## Exit Ticket Sample Solutions

1. Fill in the chart converting between fractions, decimals, and percents. Show work in the space provided.

| Fraction | Decimal | Percent |
| :---: | :---: | :---: |
| $\frac{1}{8}$ | $1 \div 8=0.125$ | $0.125 \times 100=12.5 \%$ |
| $1 \frac{125}{1000}=1 \frac{1}{8}$ | 1.125 | $1.125 \times 100=112.5 \%$ |
| $\left.\frac{2}{5}=\frac{1}{100}=12 \div 5\right) \div 100=0.004$ | $\frac{2}{5} \%$ |  |

2. Using the values from the chart in Problem 1, which is the least and which is the greatest? Explain how you arrived at your answers.

The least of the values is $\frac{2}{5} \%$, and the greatest is 1.125 . To determine which value is the least and which is the greatest, compare all three values in decimal form, fraction form, or percents. When comparing the three decimals, $0.125,1.125$, and 0.004 , one can note that 0.004 is the smallest value, so $\frac{2}{5} \%$ is the least of the values and 1. 125 is the greatest.

## Problem Set Sample Solutions

1. Create a model to represent the following percents.
a. $\quad \mathbf{9 0} \%$

b. $0.9 \%$

c. $\mathbf{9 0 0} \%$

d. $\frac{9}{10} \%$

2. Benjamin believes that $\frac{\mathbf{1}}{2} \%$ is equivalent to $\mathbf{5 0} \%$. Is he correct? Why or why not?

Benjamin is not correct because $\frac{1}{2} \%$ is equivalent to $0.50 \%$, which is equal to $\frac{\frac{1}{2}}{100}$. The second percent is equivalent to $\frac{50}{100}$. These percents are not equivalent.
3. Order the following from least to greatest.
$100 \%, \frac{1}{100}, 0.001 \%, \frac{1}{10}, 0.001,1.1,10$, and $\frac{10,000}{100}$
$0.001 \%, 0.001, \frac{1}{100}, \frac{1}{10}, 100 \%, 1.1,10$, and $\frac{10,000}{100}$
4. Fill in the chart by converting between fractions, decimals, and percents. Show work in the space below.

| Fraction | Decimal | Percent |
| :---: | :---: | :---: |
| $\frac{1}{1}$ | 1. 0 | 100\% |
| $\frac{33}{400}$ | 0.0825 | 8. 25\% |
| $6 \frac{1}{4}$ | 6.25 | 625\% |
| $\begin{gathered} \frac{1}{8} \\ \hline 100 \\ \hline \end{gathered}$ | 0.00125 | $\frac{1}{8} \%$ |
| $\frac{2}{300}$ | $0.00 \overline{6}$ | $\frac{2}{3} \%$ |
| $\frac{333}{1,000}$ | 0.333 | 33.3\% |
| $\frac{\frac{3}{4}}{100}$ | 0.0075 | $\frac{3}{4} \%$ |
| $2 \frac{1}{2}$ | 2.50 | 250\% |
| $\frac{1}{200}$ | 0.005 | $\frac{1}{2} \%$ |
| $\frac{150}{100}$ | 1.5 | 150\% |
| $\frac{5 \frac{1}{2}}{100}$ | 0.055 | $5 \frac{1}{2} \%$ |

## Exit Ticket Sample Solutions

1. On a recent survey, $60 \%$ of those surveyed indicated that they preferred walking to running.
a. If $\mathbf{5 4 0}$ people preferred walking, how many people were surveyed?

Let $n$ represent the number of people surveyed.
$0.60 n$ is the number of people who preferred walking.
Since 540 people preferred walking,

$$
\begin{aligned}
0.60 n & =540 \\
n & =\frac{540}{0.6}=\frac{5,400}{6}=900
\end{aligned}
$$

Therefore, 900 people were surveyed.
b. How many people preferred running?

Subtract 540 from 900.
$900-540=360$
Therefore, 360 people preferred running.
2. Which is greater: $\mathbf{2 5} \%$ of $\mathbf{1 5}$ or $\mathbf{1 5} \%$ of $\mathbf{2 5}$ ? Explain your reasoning using algebraic representations or visual models.

They are the same.
$0.25 \times 15=\frac{25}{100} \times 15=3.75$
$0.15 \times 25=\frac{15}{100} \times 25=3.75$
Also, you can see they are the same without actually computing the product because of any order, any grouping of multiplication.
$\frac{25}{100} \times 15=25 \times \frac{1}{100} \times 15=25 \times \frac{15}{100}$

## Problem Set Sample Solutions

Students should be encouraged to solve these problems using an algebraic approach.

1. Represent each situation using an equation. Check your answer with a visual model or numeric method.
a. What number is $\mathbf{4 0 \%}$ of $\mathbf{9 0}$ ?
$n=0.40(90)$
$n=36$
b. What number is $\mathbf{4 5} \%$ of $\mathbf{9 0}$ ?
$n=0.45(90)$
$n=40.5$
c. $\mathbf{2 7}$ is $\mathbf{3 0} \%$ of what number?
$27=0.3 n$
$\frac{27}{0.3}=1 n$
$90=n$
d. $\quad 18$ is $30 \%$ of what number?
$0.30 n=18$
$1 n=\frac{18}{0.3}$
$n=60$
e. $\quad 25.5$ is what percent of 85 ?
$25.5=p(85)$
$\frac{25.5}{85}=1 p$
$0.3=p$

$$
0.3=\frac{30}{100}=30 \%
$$

f. $\quad 21$ is what percent of 60 ?
$21=p(60)$
$\frac{21}{60}=1 p$
$0.35=p$

$$
0.35=\frac{35}{100}=35 \%
$$

2. $\mathbf{4 0} \%$ of the students on a field trip love the museum. If there are $\mathbf{2 0}$ students on the field trip, how many love the museum?

Let s represent the number of students who love the museum.

$$
\begin{aligned}
& s=0.40(20) \\
& s=8
\end{aligned}
$$

Therefore, 8 students love the museum.
3. Maya spent $\mathbf{4 0} \%$ of her savings to pay for a bicycle that cost her $\$ \mathbf{8 5}$.
a. How much money was in her savings to begin with?

Let $s$ represent the unknown amount of money in Maya's savings.

$$
\begin{aligned}
85 & =0.4 s \\
212.5 & =s
\end{aligned}
$$

Maya originally had $\$ \mathbf{2 1 2 . 5 0}$ in her savings.
b. How much money does she have left in her savings after buying the bicycle?
$\$ 212.50-\$ 85.00=\$ 127.50$
She has $\$ 127.50$ left in her savings after buying the bicycle.
4. Curtis threw $\mathbf{1 5}$ darts at a dartboard. $\mathbf{4 0} \%$ of his darts hit the bull's-eye. How many darts did not hit the bull's-eye? Let $d$ represent the number of darts that hit the bull's-eye.
$d=0.4(15)$
$d=6$
6 darts hit the bull's-eye. $15-6=9$
Therefore, 9 darts did not hit the bull's-eye.
5. A tool set is on sale for $\$ 424.15$. The original price of the tool set was $\$ 499.00$. What percent of the original price is the sale price?

Let p represent the unknown percent.
$424.15=p(499)$
$0.85=p$
The sale price is $85 \%$ of the original price.
6. Matthew scored a total of 168 points in basketball this season. He scored 147 of those points in the regular season and the rest were scored in his only playoff game. What percent of his total points did he score in the playoff game?

Matthew scored 21 points during the playoff game because $168-147=21$.
Let p represent the unknown percent.
$21=p(168)$
$0.125=p$
The points that Matthew scored in the playoff game were $12.5 \%$ of his total points scored in basketball this year.
7. Brad put 10 crickets in his pet lizard's cage. After one day, Brad's lizard had eaten $20 \%$ of the crickets he had put in the cage. By the end of the next day, the lizard had eaten $25 \%$ of the remaining crickets. How many crickets were left in the cage at the end of the second day?
Let $n$ represent the number of crickets eaten.
Day 1:
$n=0.2(10)$
$n=2$
At the end of the first day, Brad's lizard had eaten 2 of the crickets.
Day 2:
$n=0.25(10-2)$
$n=0.25(8)$
$n=2$
At the end of the second day, Brad's lizard had eaten a total of 4 crickets, leaving 6 crickets in the cage.
8. A furnace used $\mathbf{4 0} \%$ of the fuel in its tank in the month of March and then used $\mathbf{2 5} \%$ of the remaining fuel in the month of April. At the beginning of March, there were 240 gallons of fuel in the tank. How much fuel (in gallons) was left at the end of April?

March:
$n=0.4(240)$
$n=96$
Therefore, 96 gallons were used during the month of March, which means 144 gallons remain.
April:
$n=0.25(144)$
$n=36$
Therefore, 36 gallons were used during the month of April, which means 108 gallons remain.
There were 144 gallons of fuel remaining in the tank at the end of March and 108 gallons of fuel remaining at the end of April.
9. In Lewis County, there were 2,277 student athletes competing in spring sports in 2014. That was $110 \%$ of the number from 2013, which was $\mathbf{9 0} \%$ of the number from the year before. How many student athletes signed up for a spring sport in 2012?

2013:
$2,277=1.10 a$
$2,070=a$
Therefore, 2, 070 student athletes competed in spring sports in 2013.
2012:
$2,070=0.9 a$
$2,300=a$
Therefore, 2, 300 student athletes competed in spring sports in 2012.
There were 2, 070 students competing in spring sports in 2013 and 2,300 students in 2012.
10. Write a real-world word problem that could be modeled by the equation below. Identify the elements of the percent equation and where they appear in your word problem, and then solve the problem.

$$
57.5=p(250)
$$

Answers will vary. Greig is buying sliced almonds for a baking project. According to the scale, his bag contains 57.5 grams of almonds. Greig needs $\mathbf{2 5 0}$ grams of sliced almonds for his project. What percent of his total weight of almonds does Greig currently have?

The quantity 57.5 represents the part of the almonds that Greig currently has on the scale, the quantity 250 represents the $\mathbf{2 5 0}$ grams of almonds that he plans to purchase, and the variable $p$ represents the unknown percent of the whole quantity that corresponds to the quantity 57.5.

$$
\begin{aligned}
57.5 & =p(250) \\
\frac{1}{250}(57.5) & =p\left(\frac{1}{250}\right)(250) \\
\frac{57.5}{250} & =p(1) \\
0.23 & =p \\
0.23 & =\frac{23}{100}=23 \%
\end{aligned}
$$

Greig currently has $\mathbf{2 3} \%$ of the total weight of almonds that he plans to buy.

## Problem Set Sample Solutions

Encourage students to solve these problems using an equation. They can check their work with a visual or arithmetic model if needed. Problem 2, part (e) is a very challenging problem, and most students will likely solve it using arithmetic reasoning rather than an equation.

1. Solve each problem using an equation.
a. $\quad 49.5$ is what percent of 33 ?

$$
\begin{aligned}
49.5 & =p(33) \\
p & =1.5=150 \%
\end{aligned}
$$

b. $\mathbf{7 2}$ is what percent of $\mathbf{1 8 0}$ ?

$$
\begin{aligned}
72 & =\boldsymbol{p}(\mathbf{1 8 0}) \\
p & =0.4=\mathbf{4 0} \%
\end{aligned}
$$

c. What percent of $\mathbf{8 0}$ is $\mathbf{9 0}$ ?

$$
\begin{aligned}
90 & =p(80) \\
p & =1.125=112.5 \%
\end{aligned}
$$

2. This year, Benny is $\mathbf{1 2}$ years old, and his mom is $\mathbf{4 8}$ years old.
a. What percent of his mom's age is Benny's age?

Let p represent the percent of Benny's age to his mom's age.

$$
\begin{aligned}
12 & =p(48) \\
p & =0.25=25 \%
\end{aligned}
$$

Benny's age is 25\% of his mom's age.
b. What percent of Benny's age is his mom's age?

Let p represent the percent of his mom's age to Benny's age.

$$
\begin{aligned}
48 & =p(12) \\
p & =4=400 \%
\end{aligned}
$$

Benny's mom's age is 400\% of Benny's age.
c. In two years, what percent of his age will Benny's mom's age be at that time?

In two years, Benny will be 14, and his mom will be 50.

$$
\begin{aligned}
14 & \rightarrow \mathbf{1 0 0} \% \\
1 & \rightarrow\left(\frac{\mathbf{1 0 0}}{14}\right) \% \\
50 & \rightarrow \mathbf{5 0}\left(\frac{\mathbf{1 0 0}}{14}\right) \% \\
50 & \rightarrow \mathbf{2 5}\left(\frac{\mathbf{1 0 0}}{7}\right) \% \\
50 & \rightarrow\left(\frac{\mathbf{2 5 0 0}}{7}\right) \% \\
50 & \rightarrow \mathbf{3 5 7} \frac{\mathbf{1}}{7} \%
\end{aligned}
$$

His mom's age will be $357 \frac{1}{7} \%$ of Benny's age at that time.
d. In 10 years, what percent will Benny's mom's age be of his age?

In 10 years, Benny will be $\mathbf{2 2}$ years old, and his mom will be 58 years old.

$$
\begin{aligned}
22 & \rightarrow \mathbf{1 0 0} \% \\
1 & \rightarrow \frac{100}{22} \% \\
58 & \rightarrow 58\left(\frac{100}{22}\right) \% \\
58 & \rightarrow 29\left(\frac{100}{11}\right) \% \\
58 & \rightarrow \frac{2900}{11} \% \\
58 & \rightarrow 263 \frac{7}{11} \%
\end{aligned}
$$

In 10 years, Benny's mom's age will be $263 \frac{7}{11} \%$ of Benny's age at that time.
e. In how many years will Benny be $\mathbf{5 0} \%$ of his mom's age?

Benny will be $\mathbf{5 0} \%$ of his mom's age when she is $\mathbf{2 0 0} \%$ of his age (or twice his age). Benny and his mom are always 36 years apart. When Benny is 36, his mom will be 72, and he will be $50 \%$ of her age. So, in 24 years, Benny will be $\mathbf{5 0} \%$ of his mom's age.
f. As Benny and his mom get older, Benny thinks that the percent of difference between their ages will decrease as well. Do you agree or disagree? Explain your reasoning.
Student responses will vary. Some students might argue that they are not getting closer since they are always 36 years apart. However, if you compare the percents, you can see that Benny's age is getting closer to $\mathbf{1 0 0} \%$ of his mom's age, even though their ages are not getting any closer.
3. This year, Benny is $\mathbf{1 2}$ years old. His brother Lenny's age is $\mathbf{1 7 5} \%$ of Benny's age. How old is Lenny? Let $L$ represent Lenny's age. Benny's age is the whole.
$L=1.75(12)$
$L=21$
Lenny is $\mathbf{2 1}$ years old.
4. When Benny's sister Penny is $\mathbf{2 4}$, Benny's age will be $125 \%$ of her age.
a. How old will Benny be then?

