} + 1\frac{1}{2} \text{ ft.}\right) \cdot 4\frac{1}{2} \text{ ft.}$ $\quad$ $B = 2 \text{ ft.} \cdot 1\frac{1}{2} \text{ ft.}$

$LA = (2 \text{ ft.} + 2 \text{ ft.} + 3 \text{ ft.}) \cdot 4\frac{1}{2} \text{ ft.}\ SA = LA + 2b$ $\quad$ $B = 3 \text{ ft}^2$

$LA = 7 \text{ ft.} \cdot 4\frac{1}{2} \text{ ft.}$ $\quad$ $SA = 31\frac{1}{2} \text{ ft}^2 + 2(3 \text{ ft}^2)$

$LA = 28 \text{ ft}^2 + 3\frac{1}{2} \text{ ft}^2$ $\quad$ $SA = 31\frac{1}{2} \text{ ft}^2 + 6 \text{ ft}^2$

$LA = 31\frac{1}{2} \text{ ft}^2$ $\quad$ $SA = 37\frac{1}{2} \text{ ft}^2$

The surface area of the right rectangular prism is $37\frac{1}{2} \text{ ft}^2$.

6. The surface area of a right rectangular prism is $68\frac{2}{3}$ in^2. The dimensions of its base are 3 in and 7 in Use the formula $SA = LA + 2B$ and $LA = Ph$ to find the unknown height h of the prism.

$SA = LA + 2B$

$SA = P \cdot h + 2B$

$68\frac{2}{3}\text{ in}^2 = 20\text{ in.}\cdot(h) + 2(21\text{ in}^2)$

$68\frac{2}{3}\text{ in}^2 = 20\text{ in.}\cdot(h) + 42\text{ in}^2$

$68\frac{2}{3}\text{ in}^2 - 42\text{ in}^2 = 20\text{ in.}\cdot(h) + 42\text{ in}^2 - 42\text{ in}^2$

$26\frac{2}{3}\text{ in}^2 = 20\text{ in.}\cdot(h) + 0\text{ in}^2$

$26\frac{2}{3}\text{ in}^2 \cdot \frac{1}{20\text{ in.}} = 20\text{ in}\cdot\frac{1}{20\text{ in.}}\cdot(h)$

$\frac{80}{3}\text{ in}^2 \cdot \frac{1}{20\text{ in}} = 1 \cdot h$

$\frac{4}{3}\text{ in.} = h$

$h = \frac{4}{3}$ in. or $1\frac{1}{3}$ in.

The height of the prism is $1\frac{1}{3}$ in.

7. A given right triangular prism has an equilateral triangular base. The height of that equilateral triangle is approximately 7.1 cm. The distance between the bases is 9 cm. The surface area of the prism is $319\frac{1}{2}$ cm^2. Find the approximate lengths of the sides of the base.

$SA = LA + 2B$ *Let x represent the number of centimeters in each side of the equilateral triangle.*

$LA = P \cdot h$

$LA = 3(x\text{ cm}) \cdot 9\text{ cm}$

$LA = 27x\text{ cm}^2$

$B = \frac{1}{2}lw$

$B = \frac{1}{2}\cdot(x\text{ cm})\cdot 7.1\text{ cm}$

$B = 3.55x\text{ cm}^2$

$319\frac{1}{2}\ cm^2 = LA + 2B$

$319\frac{1}{2}\text{ cm}^2 = 27x\text{ cm}^2 + 2(3.55x\text{ cm}^2)$

$319\frac{1}{2}\text{ cm}^2 = 27x\text{ cm}^2 + 7.1x\text{ cm}^2$

$319\frac{1}{2}\text{ cm}^2 = 34.1x\text{ cm}^2$

$319\frac{1}{2}\text{ cm}^2 = 34\frac{1}{10}x\text{ cm}^2$

$\frac{639}{2}\text{ cm}^2 = \frac{341}{10}x\text{ cm}^2$

$\frac{639}{2}\text{ cm}^2 \cdot \frac{10}{341\text{ cm}} = \frac{341}{10}x\text{ cm}^2 \cdot \frac{10}{341\text{ cm}}$

$\frac{3195}{341}\text{cm} = x$

$x = \frac{3195}{341}\text{cm}$

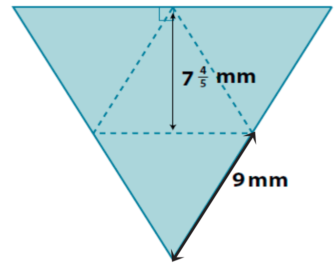
$x \approx 9.4\text{ cm}$

The lengths of the sides of the equilateral triangles are approximately 9.4 cm *each.*

Problem Set Sample Solutions

1. **For each of the following nets, draw (or describe) the solid represented by the net and find its surface area.**

   a. **The equilateral triangles are exact copies.**



The net represents a triangular pyramid where the three lateral faces are identical to each other and the triangular base.

$SA = 4B$ since the faces are all the same size and shape.

$B = \frac{1}{2}bh$

$B = \frac{1}{2} \cdot 9\text{ mm} \cdot 7\frac{4}{5}\text{ mm}$

$B = \frac{9}{2}\text{ mm} \cdot 7\frac{4}{5}\text{ mm}$

$B = \frac{63}{2}\text{ mm}^2 + \frac{36}{10}\text{ mm}^2$

$B = \frac{315}{10}\text{ mm}^2 + \frac{36}{10}\text{ mm}^2$

$B = \frac{351}{10}\text{ mm}^2$

$B = 35\frac{1}{10}\text{ mm}^2$

$SA = 4B$

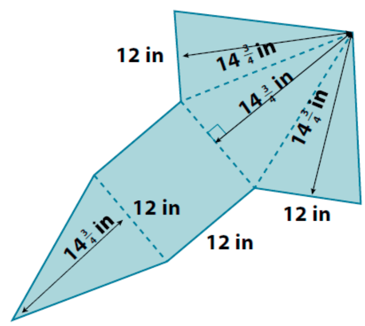
$SA = 4\left(35\frac{1}{10}\text{ mm}^2\right)$

$SA = 140\text{ mm}^2 + \frac{4}{10}\text{ mm}^2$

$SA = 140\frac{2}{5}\text{ mm}^2$

The surface area of the triangular pyramid is $140\frac{2}{5}\text{ mm}^2$.

   b. *The net represents a square pyramid that has four identical lateral faces that are triangles. The base is a square.*



$SA = LA + B$

$LA = 4 \cdot \frac{1}{2}(bh)$

$LA = 4 \cdot \frac{1}{2}\left(12\text{ in.} \cdot 14\frac{3}{4}\text{ in.}\right)$

$LA = 2\left(12\text{ in.} \cdot 14\frac{3}{4}\text{ in.}\right)$

$LA = 2(168\text{ in}^2 + 9\text{ in}^2)$

$LA = 336\text{ in}^2 + 18\text{ in}^2$

$LA = 354\text{ in}^2$

$B = s^2$

$B = (12\text{ in.})^2$

$B = 144\text{ in}^2$

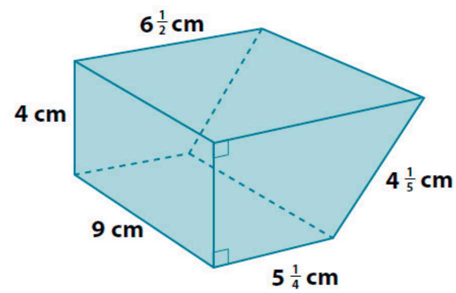
$SA = LA + B$

$SA = 354\text{ in}^2 + 144\text{ in}^2$

$SA = 498\text{ in}^2$

The surface area of the square pyramid is 498 in^2.