Let b represent Benny's age when Penny is 24.

$$
\begin{aligned}
& b=1.25(24) \\
& b=30
\end{aligned}
$$

When Penny is 24, Benny will be 30.
b. If Benny is $\mathbf{1 2}$ years old now, how old is Penny now? Explain your reasoning.

Penny is 6 years younger than Benny. If Benny is 12 now, then Penny is 6.
5. Benny's age is currently $\mathbf{2 0 0} \%$ of his sister Jenny's age. What percent of Benny's age will Jenny's age be in 4 years?

If Benny is $\mathbf{2 0 0}$ \% of Jenny's age, then he is twice her age, and she is half of his age. Half of $\mathbf{1 2}$ is $\mathbf{6}$. Jenny is currently 6 years old. In 4 years, Jenny will be 10 years old, and Benny will be 16 years old.

Quantity $=$ Percent $\times$ Whole. Let $p$ represent the unknown percent. Benny's age is the whole.

$$
\begin{aligned}
10 & =p(16) \\
p & =\frac{10}{16} \\
p & =\frac{5}{8} \\
p & =0.625=62.5 \%
\end{aligned}
$$

In 4 years, Jenny will be $\mathbf{6 2 . 5} \%$ of Benny's age.
6. At the animal shelter, there are $\mathbf{1 5}$ dogs, $\mathbf{1 2}$ cats, $\mathbf{3}$ snakes, and 5 parakeets.
a. What percent of the number of cats is the number of dogs?
$\frac{15}{12}=1.25$. That is $125 \%$. The number of dogs is $125 \%$ the number of cats.
b. What percent of the number of cats is the number of snakes?
$\frac{3}{12}=\frac{1}{4}=0.25$. There are $25 \%$ as many snakes as cats.
c. What percent less parakeets are there than dogs?
$\frac{5}{15}=\frac{1}{3}$. That is $33 \frac{1}{3} \%$. There are $66 \frac{2}{3} \%$ less parakeets than dogs.
d. Which animal has $\mathbf{8 0} \%$ of the number of another animal?
$\frac{12}{15}=\frac{4}{5}=\frac{8}{10}=0.80$. The number of cats is $\mathbf{8 0} \%$ the number of dogs.
e. Which animal makes up approximately $14 \%$ of the animals in the shelter?

Quantity $=$ Percent $\times$ Whole. The total number of animals is the whole.

$$
\begin{aligned}
& q=0.14(35) \\
& q=4.9
\end{aligned}
$$

The quantity closest to 4.9 is 5, the number of parakeets.
7. Is $\mathbf{2}$ hours and $\mathbf{3 0}$ minutes more or less than $\mathbf{1 0} \%$ of a day? Explain your answer.
$2 \mathrm{hr} .30 \mathrm{~min} . \rightarrow 2.5 \mathrm{hr}$.; 24 hours is a whole day and represents the whole quantity in this problem.
$10 \%$ of 24 hours is 2.4 hours.
$2.5>2.4$, so 2 hours and 30 minutes is more than $10 \%$ of a day.
8. A club's membership increased from 25 to $\mathbf{3 0}$ members.
a. Express the new membership as a percent of the old membership.

The old membership is the whole.
Quantity $=$ Percent $\times$ Whole. Let $p$ represent the unknown percent.

$$
\begin{aligned}
30 & =p(25) \\
p & =1.2=120 \%
\end{aligned}
$$

The new membership is $\mathbf{1 2 0} \%$ of the old membership.
b. Express the old membership as a percent of the new membership.

The new membership is the whole.

$$
\begin{aligned}
30 & \rightarrow 100 \% \\
1 & \rightarrow \frac{100}{30} \% \\
25 & \rightarrow 25 \cdot \frac{100}{30} \% \\
25 & \rightarrow 5 \cdot \frac{100}{6} \% \\
25 & \rightarrow \frac{500}{6} \%=83 \frac{1}{3} \%
\end{aligned}
$$

The old membership is $83 \frac{1}{3} \%$ of the new membership.
9. The number of boys in a school is $\mathbf{1 2 0} \%$ the number of girls at the school.
a. Find the number of boys if there are $\mathbf{3 2 0}$ girls.

The number of girls is the whole.
Quantity $=$ Percent $\times$ Whole. Let brepresent the unknown number of boys at the school.

$$
\begin{aligned}
& b=1.2(320) \\
& b=384
\end{aligned}
$$

If there are $\mathbf{3 2 0}$ girls, then there are 384 boys at the school.
b. Find the number of girls if there are 360 boys.

The number of girls is still the whole.
Quantity $=$ Percent $\times$ Whole. Let $g$ represent the unknown number of girls at the school.

$$
\begin{aligned}
360 & =1.2(g) \\
g & =300
\end{aligned}
$$

If there are 360 boys at the school, then there are 300 girls.
10. The price of a bicycle was increased from $\$ 300$ to $\$ 450$.
a. What percent of the original price is the increased price?

The original price is the whole.
Quantity $=$ Percent $\times$ Whole. Let $p$ represent the unknown percent.

$$
\begin{gathered}
450=p(300) \\
p=1.5 \\
1.5=\frac{150}{100}=150 \%
\end{gathered}
$$

The increased price is $\mathbf{1 5 0} \%$ of the original price.
b. What percent of the increased price is the original price?

The increased price, $\$ 450$, is the whole.

$$
\begin{aligned}
450 & \rightarrow \mathbf{1 0 0} \% \\
1 & \rightarrow \frac{100}{450} \% \\
300 & \rightarrow \mathbf{3 0 0}\left(\frac{\mathbf{1 0 0}}{\mathbf{4 5 0}}\right) \% \\
300 & \rightarrow 2\left(\frac{100}{3}\right) \% \\
300 & \rightarrow \frac{200}{3} \% \\
300 & \rightarrow \mathbf{6 6} \frac{2}{3} \%
\end{aligned}
$$

The original price is $66 \frac{2}{3} \%$ of the increased price.
11. The population of Appleton is $\mathbf{1 7 5} \%$ of the population of Cherryton.
a. Find the population in Appleton if the population in Cherryton is $\mathbf{4 , 0 0 0}$ people.

The population of Cherryton is the whole.
Quantity $=$ Percent $\times$ Whole. Let a represent the unknown population of Appleton.

$$
\begin{aligned}
& a=1.75(4,000) \\
& a=7,000
\end{aligned}
$$

If the population of Cherryton is 4,000 people, then the population of Appleton is 7,000 people.
b. Find the population in Cherryton if the population in Appleton is $\mathbf{1 0}, \mathbf{5 0 0}$ people.

The population of Cherryton is still the whole.
Quantity $=$ Percent $\times$ Whole. Let c represent the unknown population of Cherryton.

$$
\begin{aligned}
10,500 & =1.75 c \\
c & =10,500 \div 1.75 \\
c & =6,000
\end{aligned}
$$

If the population of Appleton is 10,500 people, then the population of Cherryton is 6,000 people.
12. A statistics class collected data regarding the number of boys and the number of girls in each classroom at their school during homeroom. Some of their results are shown in the table below.
a. Complete the blank cells of the table using your knowledge about percent.

| Number of Boys ( $x$ ) | Number of Girls ( $y$ ) | Number of Girls as a Percent of the Number of Boys |
| :---: | :---: | :---: |
| 10 | 5 | 50\% |
| 4 | 1 | 25\% |
| 18 | 12 | $66 \frac{2}{3} \%$ |
| 5 | 10 | 200\% |
| 4 | 2 | 50\% |
| 20 | 18 | 90\% |
| 4 | 10 | 250\% |
| 10 | 6 | 60\% |
| 11 | 22 | 200\% |
| 15 | 5 | $33 \frac{1}{3} \%$ |
| 15 | 3 | 20\% |
| 20 | 15 | 75\% |
| 6 | 18 | 300\% |
| 25 | 10 | 40\% |
| 10 | 11 | 110\% |
| 20 | 2 | 10\% |
| 16 | 12 | 75\% |
| 14 | 7 | 50\% |
| 3 | 6 | 200\% |
| 12 | 10 | $83 \frac{1}{3} \%$ |

b. Using a coordinate plane and grid paper, locate and label the points representing the ordered pairs $(x, y)$.

See graph to the right.
c. Locate all points on the graph that would represent classrooms in which the number of girls $\boldsymbol{y}$ is $100 \%$ of the number of boys $x$. Describe the pattern that these points make.

The points lie on a line that includes the origin; therefore, it is a proportional relationship.


Lesson 3:
d. Which points represent the classrooms in which the number of girls as a percent of the number of boys is greater than $\mathbf{1 0 0} \%$ ? Which points represent the classrooms in which the number of girls as a percent of the number of boys is less than $100 \%$ ? Describe the locations of the points in relation to the points in part (c).

All points where $y>x$ are above the line and represent classrooms where the number of girls is greater than $100 \%$ of the number of boys. All points where $y<x$ are below the line and represent classrooms where the number of girls is less than $100 \%$ of the boys.
e. Find three ordered pairs from your table representing classrooms where the number of girls is the same percent of the number of boys. Do these points represent a proportional relationship? Explain your reasoning.

There are two sets of points that satisfy this question:
$\{(3,6),(5,10)$, and $(11,22)\}$ : The points do represent a proportional relationship because there is a constant of proportionality $k=\frac{y}{x}=2$.
$\{(4,2),(10,5)$, and $(14,7)\}$ : The points do represent a proportional relationship because there is a constant of proportionality $k=\frac{y}{x}=\frac{1}{2}$.
f. Show the relationship(s) from part (e) on the graph, and label them with the corresponding equation(s).

g. What is the constant of proportionality in your equation(s), and what does it tell us about the number of girls and the number of boys at each point on the graph that represents it? What does the constant of proportionality represent in the table in part (a)?

In the equation $y=2 x$, the constant of proportionality is 2 , and it tells us that the number of girls will be twice the number of boys, or $\mathbf{2 0 0} \%$ of the number of boys, as shown in the table in part (a).

In the equation $y=\frac{1}{2} x$, the constant of proportionality is $\frac{1}{2}$, and it tells us that the number of girls will be half the number of boys, or $50 \%$ of the number of boys, as shown in the table in part (a).

## Exit Ticket Sample Solutions

Erin wants to raise her math average to a 95 to improve her chances of winning a math scholarship. Her math average for the last marking period was an 81. Erin decides she must raise her math average by $15 \%$ to meet her goal. Do you agree? Why or why not? Support your written answer by showing your math work.

No, I do not agree. $15 \%$ of 81 is $12.15 .81+12.15=93.15$, which is less than 95 . I arrived at my answer using the equation below to find $15 \%$ of 81 .

Quantity $=$ Percent $\times$ Whole
Let $G$ stand for the number of points Erin's grade will increase by after a $15 \%$ increase from 81. The whole is 81, and the percent is $15 \%$. First, I need to find $15 \%$ of 81 to arrive at the number of points represented by a $15 \%$ increase. Then, I will add that to 81 to see if it equals 95, which is Erin's goal.

$$
\begin{aligned}
G & =0.15 \times 81 \\
G & =12.15
\end{aligned}
$$

Adding the points onto her average: $81.00+12.15=93.15$
Comparing it to her goal: $93.15<95$

## Problem Set Sample Solutions

1. A store advertises $\mathbf{1 5} \%$ off an item that regularly sells for $\$ \mathbf{3 0 0}$.
a. What is the sale price of the item?
$(0.85) 300=255 ;$ the sale price is $\$ 255$.
b. How is a $15 \%$ discount similar to a $15 \%$ decrease? Explain.

In both cases, you are subtracting $15 \%$ of the whole from the whole, or finding $85 \%$ of the whole.
c. If $\mathbf{8} \%$ sales tax is charged on the sale price, what is the total with tax?
$(1.08)(255)=275.40 ;$ the total with tax is $\$ 275.40$.
d. How is 8\% sales tax like an 8\% increase? Explain.

In both cases, you are adding 8\% of the whole to the whole, or finding 108\% of the whole.
2. An item that was selling for $\$ 72.00$ is reduced to $\$ 60.00$. Find the percent decrease in price. Round your answer to the nearest tenth.

The whole is $72.72-60=12.12$ is the part. Using Quantity $=$ Percent $\times$ Whole, I get $12=p \times 72$, where $p$ represents the unknown percent, and working backward, I arrive at $\frac{12}{72}=\frac{1}{6}=0.1 \overline{6}=p$.
So, it is about a 16.7\% decrease.
3. A baseball team had 80 players show up for tryouts last year and this year had 96 players show up for tryouts. Find the percent increase in players from last year to this year.

The number of players that showed up last year is the whole; 16 players are the quantity of change since $96-80=16$.

Quantity $=$ Percent $\times$ Whole. Let prepresent the unknown percent.

$$
\begin{gathered}
16=p(80) \\
p=0.2 \\
0.2=\frac{20}{100}=20 \%
\end{gathered}
$$

The number of players this year was a $\mathbf{2 0} \%$ increase from last year.
4. At a student council meeting, there was a total of 60 students present. Of those students, 35 were female.
a. By what percent is the number of females greater than the number of males?

The number of males $(60-35=25)$ at the meeting is the whole. The part (quantity) can be represented by the number of females (35) or how many more females there are than the number of males.

$$
\begin{aligned}
\text { Quantity } & =\text { Percent } \times \text { Whole } \\
35 & =p(25) \\
p & =1.4
\end{aligned}
$$

$1.4=140 \%$, which is $\mathbf{4 0} \%$ more than $100 \%$. Therefore, there were $\mathbf{4 0} \%$ more females than males at the student council meeting.
b. By what percent is the number of males less than the number of females?

The number of females (35) at the meeting is the whole. The part (quantity) can be represented by the number of males, or the number less of males than females (10).

$$
\begin{aligned}
& \text { Quantity }=\text { Percent } \times \text { Whole } \\
& 10=p(35) \\
& p \approx 0.29 \\
& 0.29=29 \%
\end{aligned}
$$

The number of males at the meeting is approximately 29\% less than the number of females.
c. Why is the percent increase and percent decrease in parts (a) and (b) different?

The difference in the number of males and females is the same in each case, but the whole quantities in parts (a) and (b) are different.
5. Once each day, Darlene writes in her personal diary and records whether the sun is shining or not. When she looked back though her diary, she found that over a period of $\mathbf{6 0 0}$ days, the sun was shining $\mathbf{6 0} \%$ of the time. She kept recording for another $\mathbf{2 0 0}$ days and then found that the total number of sunny days dropped to $\mathbf{5 0} \%$. How many of the final $\mathbf{2 0 0}$ days were sunny days?