2. **Find the surface area of the following prism.**



$SA = LA + 2B$

$LA = P \cdot h$

$LA = \left(4 \text{ cm} + 6\frac{1}{2} \text{ cm} + 4\frac{1}{5} \text{ cm} + 5\frac{1}{4} \text{ cm}\right) \cdot 9 \text{ cm}$

$LA = \left(19 \text{ cm} + \frac{1}{2} \text{ cm} + \frac{1}{5} \text{ cm} + \frac{1}{4} \text{ cm}\right) \cdot 9 \text{ cm}$

$LA = \left(19 \text{ cm} + \frac{10}{20} \text{ cm} + \frac{4}{20} \text{ cm} + \frac{5}{20} \text{ cm}\right) \cdot 9 \text{ cm}$

$LA = \left(19 \text{ cm} + \frac{19}{20} \text{ cm}\right) \cdot 9 \text{ cm}$

$LA = 171 \text{ cm}^2 + \frac{171}{20} \text{ cm}^2$

$LA = 171 \text{ cm}^2 + 8\frac{11}{20} \text{ cm}^2$

$LA = 179\frac{11}{20} \text{ cm}^2$

$B = A_{\text{rectangle}} + A_{\text{triangle}}$ $SA = LA + 2B$

$B = \left(5\frac{1}{4} \text{ cm} \cdot 4 \text{ cm}\right) + \frac{1}{2}\left(4 \text{ cm} \cdot 1\frac{1}{4} \text{ cm}\right)$ $SA = 179\frac{11}{20} \text{ cm}^2 + 2\left(23\frac{1}{2} \text{ cm}^2\right)$

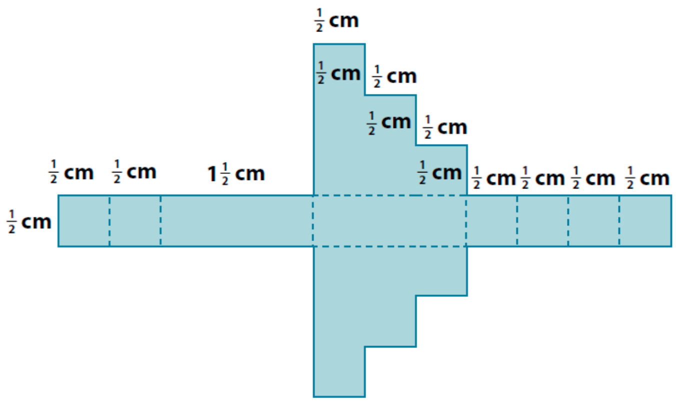
$B = (20 \text{ cm}^2 + 1 \text{ cm}^2) + \left(2 \text{ cm} \cdot 1\frac{1}{4} \text{ cm}\right)$ $SA = 179\frac{11}{20} \text{ cm}^2 + 47 \text{ cm}^2$

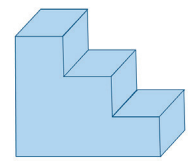
$B = 21 \text{ cm}^2 + 2\frac{1}{2} \text{ cm}^2$ $\text{SA} = 226\frac{11}{20} \text{ cm}^2$

$B = 23\frac{1}{2} \text{ cm}^2$

The surface area of the prism is $226\frac{11}{20} \text{ cm}^2$.

3. The net below is for a specific object. The measurements shown are in meters. Sketch (or describe) the object, and then find its surface area.




(3-Dimensional Form)

$SA = LA + 2B$

$LA = P \cdot h$ $\quad B = \left(\frac{1}{2}\text{ cm} \cdot \frac{1}{2}\text{ cm}\right) + \left(\frac{1}{2}\text{ cm} \cdot 1\text{ cm}\right) + \left(\frac{1}{2}\text{ cm} \cdot 1\frac{1}{2}\text{ cm}\right)$ $\quad SA = LA + 2B$

$LA = 6\text{ cm} \cdot \frac{1}{2}\text{ cm}$ $\quad B = \left(\frac{1}{4}\text{ cm}^2\right) + \left(\frac{1}{2}\text{ cm}^2\right) + \left(\frac{3}{4}\text{ cm}^2\right)$ $\quad SA = 3\text{ cm}^2 + 2\left(1\frac{1}{2}\text{ cm}^2\right)$

$LA = 3\text{ cm}^2$ $\quad B = \left(\frac{1}{4}\text{ cm}^2\right) + \left(\frac{2}{4}\text{ cm}^2\right) + \left(\frac{3}{4}\text{ cm}^2\right)$ $\quad SA = 3\text{ cm}^2 + 3\text{ cm}^2$

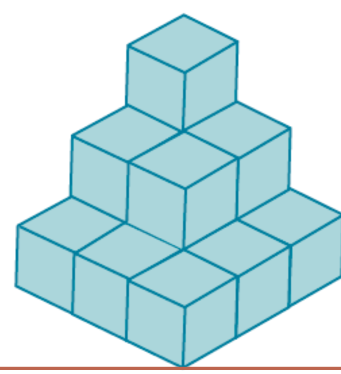
$B = \frac{6}{4}\text{ cm}^2$ $\quad SA = 6\text{ cm}^2$

$B = 1\frac{1}{2}\text{ cm}^2$

The surface area of the object is 6 cm^2.

4. In the diagram, there are 14 cubes glued together to form a solid. Each cube has a volume of $\frac{1}{8}\text{ in}^3$. Find the surface area of the solid.

The volume of a cube is s^3, *and* $\frac{1}{8}\text{ in}^3$ *is the same as* $\left(\frac{1}{2}\text{ in.}\right)^3$, *so the cubes have edges that are* $\frac{1}{2}$ in. *long. The cube faces have area* s^2, *or* $\left(\frac{1}{2}\text{ in.}\right)^2$, *or* $\frac{1}{4}\text{ in}^2$. *There are* 42 *cube faces that make up the surface of the solid.*



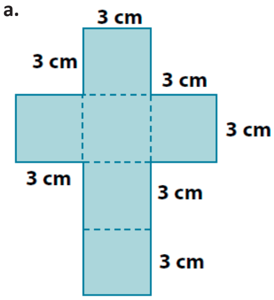
$SA = \frac{1}{4}\text{ in}^2 \cdot 42$

$SA = 10\frac{1}{2}\text{ in}^2$

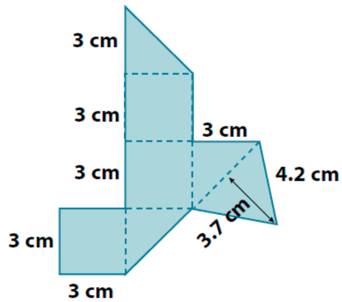
The surface area of the solid is $10\frac{1}{2}\text{ in}^2$.

5. The nets below represent three solids. Sketch (or describe) each solid, and find its surface area.

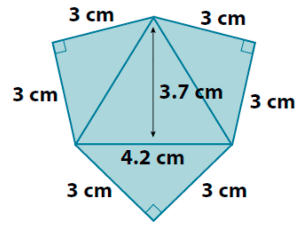
a.



b.



c.



$SA = LA + 2B$

$LA = P \cdot h$

$LA = 12 \cdot 3$

$LA = 36\ cm^2$

$B = s^2$

$B = (3\text{ cm})^2$

$B = 9\text{ cm}^2$

$SA =$
$36\text{ cm}^2 + 2(9\text{ cm}^2)$

$SA = 36\text{ cm}^2 + 18\text{ cm}^2$

$SA = 54\text{ cm}^2$

$SA = 3A_{\text{square}} + 3A_{\text{rt triangle}} + A_{\text{equ triangle}}$

$A_{\text{square}} = s^2$

$A_{\text{square}} = (3\text{ cm})^2$

$A_{\text{square}} = 9\text{ cm}^2$

$A_{\text{rt triangle}} = \frac{1}{2}bh$

$A_{\text{rt triangle}} = \frac{1}{2} \cdot 3\text{ cm} \cdot 3\text{ cm}$

$A_{\text{rt triangle}} = \frac{9}{2}$

$A_{\text{rt triange}} = 4\frac{1}{2}\text{ cm}^2$

$A_{\text{equ triangle}} = \frac{1}{2}bh$

$A_{\text{equ triangle}} = \frac{1}{2} \cdot \left(4\frac{1}{5}\text{ cm}\right) \cdot \left(3\frac{7}{10}\text{ cm}\right)$

$A_{\text{equ triangle}} = 2\frac{1}{10}\text{ cm} \cdot 3\frac{7}{10}\text{ cm}$

$A_{\text{equ triangle}} = \frac{21}{10}\text{ cm} \cdot \frac{37}{10}\text{ cm}$

$A_{\text{equ triangle}} = \frac{777}{100}\text{ cm}^2$

$A_{\text{equ triangle}} = 7\frac{77}{100}\text{ cm}^2$

$SA = 3(9\text{ cm}^2) + 3\left(4\frac{1}{2}\text{ cm}^2\right) + 7\frac{77}{100}\text{ cm}^2$

$SA = 27\text{ cm}^2 + \left(12 + \frac{3}{2}\right)\text{ cm}^2 + 7\frac{77}{100}\text{ cm}^2$

$SA = 47\text{ cm}^2 + \frac{1}{2}\text{ cm}^2 + \frac{77}{100}\text{ cm}^2$

$SA = 47\text{ cm}^2 + \frac{50}{100}\text{ cm}^2 + \frac{77}{100}\text{ cm}^2$

$SA = 47\text{ cm}^2 + \frac{127}{100}\text{ cm}^2$

$SA = 47\text{ cm}^2 + 1\text{ cm}^2 + \frac{27}{100}\text{ cm}^2$

$SA = 48\frac{27}{100}\text{ cm}^2$

$SA = 3A_{\text{rt triangle}} + A_{\text{equ triangle}}$

$SA = 3\left(4\frac{1}{2}\right)\text{ cm}^2 + 7\frac{77}{100}\text{ cm}^2$

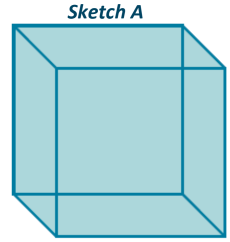
$SA = 12\text{ cm}^2 + \frac{3}{2}\text{ cm}^2 + 7\text{ cm}^2 + \frac{77}{100}\text{ cm}^2$

$SA = 20\text{ cm}^2 + \frac{1}{2}\text{ cm}^2 + \frac{77}{100}\text{ cm}^2$

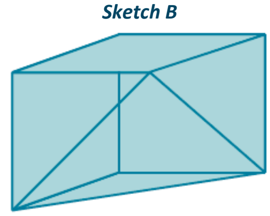
$SA = 20\text{ cm}^2 + 1\text{ cm}^2 + \frac{27}{100}\text{ cm}^2$

$SA = 21\frac{27}{100}\text{ cm}^2$

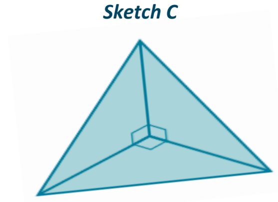
Sketch A



Sketch B



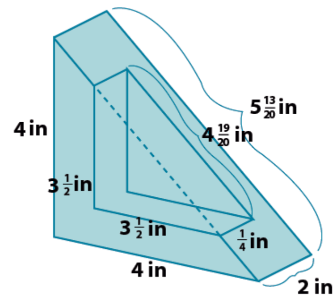
Sketch C



d. **How are figures (b) and (c) related to figure (a)?**

If the equilateral triangular faces of figures (b) and (c) were matched together, they would form the cube in part (a).

6. **Find the surface area of the solid shown in the diagram. The solid is a right triangular prism (with right triangular bases) with a smaller right triangular prism removed from it.**



$SA = LA + 2B$

$LA = P \cdot h$

$LA = \left(4 \text{ in.} + 4 \text{ in.} + 5\frac{13}{20} \text{ in.}\right) \cdot 2 \text{ in.}$

$LA = \left(13\frac{13}{20} \text{ in.}\right) \cdot 2 \text{ in.}$

$LA = 26 \text{ in}^2 + \frac{13}{10} \text{ in}^2$

$LA = 26 \text{ in}^2 + 1 \text{ in}^2 + \frac{3}{10} \text{ in}^2$

$LA = 27\frac{3}{10} \text{ in}^2$

The $\frac{1}{4}$ in. *by* $4\frac{19}{20}$ in. *rectangle has to be taken away from the lateral area:*

$A = lw$

$A = 4\frac{19}{20} \text{ in} \cdot \frac{1}{4} \text{ in}$

$A = 1 \text{ in}^2 + \frac{19}{80} \text{ in}^2$

$A = 1\frac{19}{80} \text{ in}^2$

$\text{LA} = 27\frac{3}{10} \text{ in}^2 - 1\frac{19}{80} \text{ in}^2$

$\text{LA} = 27\frac{24}{80} \text{ in}^2 - 1\frac{19}{80} \text{ in}^2$

$LA = 26\frac{5}{80} \text{ in}^2$

$LA = 26\frac{1}{16} \text{ in}^2$

Two bases of the larger and smaller triangular prisms must be added:

$SA = 26\frac{1}{16} \text{ in}^2 + 2\left(3\frac{1}{2} \text{ in} \cdot \frac{1}{4} \text{ in}\right) + 2\left(\frac{1}{2} \cdot 4 \text{ in} \cdot 4 \text{ in}\right)$

$SA = 26\frac{1}{16} \text{ in}^2 + 2 \cdot \frac{1}{4} \text{ in} \cdot 3\frac{1}{2} \text{ in} + 16 \text{ in}^2$

$SA = 26\frac{1}{16} \text{ in}^2 + \frac{1}{2} \text{ in} \cdot 3\frac{1}{2} \text{ in} + 16 \text{ in}^2$

$SA = 26\frac{1}{16} \text{ in}^2 + \left(\frac{3}{2} \text{ in}^2 + \frac{1}{4} \text{ in}^2\right) + 16 \text{ in}^2$

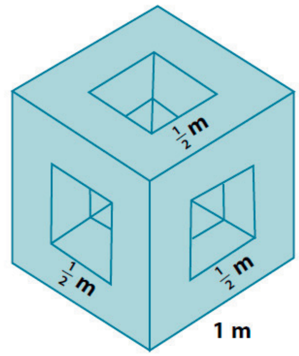
$SA = 26\frac{1}{16} \text{ in}^2 + 1 \text{ in}^2 + \frac{8}{16} \text{ in}^2 + \frac{4}{16} \text{ in}^2 + 16 \text{ in}^2$

$SA = 43\frac{13}{16} \text{ in}^2$

The surface area of the solid is $43\frac{13}{16} \text{ in}^2$.

7. **The diagram shows a cubic meter that has had three square holes punched completely through the cube on three perpendicular axes. Find the surface area of the remaining solid.**

Exterior surfaces of the cube (SA_1):



$$SA_1 = 6(1\text{ m})^2 - 6\left(\frac{1}{2}\text{ m}\right)^2$$

$$SA_1 = 6(1\text{ m}^2) - 6\left(\frac{1}{4}\text{ m}^2\right)$$

$$SA_1 = 6\text{ m}^2 - \frac{6}{4}\text{ m}^2$$

$$SA_1 = 6\text{ m}^2 - \left(1\frac{1}{2}\text{ m}^2\right)$$

$$SA_1 = 4\frac{1}{2}\text{ m}^2$$

Just inside each square hole are four intermediate surfaces that can be treated as the lateral area of a rectangular prism. Each has a height of $\frac{1}{4}$ m and perimeter of $\frac{1}{2}$ m $+ \frac{1}{2}$ m $+ \frac{1}{2}$ m $+ \frac{1}{2}$ m or 2 m.

$$SA_2 = 6(LA)$$

$$SA_2 = 6\left(2\text{ m} \cdot \frac{1}{4}\text{ m}\right)$$

$$SA_2 = 6 \cdot \frac{1}{2}\text{ m}^2$$

$$SA_2 = 3\text{ m}^2$$

The total surface area of the remaining solid is the sum of these two areas:

$$SA_T = SA_1 + SA_2.$$

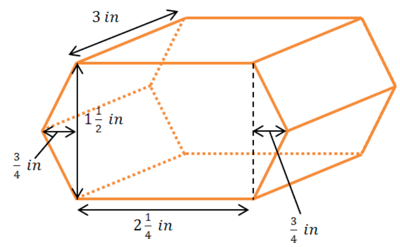
$$SA_T = 4\frac{1}{2}\text{ m}^2 + 3\text{ m}^2$$

$$SA_T = 7\frac{1}{2}\text{ m}^2$$

The surface area of the remaining solid is $7\frac{1}{2}$ m^2.

Exit Ticket Sample Solutions

The base of the right prism is a hexagon composed of a rectangle and two triangles. Find the volume of the right hexagonal prism using the formula $V = Bh$.



The area of the base is the sum of the areas of the rectangle and the two triangles.

$$B = A_{\text{rectangle}} + 2 \cdot A_{\text{triangle}}$$

$A_{\text{rectangle}} = lw$

$A_{\text{rectangle}} = 2\frac{1}{4}\text{ in.} \cdot 1\frac{1}{2}\text{ in.}$

$A_{\text{rectangle}} = \left(\frac{9}{4} \cdot \frac{3}{2}\right)\text{ in}^2$

$A_{\text{rectangle}} = \frac{27}{8}\text{ in}^2$

$A_{\text{triangle}} = \frac{1}{2}lw$

$A_{\text{triangle}} = \frac{1}{2}\left(1\frac{1}{2}\text{ in.} \cdot \frac{3}{4}\text{ in.}\right)$

$A_{\text{triangle}} = \left(\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{4}\right)\text{ in}^2$

$A_{\text{triangle}} = \frac{9}{16}\text{ in}^2$

$B = \frac{27}{8}\text{ in}^2 + 2\left(\frac{9}{16}\text{ in}^2\right)$

$B = \frac{27}{8}\text{ in}^2 + \frac{9}{8}\text{ in}^2$

$B = \frac{36}{8}\text{ in}^2$

$B = \frac{9}{2}\text{ in}^2$

$V = Bh$

$V = \left(\frac{9}{2}\text{ in}^2\right) \cdot 3\text{ in.}$

$V = \frac{27}{2}\text{ in}^3$

$V = 13\frac{1}{2}\text{ in}^3$

The volume of the hexagonal prism is $13\frac{1}{2}\text{ in}^3$.