To find the number of sunny days in the first 600 days, the total number of days is the whole.
Quantity $=$ Percent $\times$ Whole. Let s represent the number of sunny days.

$$
\begin{aligned}
& s=0.6(600) \\
& s=360
\end{aligned}
$$

There were $\mathbf{3 6 0}$ sunny days in the first $\mathbf{6 0 0}$ days.
The total number of days that Darlene observed was 800 days because $600+200=800$.

$$
\begin{aligned}
& d=0.5(800) \\
& d=400
\end{aligned}
$$

There was a total of $\mathbf{4 0 0}$ sunny days out of the $\mathbf{8 0 0}$ days.
The number of sunny days in the final 200 days is the difference of 400 days and 360 days.
$400-360=40$, so there were 40 sunny days of the last 200 days.
6. Henry is considering purchasing a mountain bike. He likes two bikes: One costs $\$ 500$, and the other costs $\$ 600$. He tells his dad that the bike that is more expensive is $20 \%$ more than the cost of the other bike. Is he correct? Justify your answer.
Yes. Quantity $=$ Percent $\times$ Whole. After substituting in the values of the bikes and percent, I arrive at the following equation: $600=1.2(500)$, which is a true equation.
7. State two numbers such that the lesser number is $25 \%$ less than the greater number.

Answers will vary. One solution is as follows: Greater number is 100; lesser number is 75.
8. State two numbers such that the greater number is $75 \%$ more than the lesser number.

Answers will vary. One solution is as follows: Greater number is $\mathbf{1 7 5}$; lesser number is 100.
9. Explain the difference in your thought process for Problems 7 and 8. Can you use the same numbers for each problem? Why or why not?

No. The whole is different in each problem. In Problem 7, the greater number is the whole. In Problem 8, the lesser number is the whole.
10. In each of the following expressions, $c$ represents the original cost of an item.
i. 0.90 c
ii. $0.10 c$
iii. $c-0.10 c$
a. Circle the expression(s) that represents $\mathbf{1 0} \%$ of the original cost. If more than one answer is correct, explain why the expressions you chose are equivalent.
c. Create a word problem involving a percent decrease so that the answer can be represented by expression (ii).

Answers will vary. The store's cashier told me I would get a 10\% discount on my purchase. How can I find the amount of the $\mathbf{1 0} \%$ discount?
d. Create a word problem involving a percent decrease so that the answer can be represented by expression (i).

Answers will vary. An item is on sale for $10 \%$ off. If the original price of the item is $c$, what is the final price after the $10 \%$ discount?
e. Tyler wants to know if it matters if he represents a situation involving a $25 \%$ decrease as $0.25 x$ or $(1-0.25) x$. In the space below, write an explanation that would help Tyler understand how the context of a word problem often determines how to represent the situation.

If the word problem asks you to find the amount of the $25 \%$ decrease, then $0.25 x$ would represent it. If the problem asks you to find the value after a $25 \%$ decrease, then $(1-0.25) x$ would be a correct representation.

## Exit Ticket Sample Solutions

1. A tank that is $\mathbf{4 0} \%$ full contains $\mathbf{6 4 8}$ gallons of water. Use a double number line to find the maximum capacity of the water tank.


I divided the percent line into intervals of $\mathbf{2 0} \%$ making five intervals of $\mathbf{2 0} \%$ in $\mathbf{1 0 0} \%$. I know that I have to divide $\frac{40}{2}$ to get 20 , so I divided $\frac{648}{2}$ to get 324 that corresponds with $20 \%$. Since there are five $20 \%$ intervals in $100 \%$, there are five 324 gallon intervals in the whole quantity, and $324 \cdot 5=1,620$. The capacity of the tank is 1,620 gallons.
2. Loretta picks apples for her grandfather to make apple cider. She brings him her cart with 420 apples. Her grandfather smiles at her and says "Thank you, Loretta. That is $35 \%$ of the apples that we need." Use mental math to find how many apples Loretta's grandfather needs. Describe your method.

420 is $35 \%$ of $1,200.35$ is not a factor of 100 , but 35 and 100 have a common factor of 5 . There are seven intervals of $5 \%$ in $35 \%$, so I divided 420 apples into seven intervals; $\frac{420}{7}=60$. There are 20 intervals of $5 \%$ in $100 \%$, so I multiplied as follows:

60-20
$60 \cdot 2 \cdot 10$
$120 \cdot 10$
1,200
Loretta's grandfather needs a total of 1, 200 apples to make apple cider.

## Problem Set Sample Solutions

## Use a double number line to answer Problems 1-5.

1. Tanner collected 360 cans and bottles while fundraising for his baseball team. This was $\mathbf{4 0} \%$ of what Reggie collected. How many cans and bottles did Reggie collect?


The greatest common factor of 40 and 100 is 20.
$\frac{1}{2}(40 \%)=20 \%$, and $\frac{1}{2}(360)=180$, so 180 corresponds with $20 \%$. There are five intervals of $20 \%$ in $100 \%$, and $\mathbf{5}(180)=900$, so Reggie collected 900 cans and bottles.
2. Emilio paid $\$ 287.50$ in taxes to the school district that he lives in this year. This year's taxes were a $15 \%$ increase from last year. What did Emilio pay in school taxes last year?


The greatest common factor of 100 and 115 is 5. There are 23 intervals of $5 \%$ in $115 \%$, and $\frac{287.5}{23}=12.5$, so 12.5 corresponds with $5 \%$. There are 20 intervals of $5 \%$ in $100 \%$, and $20(12.5)=250$, so Emilio paid $\$ 250$ in school taxes last year.
3. A snowmobile manufacturer claims that its newest model is $15 \%$ lighter than last year's model. If this year's model weighs 799 lb., how much did last year's model weigh?

$15 \%$ lighter than last year's model means $15 \%$ less than $100 \%$ of last year's model's weight, which is $\mathbf{8 5} \%$. The greatest common factor of 85 and 100 is 5 . There are 17 intervals of $5 \%$ in $85 \%$, and $\frac{799}{17}=47$, so 47 corresponds with $5 \%$. There are 20 intervals of $5 \%$ in $100 \%$, and $20(47)=940$, so last year's model weighed 940 pounds.
4. Student enrollment at a local school is concerning the community because the number of students has dropped to $\mathbf{5 0 4}$, which is a $\mathbf{2 0} \%$ decrease from the previous year. What was the student enrollment the previous year?


A $\mathbf{2 0} \%$ decrease implies that this year's enrollment is $\mathbf{8 0} \%$ of last year's enrollment. The greatest common factor of 80 and 100 is 20. There are 4 intervals of $20 \%$ in $80 \%$, and $\frac{504}{4}=126$, so 126 corresponds to $20 \%$. There are 5 intervals of $\mathbf{2 0} \%$ in $\mathbf{1 0 0} \%$, and $\mathbf{5}(126)=630$, so the student enrollment from the previous year was $\mathbf{6 3 0}$ students.
5. The color of paint used to paint a race car includes a mixture of yellow and green paint. Scotty wants to lighten the color by increasing the amount of yellow paint $30 \%$. If a new mixture contains 3 . 9 liters of yellow paint, how many liters of yellow paint did he use in the previous mixture?


The greatest common factor of 130 and 100 is 10. There are 13 intervals of $10 \%$ in $130 \%$, and $\frac{3.9}{13}=0.3$, so 0.3 corresponds to $10 \%$. There are 10 intervals of $10 \%$ in $100 \%$, and $10(0.3)=3$, so the previous mixture included 3 liters of yellow paint.

Use factors of 100 and mental math to answer Problems 6-10. Describe the method you used.
6. Alexis and Tasha challenged each other to a typing test. Alexis typed 54 words in one minute, which was $120 \%$ of what Tasha typed. How many words did Tasha type in one minute?

The greatest common factor of 120 and 100 is 20 , and there are 6 intervals of $20 \%$ in $\mathbf{1 2 0} \%$, so I divided 54 into 6 equal-sized intervals to find that 9 corresponds to $20 \%$. There are five intervals of $20 \%$ in $100 \%$, so there are five intervals of 9 words in the whole quantity. $9 \cdot 5=45$, so Tasha typed 45 words in one minute.
7. Yoshi is $5 \%$ taller today than she was one year ago. Her current height is $\mathbf{1 6 8} \mathbf{~ c m}$. How tall was she one year ago? 5\% taller means that Yoshi's height is 105\% of her height one year ago. The greatest common factor of 105 and 100 is 5, and there are 21 intervals of $5 \%$ in $105 \%$, so I divided 168 into 21 equal-sized intervals to find that $\mathbf{8} \mathbf{~ c m}$ corresponds to $5 \%$. There are 20 intervals of $5 \%$ in $\mathbf{1 0 0} \%$, so there are 20 intervals of $\mathbf{8 c m}$ in the whole quantity. $20 \cdot \mathbf{8 c m}=160 \mathrm{~cm}$, so Yoshi was 160 cm tall one year ago.
8. Toya can run one lap of the track in $1 \mathbf{m i n}$. $\mathbf{3} \mathbf{~ s e c} .$, which is $\mathbf{9 0} \%$ of her younger sister Niki's time. What is Niki's time for one lap of the track?
$1 \mathrm{~min} .3 \mathrm{sec}=63 \mathrm{sec}$. The greatest common factor of 90 and 100 is 10 , and there are nine intervals of 10 in 90, so I divided 63 sec. by 9 to find that 7 sec . corresponds to $10 \%$. There are 10 intervals of $10 \%$ in $100 \%$, so 10 intervals of 7 sec . represents the whole quantity, which is $70 \mathrm{sec} .70 \mathrm{sec} .=1 \mathrm{~min} .10 \mathrm{sec}$. Niki can run one lap of the track in 1 min .10 sec .
9. An animal shelter houses only cats and dogs, and there are $25 \%$ more cats than dogs. If there are 40 cats, how many dogs are there, and how many animals are there total?
$\mathbf{2 5 \%}$ more cats than dogs means that the number of cats is $125 \%$ the number of dogs. The greatest common factor of 125 and 100 is 25 . There are 5 intervals of $25 \%$ in $125 \%$, so I divided the number of cats into 5 intervals to find that 8 corresponds to $25 \%$. There are four intervals of $25 \%$ in $100 \%$, so there are four intervals of 8 in the whole quantity. $8 \cdot 4=32$. There are 32 dogs in the animal shelter.

The number of animals combined is $32+40=72$, so there are 72 animals in the animal shelter.
10. Angie scored 91 points on a test but only received a $65 \%$ grade on the test. How many points were possible on the test?

The greatest common factor of 65 and 100 is 5 . There are 13 intervals of $5 \%$ in $65 \%$, so I divided 91 points into 13 intervals and found that 7 points corresponds to $5 \%$. There are 20 intervals of $5 \%$ in $\mathbf{1 0 0} \%$, so $I$ multiplied 7 points times 20, which is 140 points. There were 140 points possible on Angie's test.

For Problems 11-17, find the answer using any appropriate method.
11. Robbie owns 15\% more movies than Rebecca, and Rebecca owns $\mathbf{1 0} \%$ more movies than Joshua. If Rebecca owns $\mathbf{2 2 0}$ movies, how many movies do Robbie and Joshua each have?

Robbie owns 253 movies, and Joshua owns 200 movies.
12. $20 \%$ of the seventh-grade students have math class in the morning. $16 \frac{2}{3} \%$ of those students also have science class in the morning. If 30 seventh-grade students have math class in the morning but not science class, find how many seventh-grade students there are.

There are 180 seventh-grade students.
13. The school bookstore ordered three-ring notebooks. They put $75 \%$ of the order in the warehouse and sold $\mathbf{8 0} \%$ of the rest in the first week of school. There are 25 notebooks left in the store to sell. How many three-ring notebooks did they originally order?

The store originally ordered 500 three-ring notebooks.
14. In the first game of the year, the modified basketball team made $62.5 \%$ of their foul shot free throws. Matthew made all 6 of his free throws, which made up $\mathbf{2 5} \%$ of the team's free throws. How many free throws did the team miss altogether?

The team attempted 24 free throws, made 15 of them, and missed 9.
15. Aiden's mom calculated that in the previous month, their family had used $\mathbf{4 0} \%$ of their monthly income for gasoline, and $63 \%$ of that gasoline was consumed by the family's SUV. If the family's SUV used $\$ 261.45$ worth of gasoline last month, how much money was left after gasoline expenses?

The amount of money spent on gasoline was $\$ 415$; the monthly income was $\$ 1,037.50$. The amount left over after gasoline expenses was $\$ 622.50$.
16. Rectangle $A$ is a scale drawing of Rectangle $B$ and has $25 \%$ of its area. If Rectangle $A$ has side lengths of 4 cm and 5 cm , what are the side lengths of Rectangle $B$ ?

Area ${ }_{A}=$ length $\times$ width
Area $_{A}=(5 \mathrm{~cm})(4 \mathrm{~cm})$
Area $_{A}=20$ cm $^{2}$


5 cm
The area of Rectangle A is 25\% of the area of Rectangle B.
$25 \% \times 4=100 \%$
$20 \times 4=80$
So, the area of Rectangle $B$ is $\mathbf{8 0} \mathbf{~ c m}^{2}$.
The value of the ratio of area $A$ to area $B$ is the square of the scale factor of the side lengths $A: B$.
The value of the ratio of area $A: B$ is $\frac{\mathbf{2 0}}{\mathbf{8 0}}=\frac{1}{4}$, and $\frac{1}{4}=\left(\frac{1}{2}\right)^{2}$, so the scale factor of the side lengths $A: B$ is $\frac{1}{2}$.
So, using the scale factor:
$\frac{1}{2}\left(\right.$ length $\left._{B}\right)=5 \mathrm{~cm} ;$ length $_{B}=10 \mathrm{~cm}$
$\frac{1}{2}\left(\right.$ width $\left._{B}\right)=4 \mathrm{~cm} ;$ width $_{B}=8 \mathrm{~cm}$
The dimensions of Rectangle B are $\mathbf{8 c m}$ and 10 cm .
17. Ted is a supervisor and spends $20 \%$ of his typical work day in meetings and $20 \%$ of that meeting time in his daily team meeting. If he starts each day at 7:30 a.m., and his daily team meeting is from 8:00 a.m. to 8:20 a.m., when does Ted's typical work day end?


20 minutes is $\frac{1}{3}$ of an hour since $\frac{20}{60}=\frac{1}{3}$.
Ted spends $\frac{1}{3}$ hour in his daily team meeting, so $\frac{1}{3}$ corresponds to $20 \%$ of his meeting time. There are 5 intervals of $20 \%$ in $100 \%$, and $5\left(\frac{1}{3}\right)=\frac{5}{3}$, so Ted spends $\frac{5}{3}$ hours in meetings.
$\frac{5}{3}$ of an hour corresponds to $20 \%$ of Ted's work day.


There are 5 intervals of $20 \%$ in $100 \%$, and $5\left(\frac{5}{3}\right)=\frac{25}{3}$, so Ted spends $\frac{25}{3}$ hours working. $\frac{25}{3}$ hours $=8 \frac{1}{3}$ hours. Since $\frac{1}{3}$ hour $=20$ minutes, Ted works a total of 8 hours 20 minutes. If he starts at 7:30 a.m., he works 4 hours 30 minutes until 12:00 p.m., and since $8 \frac{1}{3}-4 \frac{1}{2}=3 \frac{5}{6}$, Ted works another $3 \frac{5}{6}$ hours after 12:00 p.m. $\frac{1}{6}$ hour $=10$ minutes, and $\frac{5}{6}$ hour $=50$ minutes, so Ted works 3 hours 50 minutes after 12:00 p.m., which is 3:50 p.m. Therefore, Ted's typical work day ends at 3:50 p.m.