Problem Set Sample Solutions

1. **Calculate the volume of each solid using the formula $V = Bh$ (all angles are 90 degrees).**

   a. $V = Bh$

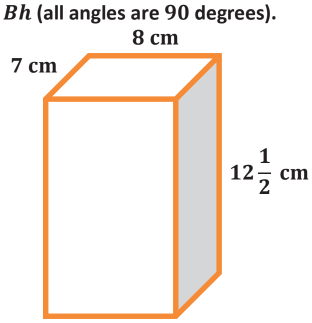
   $V = (8\text{ cm} \cdot 7\text{ cm}) \cdot 12\frac{1}{2}\text{cm}$

   $V = \left(56 \cdot 12\frac{1}{2}\right)\text{ cm}^3$

   $V = 672\text{ cm}^3 + 28\text{ cm}^3$

   $V = 700\text{ cm}^3$

   The volume of the solid is 700 cm^3.



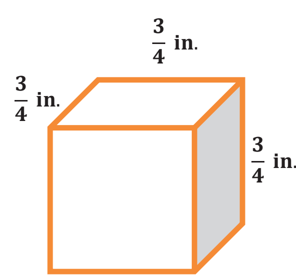
   b. $V = Bh$

   $V = \left(\frac{3}{4}\text{ in.} \cdot \frac{3}{4}\text{ in.}\right) \cdot \frac{3}{4}\text{ in.}$

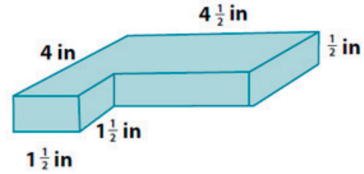
   $V = \left(\frac{9}{16}\right) \cdot \frac{3}{4}\text{in}^3$

   $V = \frac{27}{64}\text{ in}^3$

   The volume of the cube is $\frac{27}{64}\text{ in}^3$.



c. $V = Bh$



$B = A_{\text{rectangle}} + A_{\text{square}}$

$B = lw + s^2$

$B = \left(2\frac{1}{2}\text{ in.} \cdot 4\frac{1}{2}\text{ in.}\right) + \left(1\frac{1}{2}\text{ in.}\right)^2$

$B = \left(10\text{ in}^2 + 1\frac{1}{4}\text{ in}^2\right) + \left(1\frac{1}{2}\text{ in.} \cdot 1\frac{1}{2}\text{ in.}\right)$

$B = 11\frac{1}{4}\text{ in}^2 + \left(1\frac{1}{2}\text{ in}^2 + \frac{3}{4}\text{ in}^2\right)$

$B = 11\frac{1}{4}\text{ in}^2 + \frac{3}{4}\text{ in}^2 + 1\frac{1}{2}\text{ in}^2$

$B = 12\text{ in}^2 + 1\frac{1}{2}\text{ in}^2$

$B = 13\frac{1}{2}\text{ in}^2$

$V = Bh$

$V = 13\frac{1}{2}\text{ in}^2 \cdot \frac{1}{2}\text{ in.}$

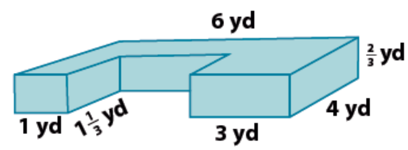
$V = \frac{13}{2}\text{ in}^3 + \frac{1}{4}\text{ in}^3$

$V = 6\text{ in}^3 + \frac{1}{2}\text{ in}^3 + \frac{1}{4}\text{ in}^3$

$V = 6\frac{3}{4}\text{ in}^3$

The volume of the solid is $6\frac{3}{4}\text{ in}^3$.

d. $V = Bh$



$B = (A_{\text{lg rectangle}}) - (A_{\text{sm rectangle}})$

$B = (lw)_1 - (lw)_2$

$B = (6\text{ yd.} \cdot 4\text{ yd.}) - \left(1\frac{1}{3}\text{ yd.} \cdot 2\text{ yd.}\right)$

$B = 24\text{ yd}^2 - \left(2\text{ yd}^2 + \frac{2}{3}\text{ yd}^2\right)$

$B = 24\text{ yd}^2 - 2\text{ yd}^2 - \frac{2}{3}\text{ yd}^2$

$B = 22\text{ yd}^2 - \frac{2}{3}\text{ yd}^2$

$B = 21\frac{1}{3}\text{ yd}^2$

$V = Bh$

$V = \left(21\frac{1}{3}\text{ yd}^2\right) \cdot \frac{2}{3}\text{ yd.}$

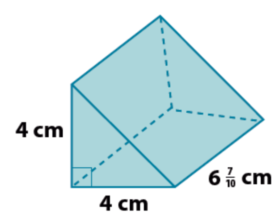
$V = 14\text{ yd}^3 + \left(\frac{1}{3}\text{ yd}^2 \cdot \frac{2}{3}\text{ yd.}\right)$

$V = 14\text{ yd}^3 + \frac{2}{9}\text{ yd}^3$

$V = 14\frac{2}{9}\text{ yd}^3$

The volume of the solid is $14\frac{2}{9}\text{ yd}^3$.

e. $V = Bh_{\text{prism}}$



$B = \frac{1}{2}bh_{\text{triangle}}$

$B = \frac{1}{2} \cdot 4\text{ cm} \cdot 4\text{ cm}$

$B = 2 \cdot 4\text{ cm}^2$

$B = 8\text{ cm}^2$

$V = Bh$

$V = 8\text{ cm}^2 \cdot 6\frac{7}{10}\text{ cm}$

$V = 48\text{ cm}^3 + \frac{56}{10}\text{ cm}^3$

$V = 48\text{ cm}^3 + 5\text{ cm}^3 + \frac{6}{10}\text{ cm}^3$

$V = 53\text{ cm}^3 + \frac{3}{5}\text{ cm}^3$

$V = 53\frac{3}{5}\text{ cm}^3$

The volume of the solid is $53\frac{3}{5}\text{ cm}^3$.

f. $V = Bh_{prism}$

$B = \frac{1}{2}bh_{\text{triangle}}$ $V = Bh$

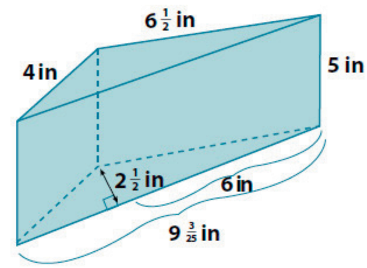
$B = \frac{1}{2} \cdot 9\frac{3}{25} \text{ in.} \cdot 2\frac{1}{2} \text{ in.}$ $V = \left(\frac{57}{5} \text{ in}^2\right) \cdot 5 \text{ in.}$

$B = \frac{1}{2} \cdot 2\frac{1}{2} \text{ in.} \cdot 9\frac{3}{25} \text{ in.}$ $V = 57 \text{ in}^3$

$B = \left(1\frac{1}{4}\right) \cdot \left(9\frac{3}{25}\right) \text{in}^2$

$B = \left(\frac{5}{4} \cdot \frac{228}{25}\right) \text{in}^2$

$B = \frac{57}{5} \text{ in}^2$ *The volume of the solid is* 57 in^3.



g. $V = Bh$

$B = A_{\text{rectangle}} + A_{\text{triangle}}$ $V = Bh$

$B = lw + \frac{1}{2}bh$ $V = 23\frac{1}{2} \text{ cm}^2 \cdot 9 \text{ cm}$

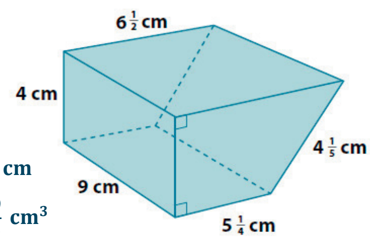
$B = \left(5\frac{1}{4} \text{ cm} \cdot 4 \text{ cm}\right) + \frac{1}{2}\left(4 \text{ cm} \cdot 1\frac{1}{4} \text{ cm}\right)$ $V = 207 \text{ cm}^3 + \frac{9}{2} \text{ cm}^3$

$B = (20 \text{ cm}^2 + 1 \text{ cm}^2) + \left(2 \text{ cm} \cdot 1\frac{1}{4} \text{ cm}\right)$ $V = 207 \text{ cm}^3 + 4 \text{ cm}^3 + \frac{1}{2} \text{ cm}^3$

$B = 21 \text{ cm}^2 + 2 \text{ cm}^2 + \frac{1}{2} \text{ cm}^2$ $V = 211\frac{1}{2} \text{ cm}^3$

$B = 23 \text{ cm}^2 + \frac{1}{2} \text{ cm}^2$

$B = 23\frac{1}{2} \text{ cm}^2$ *The volume of the solid is* $211\frac{1}{2} \text{ cm}^3$.



h. $V = Bh$

$B = A_{\text{rectangle}} + 2A_{\text{triangle}}$ $V = Bh$

$B = lw + 2 \cdot \frac{1}{2}bh$ $V = \frac{1}{8} \text{ in}^2 \cdot 2 \text{ in.}$

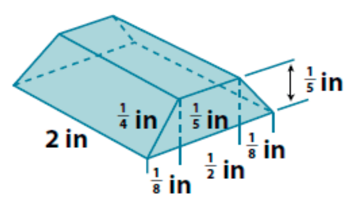
$B = \left(\frac{1}{2} \text{ in.} \cdot \frac{1}{5} \text{ in.}\right) + \left(1 \cdot \frac{1}{8} \text{ in.} \cdot \frac{1}{5} \text{ in.}\right)$ $V = \frac{1}{4} \text{ in}^3$

$B = \frac{1}{10} \text{ in}^2 + \frac{1}{40} \text{ in}^2$

$B = \frac{4}{40} \text{ in}^2 + \frac{1}{40} \text{ in}^2$ *The volume of the solid is* $\frac{1}{4} \text{ in}^3$.

$B = \frac{5}{40} \text{ in}^2$

$B = \frac{1}{8} \text{ in}^2$



2. Let l represent the length, w the width, and h the height of a right rectangular prism. Find the volume of the prism when

a. $l = 3$ cm, $w = 2\frac{1}{2}$ cm, and $h = 7$ cm.

$V = lwh$

$V = 3 \text{ cm} \cdot 2\frac{1}{2} \text{ cm} \cdot 7 \text{ cm}$

$V = 21 \cdot \left(2\frac{1}{2}\right) \text{ cm}^3$

$V = 52\frac{1}{2} \text{ cm}^3$ *The volume of the prism is* $52\frac{1}{2} \text{ cm}^3$.

b. $l = \frac{1}{4}$ cm, $w = 4$ cm, and $h = 1\frac{1}{2}$ cm.

$V = lwh$

$V = \frac{1}{4} \text{ cm} \cdot 4 \text{ cm} \cdot 1\frac{1}{2} \text{ cm}$

$V = 1\frac{1}{2} \text{ cm}^3$ *The volume of the prism is* $1\frac{1}{2} \text{ cm}^3$.

3. Find the length of the edge indicated in each diagram.

a. $V = Bh$ *Let* h *represent the number of inches in the height of the prism.*

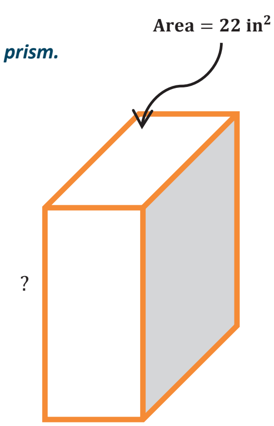
$93\frac{1}{2} \text{ in}^3 = 22 \text{ in}^2 \cdot h$

$93\frac{1}{2} \text{ in}^3 = 22h \text{ in}^2$

$22h = 93.5 \text{ in}$

$h = 4.25 \text{ in}$

The height of the right rectangular prism is $4\frac{1}{4}$ in.



What are possible dimensions of the base?

11 in *by* 2 in, *or* 22 in *by* 1 in

b. $V = Bh$ *Let* h *represent the number of meters in the height of the triangular base of the prism.*

$V = \left(\frac{1}{2}bh_{triangle}\right) \cdot h_{prism}$

$4\frac{1}{2} \text{ m}^3 = \left(\frac{1}{2} \cdot 3 \text{ m} \cdot h\right) \cdot 6 \text{ m}$

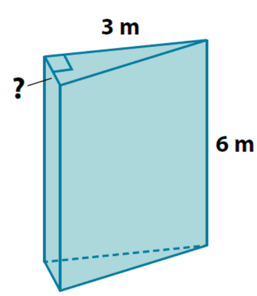
$4\frac{1}{2} \text{ m}^3 = \frac{1}{2} \cdot 18 \text{ m}^2 \cdot h$

$4\frac{1}{2} \text{ m}^3 = 9h \text{ m}^2$

$9h = 4.5 \text{ m}$

$h = 0.5 \text{ m}$

The height of the triangle is $\frac{1}{2}$ m.



Volume = 4½ m³

4. **The volume of a cube is $3\frac{3}{8}$ in^3. Find the length of each edge of the cube.**

$V = s^3$, and since the volume is a fraction, the edge length must also be fractional.

$3\frac{3}{8}\ \text{in}^3 = \frac{27}{8}\ \text{in}^3$

$3\frac{3}{8}\ \text{in}^3 = \frac{3}{2}\ \text{in.} \cdot \frac{3}{2}\ \text{in.} \cdot \frac{3}{2}\ \text{in.}$

$3\frac{3}{8}\ \text{in}^3 = \left(\frac{3}{2}\ \text{in.}\right)^3$

The lengths of the edges of the cube are $\frac{3}{2}$ in., or $1\frac{1}{2}$ in.

5. **Given a right rectangular prism with a volume of $7\frac{1}{2}$ ft^3, a length of 5 ft., and a width of 2 ft., find the height of the prism.**

$V = Bh$

$V = (lw)h$ *Let h represent the number of feet in the height of the prism.*

$7\frac{1}{2}\ \text{ft}^3 = (5\text{ft.} \cdot 2\text{ft.}) \cdot h$

$7\frac{1}{2}\ \text{ft}^3 = 10\ \text{ft}^2 \cdot h$

$7.5\ \text{ft}^3 = 10h\ \text{ft}^2$

$h = 0.75\ \text{ft.}$

The height of the right rectangular prism is $\frac{3}{4}$ ft. (or 9 in.).

Problem Set Sample Solutions

1. **Mark wants to put some fish and decorative rocks in his new glass fish tank. He measured the outside dimensions of the right rectangular prism and recorded a length of 55 cm, width of 42 cm, and height of 38 cm. He calculates that the tank will hold 87.78 L of water. Why is Mark's calculation of volume incorrect? What is the correct volume? Mark also failed to take into account the fish and decorative rocks he plans to add. How will this affect the volume of water in the tank? Explain.**

   $V = Bh = (lw)h$

   $V = 55\text{ cm} \cdot 42\text{ cm} \cdot 38\text{ cm}$

   $V = 2{,}310\text{ cm}^2 \cdot 38\text{ cm}$

   $V = 87{,}780\text{ cm}^3$

   $87{,}780\text{ cm}^3 = 87.78\text{ L}$

   Mark measured only the outside dimensions of the fish tank and did not account for the thickness of the sides of the tank. If he fills the tank with 87.78 L of water, the water will overflow the sides. Mark also plans to put fish and rocks in the tank, which will force water out of the tank if it is filled to capacity.

2. **Leondra bought an aquarium that is a right rectangular prism. The inside dimensions of the aquarium are 90 cm long, by 48 cm wide, by 60 cm deep. She plans to put water in the aquarium before purchasing any pet fish. How many liters of water does she need to put in the aquarium so that the water level is 5 cm below the top?**

   If the aquarium is 60 cm deep, then she wants the water to be 55 cm deep. Water takes on the shape of its container, so the water will form a right rectangular prism with a length of 90 cm, a width of 48 cm, and a height of 55 cm.

   $V = Bh = (lw)h$

   $V = (90\text{ cm} \cdot 48\text{ cm}) \cdot 55\text{ cm}$

   $V = 4{,}320\text{ cm}^2 \cdot 55\text{ cm}$

   $V = 237{,}600\text{ cm}^3$

   $237{,}600\text{ cm}^3 = 237.6\text{ L}$

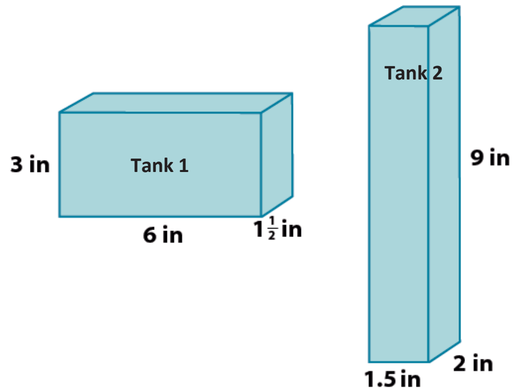
   The volume of water needed is 237.6 L.