## Problem Set Sample Solutions

This problem set is a compilation of all types of percent problems from Lessons 2-6. For each problem, students should choose an appropriate strategy to find a solution. Students may also be asked to describe the mental math they used to solve the problem.

1. Micah has 294 songs stored in his phone, which is $\mathbf{7 0} \%$ of the songs that Jorge has stored in his phone. How many songs are stored on Jorge's phone?

Quantity $=$ Percent $\times$ Whole. Let s represent the number of songs on Jorge's phone.

$$
\begin{aligned}
294 & =\frac{70}{100} \cdot s \\
294 & =\frac{7}{10} \cdot s \\
294 \cdot \frac{10}{7} & =\frac{7}{10} \cdot \frac{10}{7} \cdot s \\
42 \cdot 10 & =1 \cdot s \\
420 & =s
\end{aligned}
$$

There are 420 songs stored on Jorge's phone.
2. Lisa sold 81 magazine subscriptions, which is $27 \%$ of her class's fundraising goal. How many magazine subscriptions does her class hope to sell?

Quantity $=$ Percent $\times$ Whole. Let $s$ represent the number of magazine subscriptions Lisa's class wants to sell.

$$
\begin{aligned}
81 & =\frac{27}{100} \cdot s \\
81 \cdot \frac{100}{27} & =\frac{27}{100} \cdot \frac{100}{27} \cdot s \\
3 \cdot 100 & =1 \cdot s \\
300 & =s
\end{aligned}
$$

Lisa's class hopes to sell 300 magazine subscriptions.
3. Theresa and Isaiah are comparing the number of pages that they read for pleasure over the summer. Theresa read 2,210 pages, which was $85 \%$ of the number of pages that Isaiah read. How many pages did Isaiah read?

Quantity $=$ Percent $\times$ Whole. Let $p$ represent the number of pages that Isaiah read.

$$
\begin{aligned}
2,210 & =\frac{85}{100} \cdot p \\
2,210 & =\frac{17}{20} \cdot p \\
2,210 \cdot \frac{20}{17} & =\frac{17}{20} \cdot \frac{20}{17} \cdot p \\
130 \cdot 20 & =1 \cdot p \\
2,600 & =p
\end{aligned}
$$

Isaiah read 2, 600 pages over the summer.
4. In a parking garage, the number of SUVs is $\mathbf{4 0} \%$ greater than the number of non-SUVs. Gina counted $\mathbf{9 8}$ SUVs in the parking garage. How many vehicles were parked in the garage?
$\mathbf{4 0} \%$ greater means $100 \%$ of the non-SUVs plus another $40 \%$ of that number, or $140 \%$.
Quantity $=$ Percent $\times$ Whole. Let $d$ represent the number of non-SUVs in the parking garage.

$$
\begin{aligned}
98 & =\frac{140}{100} \cdot d \\
98 & =\frac{7}{5} \cdot d \\
98 \cdot \frac{5}{7} & =\frac{7}{5} \cdot \frac{5}{7} \cdot d \\
14 \cdot 5 & =1 \cdot d \\
70 & =d
\end{aligned}
$$

There are 70 non-SUVs in the parking garage.
The total number of vehicles is the sum of the number of the SUVs and non-SUVs.
$70+98=168$. There is a total of 168 vehicles in the parking garage.
5. The price of a tent was decreased by $15 \%$ and sold for $\$ 76.49$. What was the original price of the tent in dollars?

If the price was decreased by $15 \%$, then the sale price is $15 \%$ less than $100 \%$ of the original price, or $85 \%$. Quantity $=$ Percent $\times$ Whole. Let $t$ represent the original price of the tent.

$$
\begin{aligned}
76.49 & =\frac{85}{100} \cdot t \\
76.49 & =\frac{17}{20} \cdot t \\
76.49 \cdot \frac{20}{17} & =\frac{17}{20} \cdot \frac{20}{17} \cdot t \\
\frac{1,529.8}{17} & =1 \cdot t \\
89.988 & \approx t
\end{aligned}
$$

Because this quantity represents money, the original price was $\$ 89.99$ after rounding to the nearest hundredth.
6. $\mathbf{4 0} \%$ of the students at Rockledge Middle School are musicians. $\mathbf{7 5} \%$ of those musicians have to read sheet music when they play their instruments. If 38 of the students can play their instruments without reading sheet music, how many students are there at Rockledge Middle School?

Let $m$ represent the number of musicians at the school, and let $s$ represent the total number of students. There are two whole quantities in this problem. The first whole quantity is the number of musicians. The 38 students who can play an instrument without reading sheet music represent $25 \%$ of the musicians.

$$
\begin{aligned}
\text { Quantity } & =\text { Percent } \times \text { Whole } \\
38 & =\frac{25}{100} \cdot m \\
38 & =\frac{1}{4} \cdot m \\
38 \cdot \frac{4}{1} & =\frac{1}{4} \cdot \frac{4}{1} \cdot m \\
\frac{152}{1} & =1 \cdot m \\
152 & =m
\end{aligned}
$$

There are 152 musicians in the school.

$$
\begin{aligned}
\text { Quantity } & =\text { Percent } \times \text { Whole } \\
152 & =\frac{40}{100} \cdot s \\
152 & =\frac{2}{5} \cdot s \\
152 \cdot \frac{5}{2} & =\frac{2}{5} \cdot \frac{5}{2} \cdot s \\
\frac{760}{2} & =1 \cdot s \\
380 & =s
\end{aligned}
$$

There are 380 students at Rockledge Middle School.
7. At Longbridge Middle School, 240 students said that they are an only child, which is $48 \%$ of the school's student enrollment. How many students attend Longbridge Middle School?

$$
\begin{aligned}
& \text { Quantity } \rightarrow \mathbf{1 0 0} \% \\
& 240 \rightarrow \mathbf{4 8} \% \\
& \frac{240}{48} \rightarrow 1 \% \\
& \frac{240}{48}(100) \rightarrow \mathbf{1 0 0} \% \\
& 5(100) \rightarrow \mathbf{1 0 0} \% \\
& 500 \rightarrow \mathbf{1 0 0} \%
\end{aligned}
$$

There are 500 students attending Longbridge Middle School.
8. Grace and her father spent $4 \frac{1}{2}$ hours over the weekend restoring their fishing boat. This time makes up $6 \%$ of the time needed to fully restore the boat. How much total time is needed to fully restore the boat?

$$
\begin{aligned}
& \text { Quantity } \rightarrow \% \\
& 4 \frac{1}{2} \rightarrow 6 \% \\
& \frac{9}{2} \rightarrow 6 \% \\
& \frac{9}{\frac{2}{6}} \rightarrow 1 \% \\
& \frac{9}{\frac{2}{6}}(100) \rightarrow 100 \% \\
& \left(\frac{9}{2}\right)\left(\frac{1}{6}\right) 100 \rightarrow 100 \% \\
& \left(\frac{9}{12}\right) 100 \rightarrow 100 \% \\
& \left(\frac{3}{4}\right) 100 \rightarrow 100 \% \\
& \frac{300}{4} \rightarrow 100 \% \\
& 75 \rightarrow 100 \%
\end{aligned}
$$

The total amount of time to restore the boat is 75 hours.
9. Bethany's mother was upset with her because Bethany's text messages from the previous month were $\mathbf{2 1 8} \%$ of the amount allowed at no extra cost under her phone plan. Her mother had to pay for each text message over the allowance. Bethany had 5, 450 text messages last month. How many text messages is she allowed under her phone plan at no extra cost?

$$
\begin{aligned}
& \text { Quantity } \rightarrow \% \\
& 5,450 \rightarrow 218 \% \\
& \frac{5,450}{218} \rightarrow 1 \% \\
& \frac{5,450}{218}(\mathbf{1 0 0}) \rightarrow \mathbf{1 0 0} \% \\
& 25(100) \rightarrow \mathbf{1 0 0} \% \\
& 2,500 \rightarrow 100 \%
\end{aligned}
$$

Bethany is allowed 2,500 text messages without extra cost.
10. Harry used $84 \%$ of the money in his savings account to buy a used dirt bike that cost him $\$ \mathbf{1}, 050$. How much money is left in Harry's savings account?
Quantity $\rightarrow$ \%
$1,050 \rightarrow 84 \%$
$\frac{1,050}{84} \rightarrow 1 \%$
$\frac{1.050}{84}(100) \rightarrow 100 \%$
12.5(100) $\rightarrow$ 100\%
$1,250 \rightarrow \mathbf{1 0 0} \%$

Harry started with $\$ 1,250$ in his account but then spent $\$ 1,050$ of it on the dirt bike.
$1,250-1,050=200$
Harry has $\$ 200$ left in his savings account.
11. 15\% of the students in Mr. Riley's social studies classes watch the local news every night. Mr. Riley found that 136 of his students do not watch the local news. How many students are in Mr. Riley's social studies classes?

If 15\% of his students do watch their local news, then 85\% do not.

$$
\begin{aligned}
& \text { Quantity } \rightarrow \% \\
& 136 \rightarrow 85 \% \\
& \frac{136}{85} \rightarrow 1 \% \\
& \left(\frac{136}{85}\right)(100) \rightarrow 100 \%
\end{aligned}
$$

1. $6(100) \rightarrow 100 \%$
$160 \rightarrow 100 \%$
There are $\mathbf{1 6 0}$ total students in Mr. Riley's social studies classes.
2. Grandma Bailey and her children represent about $9.1 \%$ of the Bailey family. If Grandma Bailey has 12 children, how many members are there in the Bailey family?

Quantity $\rightarrow$ \%
$13 \rightarrow 9.1 \%$
$\left(\frac{1}{9.1}\right)(13) \rightarrow 1 \%$
$100\left(\frac{13}{9.1}\right) \rightarrow 100 \%$
$\frac{1,300}{9.1} \rightarrow \mathbf{1 0 0} \%$
$142.857 \ldots$ 100\%
The Bailey family has 143 members.
13. Shelley earned $20 \%$ more money in tips waitressing this week than last week. This week she earned $\$ 72.00$ in tips waitressing. How much money did Shelley earn last week in tips?

Quantity $=$ Percent $\times$ Whole. Let $m$ represent the number of dollars Shelley earned waitressing last week.

$$
\begin{aligned}
72 & =\frac{120}{100} m \\
72\left(\frac{100}{120}\right) & =\frac{120}{100}\left(\frac{100}{120}\right) m \\
60 & =m
\end{aligned}
$$

Shelley earned $\$ 60$ waitressing last week.
14. Lucy's savings account has $35 \%$ more money than her sister Edy's. Together, the girls have saved a total of $\$ 206$. 80. How much money has each girl saved?

The money in Edy's account corresponds to 100\%. Lucy has 35\% more than Edy, so the money in Lucy's account corresponds to $\mathbf{1 3 5} \%$. Together, the girls have a total of $\$ \mathbf{2 0 6 . 8 0}$, which is $\mathbf{2 3 5} \%$ of Edy's account balance.
Quantity $=$ Pecent $\times$ Whole. Let brepresent Edy's savings account balance in dollars.

$$
\begin{aligned}
206.8 & =\frac{235}{100} \cdot b \\
206.8 & =\frac{47}{20} \cdot b \\
206.8 \cdot \frac{20}{47} & =\frac{47}{20} \cdot \frac{20}{47} \cdot b \\
\frac{4,136}{47} & =1 \cdot b \\
88 & =b
\end{aligned}
$$

Edy has saved $\$ 88$ in her account. Lucy has saved the remainder of the $\$ 206.80$, so $206.8-88=118.8$.
Therefore, Lucy has $\$ 118.80$ saved in her account.
15. Bella spent $15 \%$ of her paycheck at the mall, and $40 \%$ of that was spent at the movie theater. Bella spent a total of $\$ 13.74$ at the movie theater for her movie ticket, popcorn, and a soft drink. How much money was in Bella's paycheck?

$$
\begin{aligned}
& \$ 13.74 \rightarrow 40 \% \\
& \$ 3.435 \rightarrow \mathbf{1 0} \% \\
& \$ 34.35 \rightarrow \mathbf{1 0 0} \%
\end{aligned}
$$

Bella spent $\$ 34.35$ at the mall.

$$
\begin{gathered}
\$ 34.35 \rightarrow 15 \% \\
\$ 11.45 \rightarrow 5 \% \\
\$ 229 \rightarrow 100 \%
\end{gathered}
$$

Bella's paycheck was \$229.
16. On a road trip, Sara's brother drove $47.5 \%$ of the trip, and Sara drove $\mathbf{8 0} \%$ of the remainder. If Sara drove for 4 hours and 12 minutes, how long was the road trip?

There are two whole quantities in this problem. First, Sara drove $\mathbf{8 0} \%$ of the remainder of the trip; the remainder is the first whole quantity. 4 hr .12 min . is equivalent to $4 \frac{12}{60} \mathrm{hr} .=4.2 \mathrm{hr}$.

$$
\begin{gathered}
\text { Quantity } \rightarrow \% \\
4.2 \rightarrow \mathbf{8 0} \% \\
\frac{4.2}{80} \rightarrow 1 \% \\
\frac{4.2}{80}(100) \rightarrow 100 \% \\
\frac{420}{80} \rightarrow 100 \% \\
\frac{42}{8} \rightarrow 100 \% \\
5.25
\end{gathered} \rightarrow 100 \%<10
$$

The remainder of the trip that Sara's brother did not drive was 5.25 hours. He drove $47.5 \%$ of the trip, so the remainder of the trip was $52.5 \%$ of the trip, and the whole quantity is the time for the whole road trip.

$$
\begin{gathered}
\text { Quantity } \rightarrow \% \\
5.25 \rightarrow 52.5 \% \\
\frac{5.25}{52.5} \rightarrow 1 \% \\
\left(\frac{5.25}{52.5}\right)(100) \rightarrow 100 \% \\
\frac{525}{52.5} \rightarrow \mathbf{1 0 0} \% \\
10 \rightarrow 100 \%
\end{gathered}
$$

The road trip was a total of 10 hours.

## Exit Ticket Sample Solutions

A store that sells skis buys them from a manufacturer at a wholesale price of $\$ 57$. The store's markup rate is $\mathbf{5 0} \%$.
a. What price does the store charge its customers for the skis?
$57 \times(1+0.50)=85.50$. The store charges $\$ 85.50$ for the skis.
b. What percent of the original price is the final price? Show your work.

Quantity $=$ Percent $\times$ Whole. Let $P$ represent the unknown percent.

$$
\begin{aligned}
85.50 & =P(57) \\
85.50\left(\frac{1}{57}\right) & =P(57)\left(\frac{1}{57}\right) \\
1.50 & =P
\end{aligned}
$$

$1.50=\frac{150}{100}=150 \%$. The final price is $150 \%$ of the original price.
c. What is the percent increase from the original price to the final price?