3. **The inside space of two different water tanks are shown below. Which tank has a greater capacity? Justify your answer.**

$V_1 = Bh = (lw)h$

$V_1 = \left(6 \text{ in.} \cdot 1\frac{1}{2} \text{ in.}\right) \cdot 3 \text{ in.}$

$V_1 = (6$



$V_1 = 9 \text{ in}^2 \cdot 3 \text{ in.}$

$V_1 = 27 \text{ in}^3$

$V_2 = Bh = (lw)h$

$V_2 = \left(1\frac{1}{2} \text{ in.} \cdot 2 \text{ in.}\right) \cdot 9 \text{ in.}$

$V_2 = (2 \text{ in}^2 + 1 \text{ in}^2) \cdot 9 \text{ in .}$

$V_2 = 3 \text{ in}^2 \cdot 9 \text{ in.}$

$V_2 = 27 \text{ in}^3$

The tanks have the same volume, 27 in^3*. Each prism has a face with an area of* 18 in^2 *(base) and a height that is* $1\frac{1}{2}$ in.

4. **The inside of a tank is in the shape of a right rectangular prism. The base of that prism is 85 cm by 64 cm. What is the minimum height inside the tank if the volume of the liquid in the tank is 92 L ?**

$V = Bh = (lw)h$

$$92{,}000 \text{ cm}^3 = (85 \text{ cm} \cdot 64 \text{ cm}) \cdot h$$

$$92{,}000 \text{ cm}^3 = 5{,}440 \text{ cm}^2 \cdot h$$

$$92{,}000 \text{ cm}^3 \cdot \frac{1}{5{,}440 \text{ cm}^2} = 5{,}440 \text{ cm}^2 \cdot \frac{1}{5{,}440 \text{ cm}^2} \cdot h$$

$$\frac{92{,}000}{5{,}440} \text{ cm} = 1 \cdot h$$

$$16\frac{31}{34} \text{ cm} = h$$

The minimum height of the inside of the tank is $16\frac{31}{34}$ cm.

5. **An oil tank is the shape of a right rectangular prism. The inside of the tank is 36.5 cm long, 52 cm wide, and 29 cm high. If 45 liters of oil have been removed from the tank since it was full, what is the current depth of oil left in the tank?**

$V = Bh = (lw)h$

$V = (36.5\text{ cm} \cdot 52\text{ cm}) \cdot 29\text{ cm}$

$V = 1,898\text{ cm}^2 \cdot 29\text{ cm}$

$V = 55,042\text{ cm}^3$

The tank has a capacity of $55,042\text{ cm}^3$*, or* 55.042 L.

$55.042\text{ L} - 45\text{ L} = 10.042\text{ L}$

If 45 L *of oil have been removed from the tank, then* 10.042 L *are left in the tank.*

$V = Bh = (lw)h$

$$\begin{aligned}
10,042\text{ cm}^3 &= (36.5\text{ cm} \cdot 52\text{ cm}) \cdot h \\
10,042\text{ cm}^3 &= 1,898\text{ cm}^2 \cdot h \\
10,042\text{ cm}^3 \cdot \frac{1}{1,898\text{ cm}^2} &= 1,898\text{ cm}^2 \cdot \frac{1}{1,898\text{ cm}^2} \cdot h \\
\frac{10,042}{1,898}\text{ cm} &= 1 \cdot h \\
5.29\text{ cm} &\approx h
\end{aligned}$$

The depth of oil left in the tank is approximately 5.29 cm.

6. **The inside of a right rectangular prism-shaped tank has a base that is 14 cm by 24 cm and a height of 60 cm. The tank is filled to its capacity with water, and then 10.92 L of water is removed. How far did the water level drop?**

$V = Bh = (lw)h$

$V = (14\text{ cm} \cdot 24\text{ cm}) \cdot 60\text{ cm}$

$V = 336\text{ cm}^2 \cdot 60\text{ cm}$

$V = 20,160\text{ cm}^3$

The capacity of the tank is $20,160\text{ cm}^3$ *or* 20.16 L.

$20,160\text{ cm}^3 - 10,920\text{ cm}^3 = 9,240\text{ cm}^3$

When 10.92 L *or* $10,920\text{ cm}^3$ *of water is removed from the tank, there remains* $9,240\text{ cm}^3$ *of water in the tank.*

$V = Bh = (lw)h$

$$\begin{aligned}
9,240\text{ cm}^3 &= (14\text{ cm} \cdot 24\text{ cm}) \cdot h \\
9,240\text{ cm}^3 &= 336\text{ cm}^2 \cdot h \\
9,240\text{ cm}^3 \cdot \frac{1}{336\text{ cm}^2} &= 336\text{ cm}^2 \cdot \frac{1}{336\text{ cm}^2} \cdot h \\
\frac{9,240}{336}\text{ cm} &= 1 \cdot h \\
27\frac{1}{2}\text{ cm} &= h
\end{aligned}$$

The depth of the water left in the tank is $27\frac{1}{2}\text{ cm}$.

$60\text{ cm} - 27\frac{1}{2}\text{ cm} = 32\frac{1}{2}\text{ cm}$

This means that the water level has dropped $32\frac{1}{2}\text{ cm}$.

7. A right rectangular prism-shaped container has inside dimensions of $7\frac{1}{2}$ cm long and $4\frac{3}{5}$ cm wide. The tank is $\frac{3}{5}$ full of vegetable oil. It contains 0.414 L of oil. Find the height of the container.

$V = Bh = (lw)h$

$$414\text{ cm}^3 = \left(7\frac{1}{2}\text{ cm} \cdot 4\frac{3}{5}\text{ cm}\right) \cdot h$$

$$414\text{ cm}^3 = 34\frac{1}{2}\text{ cm}^2 \cdot h$$

$$414\text{ cm}^3 = \frac{69}{2}\text{ cm}^2 \cdot h$$

$$414\text{ cm}^3 \cdot \frac{2}{69\text{ cm}^2} = \frac{69}{2}\text{ cm}^2 \cdot \frac{2}{69\text{cm}^2} \cdot h$$

$$\frac{828}{69}\text{ cm} = 1 \cdot h$$

$$12\text{ cm} = h$$

The vegetable oil in the container is 12 cm deep, but this is only $\frac{3}{5}$ of the container's depth. Let d represent the depth of the container in centimeters.

$$12\text{ cm} = \frac{3}{5} \cdot d$$

$$12\text{ cm} \cdot \frac{5}{3} = \frac{3}{5} \cdot \frac{5}{3} \cdot d$$

$$\frac{60}{3}\text{ cm} = 1 \cdot d$$

$$20\text{ cm} = d$$

The depth of the container is 20 cm.

8. **A right rectangular prism with length of 10 in, width of 16 in, and height of 12 in is $\frac{2}{3}$ filled with water. If the water is emptied into another right rectangular prism with a length of 12 in, a width of 12 in, and height of 9 in, will the second container hold all of the water? Explain why or why not. Determine how far (above or below) the water level would be from the top of the container.**

$\frac{2}{3} \cdot 12 \text{ in} = \frac{24}{3} \text{ in} = 8 \text{ in}$ *The height of the water in the first prism is* 8 in.

$V = Bh = (lw)h$

$V = (10 \text{ in} \cdot 16 \text{ in}) \cdot 8 \text{ in}$

$V = 160 \text{ in}^2 \cdot 8 \text{ in}$

$V = 1{,}280 \text{ in}^3$

The volume of water is $1{,}280 \text{ in}^3$.

$V = Bh = (lw)h$

$V = (12 \text{ in} \cdot 12 \text{ in}) \cdot 9 \text{ in}$

$V = 144 \text{ in}^2 \cdot 9 \text{ in}$

$V = 1{,}296 \text{ in}^3$

The capacity of the second prism is $1{,}296 \text{ in}^3$*, which is greater than the volume of water, so the water will fit in the second prism.*

$V = Bh = (lw)h$ *Let* h *represent the depth of the water in the second prism in inches.*

$1{,}280 \text{ in}^3 = (12 \text{ in} \cdot 12 \text{ in}) \cdot h$

$1{,}280 \text{ in}^3 = (144 \text{ in}^2) \cdot h$

$1{,}280 \text{ in}^3 \cdot \frac{1}{144 \text{ in}^2} = 144 \text{ in}^2 \cdot \frac{1}{144 \text{ in}^2} \cdot h$

$\frac{1{,}280}{144} \text{ in} = 1 \cdot h$

$8\frac{128}{144} \text{ in} = h$

$8\frac{8}{9} \text{ in} = h$

The depth of the water in the second prism is $8\frac{8}{9}$ in.

$9 \text{ in} - 8\frac{8}{9} \text{ in} = \frac{1}{9} \text{ in}$

The water level will be $\frac{1}{9}$ in *from the top of the second prism.*

Exit Ticket Sample Solutions

Melody is planning a raised bed for her vegetable garden.

a. **How many square feet of wood does she need to create the bed?**

$2(4\text{ ft})(1.25\text{ ft}) + 2(2.5\text{ ft})(1.25\text{ ft}) = 16.25\text{ ft}^2$

The dimensions in feet are 4 ft. *by* 1.25 ft. *by* 2.5 ft. *The lateral area is* 16.25 ft^2.

b. **She needs to add soil. Each bag contains 1.5 cubic feet. How many bags will she need to fill the vegetable garden?**

$V = 4\text{ ft} \cdot 1.25\text{ ft} \cdot 2.5\text{ ft} = 12.5\text{ ft}^3$

The volume is 12.5 ft^3. *Divide the total cubic feet by* 1.5 ft^3 *to determine the number of bags.*

$12.5\text{ ft}^3 \div 1.5\text{ ft}^3 = 8\frac{1}{3}$

Melody will need to purchase 9 *bags of soil to fill the garden bed.*

Note that if students fail to recognize the need to round up to nine bags, this should be addressed. Also, if the thickness of the wood were given, then there would be soil left over, and possibly only 8 bags would be needed, depending on the thickness.

Problem Set Sample Solutions

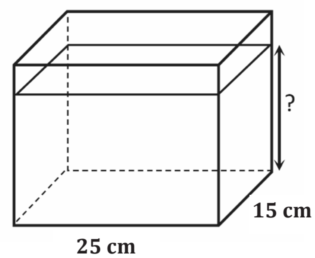
1. **The dimensions of several right rectangular fish tanks are listed below. Find the volume in cubic centimeters, the capacity in liters ($1\text{ L} = 1000\text{ cm}^3$), and the surface area in square centimeters for each tank. What do you observe about the change in volume compared with the change in surface area between the small tank and the extra-large tank?**

Tank Size	Length (cm)	Width (cm)	Height (cm)
Small	24	18	15
Medium	30	21	20
Large	36	24	25
Extra-Large	40	27	30

Tank Size	Volume (cm^3)	Capacity (L)	Surface Area (cm^2)
Small	$6,480$	6.48	$2,124$
Medium	$12,600$	12.6	$3,300$
Large	$21,600$	21.6	$4,728$
Extra-Large	$32,400$	32.4	$6,180$

While the volume of the extra-large tank is about five times the volume of the small tank, its surface area is less than three times that of the small tank.

2. A rectangular container 15 cm long by 25 cm wide contains 2.5 L of water.



   a. Find the height of the water level in the container. ($1\text{ L} = 1000\text{ cm}^3$)

   $2.5\text{ L} = 2,500\text{ cm}^3$

   To find the height of the water level, divide the volume in cubic centimeters by the area of the base.

$$\frac{2,500\text{ cm}^3}{25\text{ cm}\cdot 15\text{ cm}} = 6\frac{2}{3}\text{ cm}$$

   b. If the height of the container is 18 cm, how many more liters of water would it take to completely fill the container?

   Volume of tank: $(25\text{ cm}\times 15\text{ cm})\times 18\text{ cm} = 6,750\text{ cm}^3$

   Capacity of tank: 6.75 L

   Difference: $6.75\text{ L} - 2.5\text{ L} = 4.25\text{ L}$

   c. What percentage of the tank is filled when it contains 2.5 L of water?

$$\frac{2.5\text{ L}}{6.75\text{ L}} = 0.37 = 37\%$$

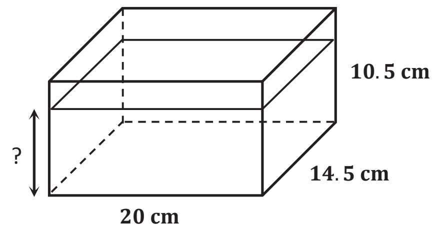
3. A rectangular container measuring 20 cm by 14.5 cm by 10.5 cm is filled with water to its brim. If 300 cm^3 are drained out of the container, what will be the height of the water level? If necessary, round to the nearest tenth.

   Volume: $(20\text{ cm}\times 14.5\text{ cm})\times 10.5\text{ cm} = 3,045\text{ cm}^3$

   Volume after draining: $2,745\text{ cm}^3$

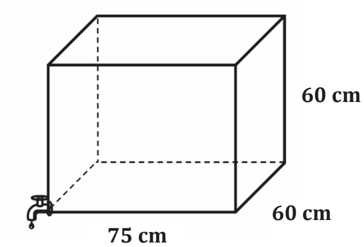
   Height (divide the volume by the area of the base):

$$\frac{2745\text{ cm}^3}{20\text{ cm}\times 14.5\text{ cm}} \approx 9.5\text{ cm}$$

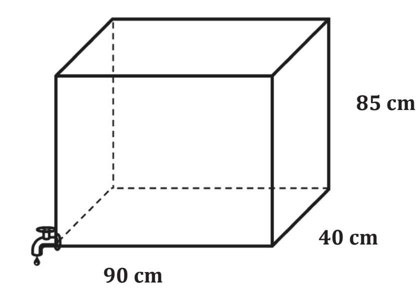


4. Two tanks are shown below. Both are filled to capacity, but the owner decides to drain them. Tank 1 is draining at a rate of 8 liters per minute. Tank 2 is draining at a rate of 10 liters per minute. Which tank empties first?

Tank 1



Tank 2



Tank 1 Volume: $75 \text{ cm} \times 60 \text{ cm} \times 60 \text{ cm} = 270,000 \text{ cm}^3$

Tank 2 Volume: $90 \text{ cm} \times 40 \text{ cm} \times 85 \text{ cm} = 306,000 \text{ cm}^3$

Tank 1 Capacity: 270 L *Tank 2 Capacity:* 306 L

To find the time to drain each tank, divide the capacity by the rate (liters per minute).

Time to drain tank 1: $\dfrac{270 \text{ L}}{8\,\frac{\text{L}}{\text{min}}} = 33.75 \text{ min}.$ *Time to drain tank 2:* $\dfrac{306 \text{ L}}{10\,\frac{\text{L}}{\text{min}}} = 30.6 \text{ min}.$

Tank 2 empties first.

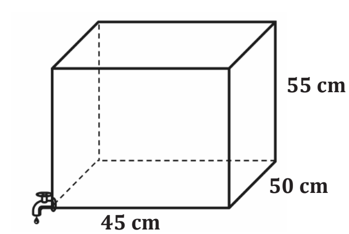
5. Two tanks are shown below. One tank is draining at a rate of 8 liters per minute into the other one, which is empty. After 10 minutes, what will be the height of the water level in the second tank? If necessary, round to the nearest minute.

Volume of the top tank: $45 \text{ cm} \times 50 \text{ cm} \times 55 \text{ cm} = 123,750 \text{ cm}^3$

Capacity of the top tank: 123.75 L

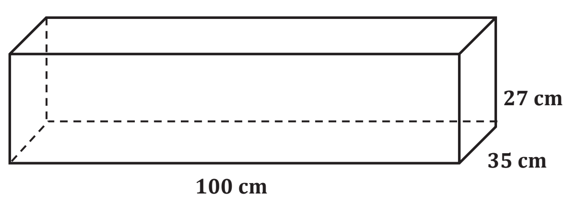
At $8\,\frac{\text{L}}{\text{min}}$ *for* 10 *minutes,* 80 L *will have drained into the bottom tank after* 10 *minutes.*

That is $80,000 \text{ cm}^3$*. To find the height, divide the volume by the area of the base.*

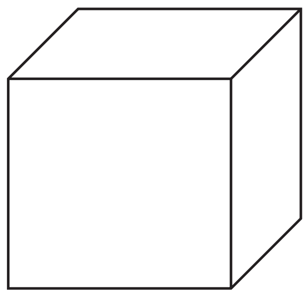


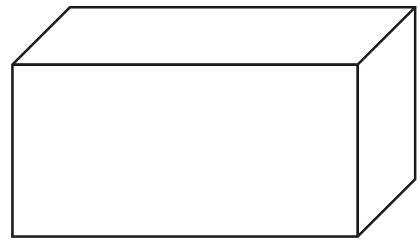
$$\frac{80,000 \text{ cm}^3}{100 \text{ cm} \cdot 35 \text{ cm}} \approx 22.9 \text{ cm}$$

After 10 *minutes, the height of the water in the bottom tank will be about* 23 *cm.*



6. Two tanks with equal volumes are shown below. The tops are open. The owner wants to cover one tank with a glass top. The cost of glass is $\$0.05$ per square inch. Which tank would be less expensive to cover? How much less?