The percent increase is $\mathbf{5 0} \%$ because $\mathbf{1 5 0} \%-\mathbf{1 0 0} \%=\mathbf{5 0} \%$.

## Problem Set Sample Solutions

In the following problems, students solve markup problems by multiplying the whole by $(1+m)$, where $m$ is the markup rate, and work backward to find the whole by dividing the markup price by $(1+m)$. They also solve markdown problems by multiplying the whole by $(1-m)$, where $m$ is the markdown rate, and work backward to find the whole by dividing the markdown price by $(1-m)$. Students also solve percent problems learned so far in the module.

1. You have a coupon for an additional $25 \%$ off the price of any sale item at a store. The store has put a robotics kit on sale for $15 \%$ off the original price of $\$ 40$. What is the price of the robotics kit after both discounts?
$(0.75)(0.85)(40)=25.50$. The price of the robotics kit after both discounts is $\$ 25.50$.
2. A sign says that the price marked on all music equipment is $30 \%$ off the original price. You buy an electric guitar for the sale price of $\$ 315$.
a. What is the original price?
$\frac{315}{1-0.30}=\frac{315}{0.70}=450$. The original price is $\$ 450$.
b. How much money did you save off the original price of the guitar?
$450-315=135$. I saved $\$ 135$ off the original price of the guitar.
c. What percent of the original price is the sale price?
$\frac{\mathbf{3 1 5}}{450}=\frac{\mathbf{7 0}}{\mathbf{1 0 0}}=\mathbf{7 0} \%$. The sale price is $70 \%$ of the original price.
3. The cost of a New York Yankee baseball cap is $\$ 24.00$. The local sporting goods store sells it for $\$ \mathbf{3 0} .00$. Find the markup rate.

Let $P$ represent the unknown percent.
$30=P(24)$
$P=\frac{\mathbf{3 0}}{\mathbf{2 4}}=1.25=(\mathbf{1 0 0} \%+25 \%)$. The markup rate is $\mathbf{2 5} \%$.
4. Write an equation to determine the selling price in dollars, $p$, on an item that is originally priced $s$ dollars after a markdown of 15\%.
$p=0.85 s$ or $p=(1-0.15) s$
a. Create and label a table showing five possible pairs of solutions to the equation.

| Price of Item Before <br> Markdown, $s$ (in dollars) | Price of Item After Markdown, <br> $p$ (in dollars) |
| :---: | :---: |
| 10 | 8.50 |
| 20 | 17.00 |
| 30 | 25.50 |
| 40 | 34.00 |
| 50 | 42.50 |

b. Create and label a graph of the equation.

c. Interpret the points $(0,0)$ and $(1, r)$.

The point $(0,0)$ means that a $\$ 0$ (free) item will cost $\$ 0$ because the $15 \%$ markdown is also \$0. The point $(1, r)$ is $(1,0.85)$, which represents the unit rate. It means that a $\$ 1.00$ item will cost $\$ 0.85$ after it is marked down by $15 \%$.
5. At the amusement park, Laura paid $\$ 6.00$ for a small cotton candy. Her older brother works at the park, and he told her they mark up the cotton candy by $\mathbf{3 0 0} \%$. Laura does not think that is mathematically possible. Is it possible, and if so, what is the price of the cotton candy before the markup?

Yes, it is possible. $\frac{6.00}{1+3}=\frac{6}{4}=1$. 50. The price of the cotton candy before the markup is $\$ 1.50$.
6. A store advertises that customers can take $25 \%$ off the original price and then take an extra $10 \%$ off. Is this the same as a $35 \%$ off discount? Explain.

No, because the $25 \%$ is taken first off the original price to get a new whole. Then, the extra $10 \%$ off is multiplied to the new whole. For example, $(1-0.25)(1-0.10)=0.675$ or $(0.75)(0.90)=0.675$. This is multiplied to the whole, which is the original price of the item. This is not the same as adding $25 \%$ and $10 \%$ to get $35 \%$ and then multiplying by ( $1-0.35$ ), or 0.65 .
7. An item that costs $\$ \mathbf{5 0 . 0 0}$ is marked $\mathbf{2 0} \%$ off. Sales tax for the item is $\mathbf{8} \%$. What is the final price, including tax? a. Solve the problem with the discount applied before the sales tax.
$(1.08)(0.80)(50)=43.20$. The final price is $\$ 43.20$.
b. Solve the problem with the discount applied after the sales tax.
$(0.80)(1.08)(50)=43.20$. The final price is $\$ 43.20$.
c. Compare your answers in parts (a) and (b). Explain.

My answers are the same. The final price is $\$ 43.20$. This is because multiplication is commutative.
8. The sale price for a bicycle is $\$ 315$. The original price was first discounted by $\mathbf{5 0} \%$ and then discounted an additional $\mathbf{1 0} \%$. Find the original price of the bicycle.
$(315 \div 0.9) \div 0.5=700$. The original price was $\$ 700$.
9. A ski shop has a markup rate of $\mathbf{5 0} \%$. Find the selling price of skis that cost the storeowner $\$ 300$.

Solution 1: Use the original price of $\$ 300$ as the whole. The markup rate is $\mathbf{5 0} \%$ of $\$ 300$ or $\$ 150$.
The selling price is $\$ 300+\$ 150=\$ 450$.
Solution 2: Multiply $\$ 300$ by 1 plus the markup rate (i.e., the selling price is $(1.5)(\$ 300)=\$ 450)$.
10. A tennis supply store pays a wholesaler $\$ 90$ for a tennis racquet and sells it for $\$ 144$. What is the markup rate?

Solution 1: Let the original price of $\$ 90$ be the whole. Quantity $=$ Percent $\times$ Whole.
$144=\operatorname{Percent}(90)$
$\frac{144}{90}=$ Percent

1. $6=160 \%$. This is a $\mathbf{6 0} \%$ increase. The markup rate is $\mathbf{6 0} \%$.

Solution 2:
Selling Price $=(1+m)($ Whole $)$

$$
\begin{aligned}
144 & =(1+m) 90 \\
1+m & =\frac{144}{90} \\
m & =1.6-1=0.6=60 \%
\end{aligned}
$$

The markup rate is $\mathbf{6 0} \%$.
11. A shoe store is selling a pair of shoes for $\$ 60$ that has been discounted by $25 \%$. What was the original selling price? Solution 1:

$$
\begin{aligned}
\$ 60 & \rightarrow \mathbf{7 5} \% \\
\$ 20 & \rightarrow \mathbf{2 5} \% \\
\$ 80 & \rightarrow \mathbf{1 0 0} \%
\end{aligned}
$$

The original price was $\$ 80$.
Solution 2: Let $\boldsymbol{x}$ be the original cost in dollars.

$$
\begin{aligned}
(1-0.25) x & =60 \\
\frac{3}{4} x & =60 \\
\left(\frac{4}{3}\right)\left(\frac{3}{4} x\right) & =\frac{4}{3}(60) \\
x & =80
\end{aligned}
$$

The original price was $\$ 80$.
12. A shoe store has a markup rate of $75 \%$ and is selling a pair of shoes for $\$ 133$. Find the price the store paid for the shoes.

Solution 1:

$$
\begin{aligned}
\$ 133 & \rightarrow \text { 175\% } \\
\$ 19 & \rightarrow \mathbf{2 5} \% \\
\$ 76 & \rightarrow \mathbf{1 0 0} \%
\end{aligned}
$$

The store paid \$76.
Solution 2: Divide the selling price by 1.75.

$$
\frac{133}{1.75}=76
$$

The store paid \$76.
13. Write $5 \frac{1}{4} \%$ as a simple fraction.
$\underline{21}$
$\overline{400}$
14. Write $\frac{3}{8}$ as a percent.
37.5\%
15. If $\mathbf{2 0} \%$ of the $\mathbf{7 0}$ faculty members at John F. Kennedy Middle School are male, what is the number of male faculty members?
$(0.20)(70)=14$. Therefore, 14 faculty members are male.
16. If a bag contains 400 coins, and $33 \frac{1}{2} \%$ are nickels, how many nickels are there? What percent of the coins are not nickels?
$(400)(0.335)=134$. Therefore, 134 of the coins are nickels. The percent of coins that are not nickels is $66 \frac{1}{2} \%$.
17. The temperature outside is $\mathbf{6 0}$ degrees Fahrenheit. What would be the temperature if it is increased by $\mathbf{2 0} \%$ ? $(60)(1.2)=72$. Therefore, the temperature would be 72 degrees Fahrenheit.

## Exit Ticket Sample Solutions

1. The veterinarian weighed Oliver's new puppy, Boaz, on a defective scale. He weighed 36 pounds. However, Boaz weighs exactly 34.5 pounds. What is the percent of error in measurement of the defective scale to the nearest tenth?

$$
\begin{aligned}
\frac{|36-34.5|}{|34.5|} \times 100 \% & =4 \frac{8}{23} \% \\
& \approx 4.3 \%
\end{aligned}
$$

2. Use the $\pi$ key on a scientific or graphing calculator to compute the percent of error of the approximation of pi , 3. 14, to the value $\pi$. Show your steps, and round your answer to the nearest hundredth of a percent.
$\frac{|3.14-\pi|}{|\boldsymbol{\pi}|} \times 100 \%=\mathbf{0 . 0 5} \%$
3. Connor and Angie helped take attendance during their school's practice fire drill. If the actual count was between 77 and 89 , inclusive, what is the most the absolute error could be? What is the most the percent error could be? Round your answer to the nearest tenth of a percent.

The most the absolute error could be is $|89-77|=|12|=12$.
The percent error will be largest when the exact value is smallest. The most the percent error could be is
$\frac{|12|}{|77|} \times 100 \%<15.6 \%$. The percent error is less than $15.6 \%$.

## Problem Set Sample Solutions

Students may choose any method to solve problems.

1. The odometer in Mr. Washington's car does not work correctly. The odometer recorded $\mathbf{1 3 . 2}$ miles for his last trip to the hardware store, but he knows the distance traveled is $\mathbf{1 5}$ miles. What is the percent error? Use a calculator and the percent error formula to help find the answer. Show your steps.

15 is the exact value, and 13.2 is the approximate value. Using the percent error formula, $\frac{|a-x|}{|x|} \times 100 \%$, the percent error is

$$
\frac{|13.2-15|}{|15|} \times 100 \%=12 \% .
$$

The percent error is equal to $12 \%$.
2. The actual length of a soccer field is 500 feet. A measuring instrument shows the length to be 493 feet. The actual width of the field is 250 feet, but the recorded width is 246.5 feet. Answer the following questions based on this information. Round all decimals to the nearest tenth.
a. Find the percent error for the length of the soccer field.

$$
\frac{|493-500|}{|500|} \times 100 \%=1.4 \%
$$



500 feet
$\boldsymbol{A}=\boldsymbol{l} \times \boldsymbol{w}$
$A=(500)(250)=125,000$
The actual area is 125,000 square feet.

Approximate area:
$\boldsymbol{A}=\boldsymbol{l} \times \boldsymbol{w}$
$A=(493)(246.5)$
The approximate area is $121,524.5$ square feet.

Percent error of the area:
$\frac{|121,524.5-125,000|}{|125,000|} \times 100 \%=2.8 \%$
c. Explain why the values from parts (a) and (b) are different.

In part (a), 1.4\% is the percent error for the length, which is one dimension of area. Part (b) is the percent error for the area, which includes two dimensions-length and width. The percent error for the width of the soccer field should be the same as the percent error for the length if the same measuring tool is used. So, $2.8 \%=1.4 \% \times 2$. However, this is not always the case. Percent error for the width is not always the same as the percent error for the length. It is possible to have an error for both the length and the width, yet the area has no error. For example: publicized length $=100$ feet, publicized width $=90$ feet, actual length $=150$ feet, and actual width $=60$ feet.
3. Kayla's class went on a field trip to an aquarium. One tank had 30 clown fish. She miscounted the total number of clown fish in the tank and recorded it as 24 fish. What is Kayla's percent error?

$$
\frac{|24-30|}{|30|} \times 100 \%=20 \%
$$

4. Sid used geometry software to draw a circle of radius 4 units on a grid. He estimated the area of the circle by counting the squares that were mostly inside the circle and got an answer of 52 square units.

a. Is his estimate too large or too small?
$A=\pi r^{2}$
$A=4^{2} \pi=16 \pi$
The exact area of the circle is $16 \pi$ square units. $16 \pi$ is approximately 50 . 3 . His estimate is too large.
b. Find the percent error in Sid's estimation to the nearest hundredth using the $\pi$ key on your calculator.

$$
\frac{|52-16 \pi|}{|16 \pi|} \times 100 \% \approx 3.45 \%
$$

5. The exact value for the density of aluminum is $2.699 \mathrm{~g} / \mathrm{cm}^{3}$. Working in the science lab at school, Joseph finds the density of a piece of aluminum to be $2.75 \mathrm{~g} / \mathrm{cm}^{3}$. What is Joseph's percent error? (Round to the nearest hundredth.)
$\frac{|2.75-2.699|}{|2.699|} \times 100 \% \approx 1.89 \%$
6. The world's largest marathon, The New York City Marathon, is held on the first Sunday in November each year. Between 2 million and 2.5 million spectators will line the streets to cheer on the marathon runners. At most, what is the percent error?

$$
\frac{|2.5-2|}{|2|} \times 100 \%=25 \%
$$

7. A circle is inscribed inside a square, which has a side length of $\mathbf{1 2 . 6} \mathbf{~ c m}$. Jared estimates the area of the circle to be about $80 \%$ of the area of the square and comes up with an estimate of $127 \mathrm{~cm}^{2}$.
a. Find the absolute error from Jared's estimate to two decimal places using the $\pi$ key on your calculator.

$$
\left|127-\pi 6.3^{2}\right| \approx 2.31
$$

The absolute error is approximately 2.31 cm .

12.6 cm
b. Find the percent error of Jared's estimate to two decimal places using the $\pi$ key on your calculator.

$$
\frac{\left|127-\pi 6.3^{2}\right|}{\left|\pi 6.3^{2}\right|} \times 100 \% \approx 1.85 \% . \text { The percent error is approximately } 1.85 \%
$$

c. Do you think Jared's estimate was reasonable?

Yes. The percent error is less than $2 \%$.
d. Would this method of computing the area of a circle always be too large?

Yes. If the circle has radius $r$, then the area of the circle is $\pi r^{2}$, and the area of the square is $4 r^{2}$.
$\frac{\pi r^{2}}{4 r^{2}}=\frac{\pi}{4}$. The area approximately equals $0.785=78.5 \%<\mathbf{8 0} \%$.
8. In a school library, $52 \%$ of the books are paperback. If there are 2,658 books in the library, how many of them are not paperback to the nearest whole number?
$100 \%-52 \%=48 \%$
Let $n$ represent the number of books that are not paperback.
$n=0.48(2,658)$
$n=1,275.84$
About 1, 276 books are not paperback.
9. Shaniqua has $\mathbf{2 5}$ \% less money than her older sister Jennifer. If Shaniqua has $\$ \mathbf{1 8 0}$, how much money does Jennifer have?
$100 \%-25 \%=75 \%$
Let j represent the amount of money that Jennifer has.

$$
\begin{aligned}
180 & =\frac{3}{4} j \\
\frac{4}{3}(180) & =\left(\frac{3}{4}\right)\left(\frac{4}{3}\right) j \\
240 & =j
\end{aligned}
$$

Jennifer has \$240.
10. An item that was selling for $\$ 1,102$ is reduced to $\$ 806$. To the nearest whole, what is the percent decrease? Let p represent the percent decrease.

$$
\begin{aligned}
& 1,102-806=296 \\
& 296=p \cdot 1,102 \\
& \frac{296}{1,102}=p \\
& 0.2686=p
\end{aligned}
$$

The percent decrease is approximately $27 \%$.
11. If $\mathbf{6 0}$ calories from fat is $75 \%$ of the total number of calories in a bag of chips, find the total number of calories in the bag of chips.

Let $t$ represent the total number of calories in a bag of chips.

$$
\begin{aligned}
60 & =\frac{3}{4} t \\
\frac{4}{3} \cdot 60 & =\frac{3}{4} \cdot \frac{4}{3} \cdot t \\
80 & =t
\end{aligned}
$$

The total number of calories in the bag of chips is $\mathbf{8 0}$ calories.

## Exit Ticket Sample Solutions

Terrence and Lee were selling magazines for a charity. In the first week, Terrence sold $\mathbf{3 0} \%$ more than Lee. In the second week, Terrence sold 8 magazines, but Lee did not sell any. If Terrence sold $50 \%$ more than Lee by the end of the second week, how many magazines did Lee sell?

Choose any model to solve the problem. Show your work to justify your answer.
Answers may vary.
Equation Model:
Let $m$ be the number of magazines Lee sold.
$150 \%-130 \%=20 \%$, so $0.2 m=8$ and $m=40$

Visual Model:


## Problem Set Sample Solutions

1. Solve each problem using an equation.
a. What is $\mathbf{1 5 0} \%$ of $\mathbf{6 2 5}$ ?
$n=1.5(625)$
$n=937.5$
b. $\mathbf{9 0}$ is $\mathbf{4 0 \%}$ of what number?
$90=0.4(n)$
$n=225$
c. What percent of 520 is 40 ? Round to the nearest hundredth of a percent.

$$
\begin{aligned}
40 & =p(520) \\
p & \approx 0.0769=7.69 \%
\end{aligned}
$$

2. The actual length of a machine is 12.25 cm . The measured length is 12.2 cm . Round the answer to part (b) to the nearest hundredth of a percent.
a. Find the absolute error.
$|12.2-12.25|=0.05$
The absolute error is $\mathbf{0 . 0 5} \mathbf{~ c m}$.
b. Find the percent error.
$\begin{aligned} & \frac{0.05}{|12.25|} \times 100 \%=0.4082 \% \\ & \text { percent error } \approx 0.41 \%\end{aligned}$
3. A rowing club has $\mathbf{6 0 0}$ members. $\mathbf{6 0} \%$ of them are women. After $\mathbf{2 0 0}$ new members joined the club, the percentage of women was reduced to $\mathbf{5 0} \%$. How many of the new members are women? 40 of the new members are women.
4. $\mathbf{4 0} \%$ of the marbles in a bag are yellow. The rest are orange and green. The ratio of the number of orange to the number of green is 4:5. If there are $\mathbf{3 0}$ green marbles, how many yellow marbles are there? Use a visual model to show your answer.
5 units $=\mathbf{3 0}$ marbles
1 unit $=30$ marbles $\div 5=6$ marbles 4 units $=4 \times 6$ marbles $=24$ marbles

$$
\begin{aligned}
30+24 & =54 \rightarrow 60 \% \\
18 & \rightarrow 20 \% \\
36 & \rightarrow 40 \%
\end{aligned}
$$

There are 36 yellow marbles because $40 \%$ of the marbles are yellow.

5. Susan has $\mathbf{5 0} \%$ more books than Michael. Michael has $\mathbf{4 0}$ books. If Michael buys $\mathbf{8}$ more books, will Susan have more or less books than Michael? What percent more or less will Susan's books be? Use any method to solve the problem.
Susan has 25\% more.
6. Harry's amount of money is $\mathbf{7 5} \%$ of Kayla's amount of money. After Harry earned $\$ \mathbf{3 0}$ and Kayla earned $\mathbf{2 5} \%$ more of her money, Harry's amount of money is $\mathbf{8 0} \%$ of Kayla's money. How much money did each have at the beginning? Use a visual model to solve the problem.


Each bar is $\$ 30$. Harry started with $\$ 90$, and Kayla started with $\$ 120$.

## Exit Ticket Sample Solutions

1. Erica's parents gave her $\$ 500$ for her high school graduation. She put the money into a savings account that earned 7. $5 \%$ annual interest. She left the money in the account for nine months before she withdrew it. How much interest did the account earn if interest is paid monthly?

$$
\begin{aligned}
& I=P r t \\
& I=(500)(0.075)\left(\frac{9}{12}\right) \\
& I=28.125
\end{aligned}
$$

The interest earned is \$28. 13.
2. If she would have left the money in the account for another nine months before withdrawing, how much interest would the account have earned?

$$
\begin{aligned}
& I=P r t \\
& I=(500)(0.075)\left(\frac{18}{12}\right) \\
& I=56.25
\end{aligned}
$$

The account would have earned \$56.25.
3. About how many years and months would she have to leave the money in the account if she wants to reach her goal of saving $\$ 750$ ?
$750-500=250 \quad$ She would need to earn \$250 in interest.

$$
\begin{aligned}
I & =\text { Prt } \\
250 & =(500)(0.075) t \\
250 & =37.5 t \\
250\left(\frac{1}{37.5}\right) & =\left(\frac{1}{37.5}\right)(37.5) t \\
6 \frac{2}{3} & =t
\end{aligned}
$$

It would take her 6 years and 8 months to reach her goal because $\frac{2}{3} \times 12$ months is $\mathbf{8}$ months.

## Problem Set Sample Solutions

1. Enrique takes out a student loan to pay for his college tuition this year. Find the interest on the loan if he borrowed $\$ 2,500$ at an annual interest rate of $\mathbf{6} \%$ for 15 years.
$I=2,500(0.06)(15)$
$I=2,250$
Enrique would have to pay $\$ 2,250$ in interest.
2. Your family plans to start a small business in your neighborhood. Your father borrows $\$ \mathbf{1 0}, \mathbf{0 0 0}$ from the bank at an annual interest rate of $8 \%$ rate for 36 months. What is the amount of interest he will pay on this loan?
$I=10,000(0.08)(3)$
$I=2,400$
He will pay $\$ 2,400$ in interest.
3. Mr. Rodriguez invests $\$ 2,000$ in a savings plan. The savings account pays an annual interest rate of $5.75 \%$ on the amount he put in at the end of each year.
a. How much will Mr. Rodriguez earn if he leaves his money in the savings plan for $\mathbf{1 0}$ years?
$I=2,000(0.0575)(10)$
$I=1,150$
He will earn \$1, 150.
b. How much money will be in his savings plan at the end of $\mathbf{1 0}$ years?

At the end of 10 years, he will have $\$ 3,150$ because $\$ 2,000+\$ 1,150=\$ 3,150$.
c. Create (and label) a graph in the coordinate plane to show the relationship between time and the amount of interest earned for 10 years. Is the relationship proportional? Why or why not? If so, what is the constant of proportionality?


Yes, the relationship is proportional because the graph shows a straight line touching the origin. The constant of proportionality is $\mathbf{1 1 5}$ because the amount of interest earned increases by $\$ \mathbf{1 1 5}$ for every one year.
d. Explain what the points $(\mathbf{0}, \mathbf{0})$ and $(1,115)$ mean on the graph.
$(0,0)$ means that no time has elapsed and no interest has been earned.
$(1,115)$ means that after 1 year, the savings plan would have earned \$115. 115 is also the constant of proportionality.
e. Using the graph, find the balance of the savings plan at the end of seven years.

From the table, the point $(7,805)$ means that the balance would be $\$ 2,000+\$ 805=\$ 2,805$.

Lesson 10:
f. After how many years will Mr. Rodriguez have increased his original investment by more than $\mathbf{5 0}$ \%? Show your work to support your answer.

Quantity $=$ Percent $\times$ Whole
Let $\mathbf{Q}$ be the account balance that is $\mathbf{5 0} \%$ more than the original investment.

$$
\begin{aligned}
& Q>(1+0.50)(2,000) \\
& Q>3,000
\end{aligned}
$$

The balance will be greater than $\$ 3,000$ beginning between 8 and 9 years because the graph shows $(8,920)$ and $(9,1035)$, so $\$ 2,000+\$ 920=\$ 2,920<\$ 3,000$, and $\$ 2,000+\$ 1,035=\$ 3,035>\$ 3,000$.

## Challenge Problem:

4. George went on a game show and won $\$ 60,000$. He wanted to invest it and found two funds that he liked. Fund $\mathbf{2 5 0}$ earns $\mathbf{1 5} \%$ interest annually, and Fund 100 earns $\mathbf{8 \%}$ interest annually. George does not want to earn more than $\$ 7,500$ in interest income this year. He made the table below to show how he could invest the money.

|  | $\boldsymbol{I}$ | $\boldsymbol{P}$ | $\boldsymbol{r}$ | $\boldsymbol{t}$ |
| :---: | :---: | :---: | :---: | :---: |
| Fund 100 | $0.08 x$ | $x$ | 0.08 | 1 |
| Fund 250 | $0.15(60000-x)$ | $60,000-x$ | 0.15 | 1 |
| Total | 7,500 | 60,000 |  |  |

a. Explain what value $x$ is in this situation.
$x$ is the principal, in dollars, that George could invest in Fund 100.
b. Explain what the expression $\mathbf{6 0}, \mathbf{0 0 0}-x$ represents in this situation.
$60,000-x$ is the principal, in dollars, that George could invest in Fund 250. It is the money he would have left over once he invests in Fund 100.
c. Using the simple interest formula, complete the table for the amount of interest earned.

See the table above.
d. Write an inequality to show the total amount of interest earned from both funds.
$0.08 x+0.15(60,000-x) \leq 7,500$
e. Use algebraic properties to solve for $x$ and the principal, in dollars, George could invest in Fund 100. Show your work.

$$
\begin{aligned}
0.08 x+9,000-0.15 x & \leq 7,500 \\
9,000-0.07 x & \leq 7,500 \\
9,000-9,000-0.07 x & \leq 7,500-9,000 \\
-0.07 x & \leq-1,500 \\
\left(\frac{1}{-0.07}\right)(-0.07 x) & \leq\left(\frac{1}{-0.07}\right)(-1,500) \\
x & \geq 21,428.57
\end{aligned}
$$

$x$ approximately equals $\$ 21,428.57$. George could invest $\$ 21,428.57$ or more in Fund 100.
f. Use your answer from part (e) to determine how much George could invest in Fund 250. He could invest $\$ 38,571$. 43 or less in Fund 250 because $60,000-21,428.57=38,571.43$.
g. Using your answers to parts (e) and (f), how much interest would George earn from each fund?

Fund 100: $0.08 \times 21,428.57 \times 1$ approximately equals $\$ 1,714.29$.
Fund 250: $0.15 \times 38,571.43 \times 1$ approximately equals $\$ 5,785.71$ or $\$ 7,500-\$ 1,714.29$.

## Exit Ticket Sample Solutions

Lee sells electronics. He earns a 5\% commission on each sale he makes.
a. Write an equation that shows the proportional relationship between the dollar amount of electronics Lee sells, $d$, and the amount of money he makes in commission, $c$.
$c=\frac{1}{20} d$ or $c=0.05 d$
b. Express the constant of proportionality as a decimal.
0.05
c. Explain what the constant of proportionality means in the context of this situation.

The constant of proportionality of 0.05 means that Lee would earn five cents for every dollar of electronics that he sells.
d. If Lee wants to make $\$ 100$ in commission, what is the dollar amount of electronics he must sell?

$$
\begin{aligned}
c & =0.05 d \\
100 & =0.05 d \\
\frac{1}{0.05}(100) & =\frac{1}{0.05}(0.05) d \\
2,000 & =d
\end{aligned}
$$

Lee must sell \$2,000 worth of electronics.

## Problem Set Sample Solutions

1. A school district's property tax rate rises from $2.5 \%$ to $2.7 \%$ to cover a $\$ \mathbf{3 0 0}, 000$ budget deficit (shortage of money). What is the value of the property in the school district to the nearest dollar? (Note: Property is assessed at $100 \%$ of its value.)

Let $W$ represent the worth of the property in the district, in dollars.

$$
300,000=0.002 W
$$

$$
300,000\left(\frac{1}{0.002}\right)=0.002\left(\frac{1}{0.002}\right) W
$$

$$
150,000,000=W
$$

The property is worth $\$ 150,000,000$.
2. Jake's older brother, Sam, has a choice of two summer jobs. He can either work at an electronics store or at the school's bus garage. The electronics store would pay him to work 15 hours per week. He would make $\$ 8$ per hour plus a $2 \%$ commission on his electronics sales. At the school's bus garage, Sam could earn $\$ 300$ per week working 15 hours cleaning buses. Sam wants to take the job that pays him the most. How much in electronics would Sam have to sell for the job at the electronics store to be the better choice for his summer job?

Let $S$ represent the amount, in dollars, sold in electronics.

$$
\begin{aligned}
& 300<8(15)+0.02(S) \\
& 300<120+0.02 S \\
& 180<0.02 S \\
& 180\left(\frac{1}{0.02}\right)<0.02\left(\frac{1}{0.02}\right) S \\
& 9,000<S
\end{aligned}
$$

Sam would have to sell more than $\$ 9,000$ in electronics for the electronics store to be the better choice.
3. Sarah lost her science book. Her school charges a lost book fee equal to $75 \%$ of the cost of the book. Sarah received a notice stating she owed the school $\$ 60$ for the lost book.
a. Write an equation to represent the proportional relationship between the school's cost for the book and the amount a student must pay for a lost book. Let $B$ represent the school's cost of the book in dollars and $N$ represent the student's cost in dollars.
$N=0.75 B$
b. What is the constant or proportionality? What does it mean in the context of this situation?