Dimensions: 12 in. long by 8 in. wide by 10 in. high

Surface area: 96 in^2

Cost: $\dfrac{\$0.05}{\text{in}^2} \cdot 96\text{ in}^2 = \$4.80$

Dimensions: 15 in. long by 8 in. wide by 8 in. high

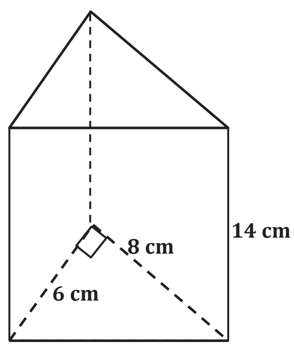
Surface area: 120 in^2

Cost: $\dfrac{\$0.05}{\text{in}^2} \cdot 120\text{ in}^2 = \$6.00$

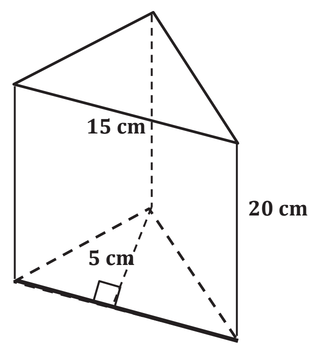
The first tank is less expensive. It is $\$1.20$ *cheaper.*

7. Each prism below is a gift box sold at the craft store.

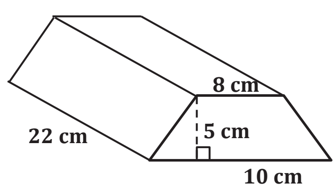
(a)



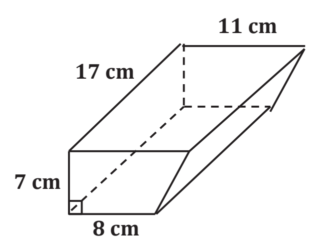
(b)



(c)



(d)



a. What is the volume of each prism?

(a) Volume $= 336\text{ cm}^3$, *(b) Volume* $= 750\text{ cm}^3$, *(c) Volume* $= 990\text{ cm}^3$, *(d) Volume* $= 1130.5\text{ cm}^3$

b. Jenny wants to fill each box with jelly beans. If one ounce of jelly beans is approximately 30 cm^3, estimate how many ounces of jelly beans Jenny will need to fill all four boxes? Explain your estimates.

Divide each volume in cubic centimeters by 30.

(a) 11.2 *ounces* *(b)* 25 *ounces* *(c)* 33 *ounces* *(d)* 37.7 *ounces*

Jenny would need a total of 106.9 *ounces.*

8. Two rectangular tanks are filled at a rate of 0.5 cubic inches per minute. How long will it take each tank to be half-full?

a. Tank 1 Dimensions: 15 in. by 10 in. by 12.5 in.

Volume: $1,875\ \text{in}^3$

Half of the volume is $937.5\ \text{in}^3$.

To find the number of minutes, divide the volume by the rate in cubic inches per minute.

Time: $1,875$ minutes.

b. Tank 2 Dimensions: $2\frac{1}{2}$ in. by $3\frac{3}{4}$ in. by $4\frac{3}{8}$ in.

Volume: $\frac{2625}{64}\ \text{in}^3$

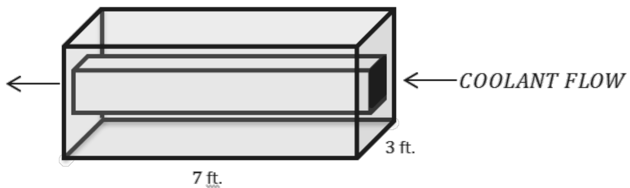
Half of the volume is $\frac{2625}{128}\ \text{in}^3$.

To find the number of minutes, divide the volume by the rate in cubic inches per minute.

Time: 41 minutes

Exit Ticket Sample Solutions

Lawrence is designing a cooling tank that is a square prism. A pipe in the shape of a smaller $2\text{ ft} \times 2\text{ ft}$ square prism passes through the center of the tank as shown in the diagram, through which a coolant will flow.



Scaffolding:
If students have mastered this concept easily, assign only parts (c) and (d).

a. **What is the volume of the tank including the cooling pipe?**

$7\text{ ft.} \times 3\text{ ft.} \times 3\text{ ft.} = 63\text{ ft}^3$

b. **What is the volume of coolant that fits inside the cooling pipe?**

$2\text{ ft.} \times 2\text{ ft.} \times 7\text{ ft.} = 28\text{ ft}^3$

c. **What is the volume of the shell (the tank not including the cooling pipe)?**

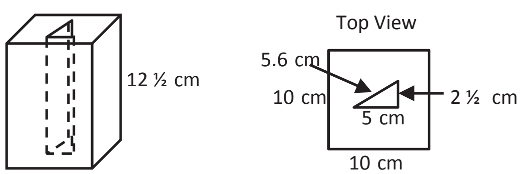
$63\text{ ft}^3 - 28\text{ ft}^3 = 35\text{ ft}^3$

d. **Find the surface area of the cooling pipe.**

$2\text{ ft.} \times 7\text{ ft.} \times 4 = 56\text{ ft}^2$

Problem Set Sample Solutions

1. **A child's toy is constructed by cutting a right triangular prism out of a right rectangular prism.**



a. **Calculate the volume of the rectangular prism.**

$10\text{ cm} \times 10\text{ cm} \times 12\frac{1}{2}\text{ cm} = 1250\text{ cm}^3$

b. **Calculate the volume of the triangular prism.**

$\frac{1}{2}\left(5\text{ cm} \times 2\frac{1}{2}\text{ cm}\right) \times 12\frac{1}{2}\text{ cm} = 78\frac{1}{8}\text{ cm}^3$

c. **Calculate the volume of the material remaining in the rectangular prism.**

$1250\text{ cm}^3 - 78\frac{1}{8}\text{ cm}^3 = 1171\frac{7}{8}\text{ cm}^3$

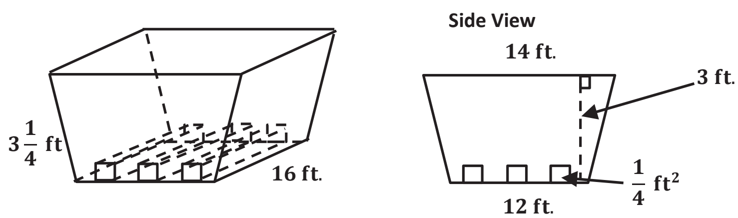
d. What is the largest number of triangular prisms that can be cut from the rectangular prism?

$$\frac{1250 \text{ cm}^3}{78\frac{1}{8}\text{ cm}^3} = 16$$

e. What is the surface area of the triangular prism (assume there is no top or bottom)?

$$5.6 \text{ cm} \times 12\frac{1}{2} \text{ cm} + 2\frac{1}{2} \text{ cm} \times 12\frac{1}{2} \text{ cm} + 5 \text{ cm} \times 12\frac{1}{2} \text{ cm} = 163\frac{3}{4} \text{ cm}^2$$

2. A landscape designer is constructing a flower bed in the shape of a right trapezoidal prism. He needs to run three identical square prisms through the bed for drainage.



a. What is the volume of the bed without the drainage pipes?

$$\frac{1}{2}(14 \text{ ft.} + 12 \text{ ft.}) \times 3 \text{ ft.} \times 16 \text{ ft.} = 624 \text{ ft}^3$$

b. What is the total volume of the three drainage pipes?

$$3\left(\frac{1}{4} \text{ ft}^2 \times 16 \text{ ft.}\right) = 12 \text{ ft}^3$$

c. What is the volume of soil if the planter is filled to $\frac{3}{4}$ of its total capacity with the pipes in place?

$$\frac{3}{4}\left(624 \text{ ft}^3\right) - 12 \text{ ft}^3 = 456 \text{ ft}^3$$

d. What is the height of the soil? If necessary, round to the nearest tenth.

$$\frac{456 \text{ ft}^3}{\frac{1}{2}(14 \text{ ft.} + 12 \text{ ft.}) \times 16 \text{ ft.}} \approx 2.2 \text{ ft.}$$

e. If the bed is made of 8 ft. $\times$ 4 ft. pieces of plywood, how many pieces of plywood will the landscape designer need to construct the bed without the drainage pipes?

$$2\left(3\frac{1}{4} \text{ ft.} \times 16 \text{ ft.}\right) + 12 \text{ ft.} \times 16 \text{ ft.} + 2\left(\frac{1}{2}(12 \text{ ft.} + 14 \text{ ft.}) \times 3 \text{ ft.}\right) = 374 \text{ ft}^2$$

$$374 \text{ ft}^2 \div \frac{(8 \text{ ft.} \times 4 \text{ ft.})}{\text{piece of plywood}} = 11.7, \text{ or } 12 \text{ pieces of plywood}$$

f. If the plywood needed to construct the bed costs $\$35$ per 8 ft. $\times$ 4 ft. piece, the drainage pipes cost $\$125$ each, and the soil costs $\$1.25$/cubic foot, how much does it cost to construct and fill the bed?

$$\frac{\$35}{\text{piece of plywood}}(12 \text{ pieces of plywood}) + \frac{\$125}{\text{pipe}}(3 \text{ pipes}) + \frac{\$1.25}{\text{ft}^3 \text{ soil}}(456 \text{ ft}^3 \text{ soil}) = \$1{,}365.00$$