The constant of proportionality is $75 \%=0.75$. It means that for every $\$ 1$ the school spends to purchase a textbook, a student must pay $\$ 0.75$ for a lost book.
c. How much did the school pay for the book?

$$
\begin{aligned}
60 & =0.75 B \\
60\left(\frac{1}{0.75}\right) & =0.75\left(\frac{1}{0.75}\right) B \\
\frac{60}{0.75} & =B \\
80 & =B
\end{aligned}
$$

The school paid \$80 for the science book.
4. In the month of May, a certain middle school has an average daily absentee rate of $8 \%$ each school day. The absentee rate is the percent of students who are absent from school each day.
a. Write an equation that shows the proportional relationship between the number of students enrolled in the middle school and the average number of students absent each day during the month of May. Let $s$ represent the number of students enrolled in school, and let $a$ represent the average number of students absent each day in May.

$$
a=0.08 s
$$

b. Use your equation to complete the table. List 5 possible values for $s$ and $a$.

| $s$ | $a$ |
| :---: | :---: |
| 100 | 8 |
| 200 | 16 |
| 300 | 24 |
| 400 | 32 |
| 500 | 40 |

c. Identify the constant of proportionality, and explain what it means in the context of this situation.

The constant of proportionality is $\mathbf{0 . 0 8} .0 .08=8 \%$, so on average, for every 100 students enrolled in school, 8 are absent from school.
d. Based on the absentee rate, determine the number of students absent on average from school during the month of May if there are 350 students enrolled in the middle school.

28 students; 350 is halfway between 300 and 400. So, I used the table of values and looked at the numbers of students absent that correspond to 300 and 400 students at the school, which are 24 and 32. Halfway between 24 and 32 is 28 .
5. The equation shown in the box below could relate to many different percent problems. Put an $X$ next to each problem that could be represented by this equation. For any problem that does not match this equation, explain why it does not. Quantity =1.05 • Whole
$\qquad$ Find the amount of an investment after 1 year with $0.5 \%$ interest paid annually.
The equation should be Quantity $=1.005 \cdot$ Whole.
$\qquad$ Write an equation to show the amount paid for an item including tax, if the tax rate is $5 \%$.
$\qquad$ A proportional relationship has a constant of proportionality equal to $105 \%$.
$\qquad$
$X$

| Whole | 0 | 100 | 200 | 300 | 400 | 500 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Quantity | 0 | 105 | 210 | 315 | 420 | 525 |

Mr. Hendrickson sells cars and earns a 5\% commission on every car he sells. Write an equation to show the relationship between the price of a car Mr. Hendrickson sold and the amount of commission he earns.

The equation should be Quantity $=0.05 \cdot$ Whole.

## Problem Set Sample Solutions

1. Use the diagram below to create a scale drawing using a scale factor of $133 \frac{1}{3} \%$. Write numerical equations to find the horizontal and vertical distances in the scale drawing.


Scale factor: $\quad \frac{133 \frac{1}{3} \cdot 3}{100 \cdot 3}=\frac{400}{300}=\frac{4}{3}$
Horizontal distance: $\quad 9\left(\frac{4}{3}\right)=12$
Vertical distance forks: $\quad 3\left(\frac{4}{3}\right)=4$
Vertical distance handle: $6\left(\frac{4}{3}\right)=8$
Scale drawing:

2. Create a scale drawing of the original drawing given below using a horizontal scale factor of $80 \%$ and a vertical scale factor of $\mathbf{1 7 5} \%$. Write numerical equations to find the horizontal and vertical distances.


Horizontal scale factor:

Horizontal segment lengths:

Horizontal distance:

Vertical scale factor:

Vertical distance:

Scale drawing:

3. The accompanying diagram shows that the length of a pencil from its eraser to its tip is 7 units and that the eraser is 1.5 units wide. The picture was placed on a photocopy machine and reduced to $66 \frac{2}{3} \%$. Find the new size of the pencil, and sketch a drawing. Write numerical equations to find the new dimensions.


Scale factor:

$$
66 \frac{2}{3} \%=\frac{66 \frac{2}{3} \cdot 3}{100 \cdot 3}=\frac{200}{300}=\frac{2}{3}
$$

Pencil length:

$$
7\left(\frac{2}{3}\right)=4 \frac{2}{3}
$$

Eraser:

$$
\left(1 \frac{1}{2}\right)\left(\frac{2}{3}\right)=\left(\frac{3}{2}\right)\left(\frac{2}{3}\right)=1
$$


4. Use the diagram to answer each question.
a. What are the corresponding horizontal and vertical distances in a scale drawing if the scale factor is $25 \%$ ? Use numerical equations to find your answers.
Horizontal distance on original drawing: 14
Vertical distance on original drawing: 10

## Scale drawing:

Scale factor:
25\%
$\frac{25}{100}=\frac{1}{4}$


$$
\begin{array}{ll}
\text { Horizontal distance: } & 14\left(\frac{1}{4}\right)=3.5 \\
\text { Vertical distance: } & 10\left(\frac{1}{4}\right)=2.5
\end{array}
$$

b. What are the corresponding horizontal and vertical distances in a scale drawing if the scale factor is $\mathbf{1 6 0} \%$ ? Use a numerical equation to find your answers.

Horizontal distance on original drawing: 14
Vertical distance on original drawing: 10
Scale drawing:
Scale factor:
160\%
$\frac{160}{100}=\frac{8}{5}$
Horizontal distance:

Vertical distance:

$$
14\left(\frac{8}{5}\right)=22.4
$$

$$
10\left(\frac{8}{5}\right)=16
$$

Lesson 12: The Scale Factor as a Percent for a Scale Drawing
5. Create a scale drawing of the original drawing below using a horizontal scale factor of $200 \%$ and a vertical scale factor of $\mathbf{2 5 0} \%$.


## Answer:


6. Using the diagram below, on grid paper sketch the same drawing using a horizontal scale factor of $50 \%$ and a vertical scale factor of $\mathbf{1 5 0} \%$.


Answer:


## Problem Set Sample Solutions

1. The scale factor from Drawing 1 to Drawing 2 is $41 \frac{2}{3} \%$. Justify why Drawing 1 is a scale drawing of Drawing 2 and why it is an enlargement of Drawing 2. Include the scale factor in your justification.


$$
\text { Quantity }=\text { Percent } \times \text { Whole }
$$

Length in Drawing $1=$ Percent $\times$ Length in Drawing 2

$$
\begin{aligned}
& 100 \%=\text { Percent } \times 41 \frac{2}{3} \% \\
& \frac{100 \%}{41 \frac{2}{3} \%}=\frac{100 \cdot 3}{41 \frac{2}{3} \cdot 3}=\frac{300}{125}=\frac{12}{5}=2.40=240 \%
\end{aligned}
$$

Drawing 1 is a scale drawing of Drawing 2 because the lengths of Drawing 1 would be larger than the corresponding lengths of Drawing 2.

Since the scale factor is greater than $100 \%$, the scale drawing is an enlargement of the original drawing.
2. The scale factor from Drawing 1 to Drawing $\mathbf{2}$ is $\mathbf{4 0 \%}$, and the scale factor from Drawing 2 to Drawing $\mathbf{3}$ is $\mathbf{3 7 . 5} \%$. What is the scale factor from Drawing 1 to Drawing 3? Explain your reasoning, and check your answer using an example.


To find the scale factor from Drawing 1 to 3, I needed to find $37.5 \%$ of $40 \%$, so $(0.375)(0.40)=0.15$. The scale factor from Drawing 1 to Drawing 3 would be 15\%.

Check: Assume the length of Drawing 1 is 10. Then, using the scale factor for Drawing 2, the corresponding length of Drawing 2 would be 4. Then, applying the scale factor to Drawing 3, Drawing 3 would be $4(0.375)=1$. 5. To go directly from Drawing 1 to Drawing 3, which was found to have a scale factor of $15 \%$, then $10(0.15)=1.5$.
3. Traci took a photograph and printed it to be a size of 4 units by 4 units as indicated in the diagram. She wanted to enlarge the original photograph to a size of 5 units by 5 units and 10 units by 10 units.
a. Sketch the different sizes of photographs.

b. What was the scale factor from the original photo to the photo that is 5 units by 5 units?

The scale factor from the original to the 5 by 5 enlargement is $\frac{5}{4}=1.25=125 \%$.
c. What was the scale factor from the original photo to the photo that is $\mathbf{1 0}$ units by $\mathbf{1 0}$ units?

The scale factor from the original to the 10 by 10 photo is $\frac{10}{4}=2.5=250 \%$.
d. What was the scale factor from the $5 \times 5$ photo to the $10 \times 10$ photo?

The scale factor from the $5 \times 5$ photo to the $10 \times 10$ photo is $\frac{10}{5}=2=200 \%$.
e. Write an equation to verify how the scale factor from the original photo to the enlarged $\mathbf{1 0} \times 10$ photo can be calculated using the scale factors from the original to the $5 \times 5$ and then from the $5 \times 5$ to the $10 \times 10$.

Scale factor original to $5 \times 5$ : ( $125 \%$ )
Scale factor $5 \times 5$ to $10 \times 10$ : (200\%)
$4(1.25)=5$
$5(2.00)=10$
Original to $10 \times 10$, scale factor $=\mathbf{2 5 0} \%$
$4(2.50)=10$
The true equation $4(1.25)(2.00)=4(2.50)$ verifies that a single scale factor of $250 \%$ is equivalent to a scale factor of $\mathbf{1 2 5} \%$ followed by a scale factor of $\mathbf{2 0 0} \%$.
4. The scale factor from Drawing 1 to Drawing 2 is $\mathbf{3 0 \%}$, and the scale factor from Drawing 1 to Drawing $\mathbf{3}$ is $\mathbf{1 7 5 \%}$. What are the scale factors of each given relationship? Then, answer the question that follows. Drawings are not to scale.
a. Drawing 2 to Drawing 3

The scale factor from Drawing 2 to Drawing 3 is
$\frac{175 \%}{30 \%}=\frac{1.75}{0.30}=\frac{175}{30}=\frac{35}{6}=5 \frac{5}{6}=583 \frac{1}{3} \%$.
b. Drawing 3 to Drawing 1

The scale factor from Drawing 3 to Drawing 1 is
$\frac{1}{1.75}=\frac{100}{175}=\frac{4}{7} \approx 57.14 \%$.
c. Drawing 3 to Drawing 2


The scale factor from Drawing 3 to Drawing 2 is
$\frac{0.3}{1.75}=\frac{30}{175}=\frac{6}{35} \approx 17.14 \%$.
d. How can you check your answers?

To check my answers, I can work backwards and multiply the scale factor from Drawing 1 to Drawing 3 of 175\% to the scale factor from Drawing 3 to Drawing 2, and I should get the scale factor from Drawing 1 to Drawing 2.
$(1.75)(0.1714) \approx 0.29995 \approx 0.30=30 \%$

## Problem Set Sample Solutions

1. The smaller train is a scale drawing of the larger train. If the length of the tire rod connecting the three tires of the larger train, as shown below, is 36 inches, write an equation to find the length of the tire rod of the smaller train. Interpret your solution in the context of the problem.


Scale factor:

$$
\begin{aligned}
\text { Smaller } & =\text { Percent } \times \text { Larger } \\
6 & =\text { Percent } \times 16 \\
\frac{6}{16} & =0.375=37.5 \%
\end{aligned}
$$

Tire rod of smaller train: $(36)(0.375)=13.5$
The length of the tire rod of the smaller train is 13.5 in .
Since the scale drawing is smaller than the original, the corresponding tire rod is the same percent smaller as the windows. Therefore, finding the scale factor using the windows of the trains allows us to then use the scale factor to find all other corresponding lengths.
2. The larger arrow is a scale drawing of the smaller arrow. If the distance around the smaller arrow is 25.66 units. What is the distance around the larger arrow? Use an equation to find the distance and interpret your solution in the context of the problem.

Horizontal distance of smaller arrow: 8 units
Horizontal distance of larger arrow: 12 units
Scale factor:

$$
\begin{aligned}
\text { Larger } & =\text { Percent } \times \text { Smaller } \\
12 & =\text { Percent } \times 8 \\
\frac{12}{8} & =1.5=150 \%
\end{aligned}
$$

Distance around larger arrow:

$$
(25.66)(1.5)=38.49
$$

The distance around the larger arrow is 38.49 units.


An equation where the distance of the smaller arrow is multiplied by the scale factor results in the distance around the larger arrow.
3. The smaller drawing below is a scale drawing of the larger. The distance around the larger drawing is 39.4 units. Using an equation, find the distance around the smaller drawing.

Vertical distance of larger drawing: 10 units
Vertical distance of smaller drawing: 4 units

## Scale factor:

$$
\begin{aligned}
\text { Smaller } & =\text { Percent } \times \text { Larger } \\
4 & =\text { Percent } \times 10 \\
\frac{4}{10} & =0.4=40 \%
\end{aligned}
$$

## Total distance:

$$
(39.4)(0.4)=15.76
$$

The total distance around the smaller drawing is 15.76 units.
4. The figure is a diagram of a model rocket and is a two-dimensional scale drawing of an actual rocket. The length of a model rocket is 2.5 feet, and the wing span is 1.25 feet. If the length of an actual rocket is 184 feet, use an equation to find the wing span of the actual rocket.

Length of actual rocket: 184 ft .
Length of model rocket: 2.5 ft .

## Scale Factor:

$$
\begin{aligned}
\text { Actual } & =\text { Percent } \times \text { Model } \\
184 & =\text { Percent } \times 2.5 \\
\frac{184}{2.5} & =73.60=7,360 \%
\end{aligned}
$$

Wing span:
Model rocket wing span: 1.25 ft .
Actual rocket wing span : $(1.25)(73.60)=92$
The wing span of the actual rocket is $\mathbf{9 2} \mathbf{f t}$.


## Exit Ticket Sample Solutions

Write an equation relating the area of the original (larger) drawing to its smaller scale drawing. Explain how you determined the equation. What percent of the area of the larger drawing is the smaller scale drawing?


Scale factor:

$$
\begin{aligned}
\text { Quantity } & =\text { Percent } \times \text { Whole } \\
\text { Scale Drawing Length } & =\text { Percent } \times \text { Original Length } \\
6 & =\text { Percent } \times 15 \\
\frac{6}{15} & =\frac{2}{5}=\frac{4}{10}=0.4
\end{aligned}
$$

The area of the scale drawing is equal to the square of the scale factor times the area of the original drawing. Using $A$ to represent the area of the original drawing, then the area of the scale is

$$
\left(\frac{4}{10}\right)^{2} A=\frac{16}{100} A
$$

As a percent, $\frac{16}{100} A=0.16 A$.
Therefore, the area of the scale drawing is $16 \%$ of the area of the original drawing.

## Problem Set Sample Solutions

1. What percent of the area of the larger circle is shaded?
a. Solve this problem using scale factors.

## Scale factors:

Shaded small circle: radius $=1$ unit
Shaded medium circle: radius $=2$ units
Large circle: radius $=3$ units, area $=\mathrm{A}$
Area of small circle:

$$
\left(\frac{1}{3}\right)^{2} A=\frac{1}{9} A
$$

Area of medium circle:
$\left(\frac{2}{3}\right)^{2} A=\frac{4}{9} A$
Area of shaded region: $\quad \frac{1}{9} A+\frac{4}{9} A=\frac{5}{9} A=\frac{5}{9} A \times 100 \%=55 \frac{5}{9} \% A$
The area of the shaded region is $55 \frac{5}{9} \%$ of the area of the entire circle.
b. Verify your work in part (a) by finding the actual areas.

Areas:
Small circle:

$$
\begin{aligned}
& A=\pi r^{2} \\
& A=\pi\left(1 \text { unit }^{2}\right. \\
& A=1 \pi \text { unit }^{2} \\
& A=\pi r^{2} \\
& A=\pi\left(2 \text { units }^{2}\right. \\
& A=4 \pi \text { units }^{2}
\end{aligned}
$$

Medium circle:

Area of shaded circles: $\quad 1 \pi$ unit $^{2}+4 \pi$ units $^{2}=5 \pi$ units $^{2}$
Large circle:
$A=\pi r^{2}$
$A=\pi(3 \text { units })^{2}$
$A=9 \pi$ units $^{2}$
Percent of shaded to large circle:

$$
\frac{5 \pi \text { units }^{2}}{9 \pi \text { units }^{2}}=\frac{5}{9}=\frac{5}{9} \times 100 \%=55 \frac{5}{9} \%
$$

2. The area of the large disk is $\mathbf{5 0 . 2 4}$ units $^{2}$.
a. Find the area of the shaded region using scale factors. Use 3.14 as an estimate for $\pi$.

Radius of small shaded circles $=1$ unit
Radius of larger shaded circle $=2$ units
Radius of large disk $=4$ units
Scale factor of shaded region:
Small shaded circles: $\frac{1}{4}$
Large shaded circle: $\frac{2}{4}$
If $A$ represents the area of the large disk, then the total shaded area:

$$
\begin{aligned}
& \left(\frac{1}{4}\right)^{2} A+\left(\frac{1}{4}\right)^{2} A+\left(\frac{2}{4}\right)^{2} A \\
= & \frac{1}{16} A+\frac{1}{16} A+\frac{4}{16} A \\
= & \frac{6}{16} A \\
= & \frac{6}{16}\left(50.24 \text { units }^{2}\right)
\end{aligned}
$$

The area of the shaded region is 18.84 units $^{2}$.
b. What percent of the large circular region is unshaded?

Area of the shaded region is 18.84 square units. Area of total is 50.24 square units. Area of the unshaded region is $\mathbf{3 1 . 4 0}$ square units. Percent of large circular region that is unshaded is

$$
\frac{31.4}{50.24}=\frac{5}{8}=0.625=62.5 \%
$$

3. Ben cut the following rockets out of cardboard. The height from the base to the tip of the smaller rocket is 20 cm . The height from the base to the tip of the larger rocket is $\mathbf{1 2 0} \mathbf{~ c m}$. What percent of the area of the smaller rocket is the area of the larger rocket?

Height of smaller rocket: 20 cm
Height of larger rocket: 120 cm
Scale factor:

$$
\text { Quantity }=\text { Percent } \times \text { Whole }
$$

Actual height of larger rocket $=$ Percent $\times$ height of smaller rocket

$$
\begin{aligned}
120 & =\text { Percent } \times 20 \\
6 & =\text { Percent }
\end{aligned}
$$

600\%

## Area of larger rocket:

(scale factor) ${ }^{2}$ (area of smaller rocket)

$(6)^{2}($ area of smaller rocket)
36A

$$
36=36 \times 100 \%=3,600 \%
$$

The area of the larger rocket is 3,600\% the area of the smaller rocket.
4. In the photo frame depicted below, three 5 inch by 5 inch squares are cut out for photographs. If these cut-out regions make up $\frac{3}{16}$ of the area of the entire photo frame, what are the dimensions of the photo frame? Since the cut-out regions make up $\frac{3}{16}$ of the entire photo frame, then each cut-out region makes up $\frac{\frac{3}{16}}{3}=\frac{1}{16}$ of the entire photo frame.

The relationship between the area of the scale drawing is
(square factor) ${ }^{2} \times$ area of original drawing.
The area of each cut-out is $\frac{1}{16}$ of the area of the original photo frame. Therefore, the
 square of the scale factor is $\frac{1}{16}$. Since $\left(\frac{1}{4}\right)^{2}=\frac{1}{16}$, the scale factor that relates the cutout to the entire photo frame is $\frac{1}{4}$, or $25 \%$.
To find the dimensions of the square photo frame:

$$
\begin{aligned}
\text { Quantity } & =\text { Percent } \times \text { Whole } \\
\text { Small square side length } & =\text { Percent } \times \text { Photo frame side length } \\
5 \mathrm{in} . & =25 \% \times \text { Photo frame side length } \\
5 \mathrm{in} . & =\frac{1}{4} \times \text { Photo frame side length } \\
4(5) \text { in. } & =4\left(\frac{1}{4}\right) \times \text { Photo frame side length } \\
20 \mathrm{in} . & =\text { Photo frame side length }
\end{aligned}
$$

The dimensions of the square photo frame are 20 in . by 20 in .
5. Kelly was online shopping for envelopes for party invitations and saw these images on a website.


The website listed the dimensions of the small envelope as 6 in . by 8 in . and the medium envelope as 10 in . by $13 \frac{1}{3} \mathrm{in}$.
a. Compare the dimensions of the small and medium envelopes. If the medium envelope is a scale drawing of the small envelope, what is the scale factor?

To find the scale factor,

$$
\begin{array}{rlrl}
\text { Quantity } & =\text { Percent } \times \text { Whole } & \text { Quantity } & =\text { Percent } \times \text { Whole } \\
\text { Medium height } & =\text { Percent } \times \text { small height } & \text { Medium width } & =\text { Percent } \times \text { Small width } \\
10 & =\text { Percent } \times 6 & 13 \frac{1}{3}=\text { Percent } \times 8 \\
\frac{10}{6} & =\frac{5}{3}=\frac{5}{3} \times 100 \%=166 \frac{2}{3} \% & \frac{13}{3} & =\frac{5}{3}=\frac{5}{3} \times 100 \%=166 \frac{2}{3} \%
\end{array}
$$

b. If the large envelope was created based on the dimensions of the small envelope using a scale factor of $\mathbf{2 5 0} \%$, find the dimensions of the large envelope.

Scale factor is $\mathbf{2 5 0} \%$, so multiply each dimension of the small envelope by $\mathbf{2 . 5 0}$.
Large envelope dimensions are as follows:

$$
(6 \mathrm{in} .)(2.5)=15 \mathrm{in} . \quad(8 \mathrm{in} .)(2.5)=20 \mathrm{in} .
$$

c. If the medium envelope was created based on the dimensions of the large envelope, what scale factor was used to create the medium envelope?

Scale factor:

$$
\left.\begin{array}{rlrl}
\text { Quantity } & =\text { Percent } \times \text { Whole } & \text { Quantity } & =\text { Percent } \times \text { Whole } \\
\text { Medium } & =\text { Percent } \times \text { Large } & \text { Medium } & =\text { Percent } \times \text { Large } \\
10 & =\text { Percent } \times 15 & 13 \frac{1}{3} & =\text { Percent } \times 20 \\
\frac{10}{15} & =\text { Percent } & 13 \frac{1}{3} & =\text { Percent } \\
\frac{2}{3} & =\frac{2}{3} \times 100 \%=66 \frac{2}{3} \% & \frac{20}{3} & \frac{2}{3}
\end{array}=\frac{2}{3} \times 100 \%=66 \frac{2}{3} \%\right)
$$

d. What percent of the area of the larger envelope is the area of the medium envelope?

Scale factor of larger to medium: $66 \frac{2}{3} \%=\frac{2}{3}$
Area: $\left(\frac{2}{3}\right)^{2}=\frac{4}{9}=\frac{4}{9} \times 100 \%=44 \frac{4}{9} \%$
The area of the medium envelope is $\mathbf{4 4} \frac{\mathbf{4}}{9} \%$ of the larger envelope.

## Exit Ticket Sample Solutions

1. Jodie spent $\mathbf{2 5}$ \% less buying her English reading book than Claudia. Gianna spent $\mathbf{9} \%$ less than Claudia. Gianna spent more than Jodie by what percent?

Let crepresent the amount Claudia spent, in dollars. The number of dollars Jodie spent was 0.75 c, and the number of dollars Gianna spent was 0.91 c. $0.91 c \div 0.75 c=\frac{91}{75} \times 100 \%=121 \frac{1}{3} \%$. Gianna spent $21 \frac{1}{3} \%$ more than Jodie.
2. Mr. Ellis is a teacher who tutors students after school. Of the students he tutors, $\mathbf{3 0} \%$ need help in computer science and the rest need assistance in math. Of the students who need help in computer science, $\mathbf{4 0} \%$ are enrolled in Mr. Ellis's class during the school day. Of the students who need help in math, 25\% are enrolled in his class during the school day. What percent of the after-school students are enrolled in Mr. Ellis's classes?
Let t represent the after-school students tutored by Mr. Ellis.
Computer science after-school students: $0.3 t$
Math after-school students: $0.7 t$

After-school computer science students who are also Mr. Ellis's students: $0.4 \times 0.3 t=0.12 t$
After-school math students who are also Mr. Ellis's students: $0.25 \times 0.7 t=0.175 t$

Number of after-school students who are enrolled in Mr. Ellis's classes: $0.12 t+0.175 t=0.295 t$
Out of all the students Mr. Ellis tutors, 29.5\% of the tutees are enrolled in his classes.

## Problem Set Sample Solutions

1. One container is filled with a mixture that is $\mathbf{3 0} \%$ acid. A second container is filled with a mixture that is $\mathbf{5 0} \%$ acid. The second container is $\mathbf{5 0} \%$ larger than the first, and the two containers are emptied into a third container. What percent of acid is the third container?

Let $t$ be the amount of mixture in the first container. Then the second container has $1.5 t$, and the third container has 2.5t.

The amount of acid in the first container is $0.3 t$, the amount of acid in the second container is $0.5(1.5 t)=0.75 t$, and the amount of acid in the third container is $\mathbf{1 . 0 5 t}$. The percent of acid in the third container is $\frac{1.05}{2.5} \times 100 \%=42 \%$.
2. The store's markup on a wholesale item is $\mathbf{4 0} \%$. The store is currently having a sale, and the item sells for $\mathbf{2 5 \%}$ off the retail price. What is the percent of profit made by the store?

Let w represent the wholesale price of an item.
Retail price: $1.4 w$
Sale price: $1.4 w-(1.4 w \times 0.25)=1.05 w$
The store still makes a 5\% profit on a retail item that is on sale.
3. During lunch hour at a local restaurant, $\mathbf{9 0} \%$ of the customers order a meat entrée and $\mathbf{1 0} \%$ order a vegetarian entrée. Of the customers who order a meat entrée, $\mathbf{8 0} \%$ order a drink. Of the customers who order a vegetarian entrée, $\mathbf{4 0} \%$ order a drink. What is the percent of customers who order a drink with their entrée?

Let e represent lunch entrées.
Meat entrées: $0.9 e$
Vegetarian entrées: $0.1 e$
Meat entrées with drinks: $0.9 e \times 0.8=0.72 e$
Vegetarian entrées with drinks: $0.1 e \times 0.4=0.04 e$
Entrées with drinks: $0.72 e+0.04 e=0.76 e$. Therefore, $76 \%$ of lunch entrées are ordered with a drink.
4. Last year's spell-a-thon spelling test for a first grade class had $15 \%$ more words with four or more letters than this year's spelling test. Next year, there will be $5 \%$ less than this year. What percent more words have four or more letters in last year's test than next year's?

Let t represent this year's amount of spell-a-thon words with four letters or more.
Last year: 1.15t
Next year: $0.95 t$

1. $15 t \div 0.95 t \times 100 \% \approx 121 \%$. There were about $21 \%$ more words with four or more letters last year than there will be next year.
2. An ice cream shop sells $75 \%$ less ice cream in December than in June. Twenty percent more ice cream is sold in July than in June. By what percent did ice cream sales increase from December to July?

Let $j$ represent sales in June.
December: 0.25j
July: 1.20j
$1.20 \div 0.25=4.8 \times 100 \%=480 \%$. Ice cream sales in July increase by $\mathbf{3 8 0} \%$ from ice cream sales in December.
6. The livestock on a small farm the prior year consisted of $\mathbf{4 0} \%$ goats, $\mathbf{1 0} \%$ cows, and $\mathbf{5 0} \%$ chickens. This year, there is a $\mathbf{5} \%$ decrease in goats, $9 \%$ increase in cows, and $15 \%$ increase in chickens. What is the percent increase or decrease of livestock this year?

Let l represent the number of livestock the prior year.
Goats decrease: $0.4 l-(0.4 l \times 0.05)=0.38 l$ or $0.95(0.4 l)=0.38 l$
Cows increase: $0.1 l+(0.1 l \times 0.09)=0.109 l$ or $1.09(0.1 l)=0.109 l$
Chickens increase: $0.5 k+(0.5 k \times 0.15)=0.575 l$ or $1.15(0.5 l)=0.575 l$
$0.38 l+0.109 l+0.575 l=1.064 l$. There is an increase of $6.4 \%$ in livestock.
7. In a pet shelter that is occupied by $55 \%$ dogs and $45 \%$ cats, $\mathbf{6 0} \%$ of the animals are brought in by concerned people who found these animals in the streets. If $\mathbf{9 0} \%$ of the dogs are brought in by concerned people, what is the percent of cats that are brought in by concerned people?

Let c represent the percent of cats brought in by concerned people.

$$
\begin{aligned}
0.55(0.9)+(0.45)(c) & =1(0.6) \\
0.495+0.45 c & =0.6 \\
0.495-0.495+0.45 c & =0.6-0.495 \\
0.45 c & =0.105 \\
0.45 c \div 0.45 & =0.105 \div 0.45 \\
c & \approx 0.233
\end{aligned}
$$

About 23\% of the cats brought into the shelter are brought in by concerned people.
8. An artist wants to make a particular teal color paint by mixing a $75 \%$ blue hue and $25 \%$ yellow hue. He mixes a blue hue that has $\mathbf{8 5} \%$ pure blue pigment and a yellow hue that has $\mathbf{6 0} \%$ of pure yellow pigment. What is the percent of pure pigment that is in the resulting teal color paint?

Let p represent the teal color paint.

$$
(0.75 \times 0.85 p)+(0.25 \times 0.6 p)=0.7875 p
$$

78.75\% of pure pigment is in the resulting teal color paint.
9. On Mina's block, $65 \%$ of her neighbors do not have any pets, and $35 \%$ of her neighbors own at least one pet. If $\mathbf{2 5 \%}$ of the neighbors have children but no pets, and $\mathbf{6 0} \%$ of the neighbors who have pets also have children, what percent of the neighbors have children?

Let $n$ represent the number of Mina's neighbors.
Neighbors who do not have pets: $0.65 n$
Neighbors who own at least one pet: $0.35 n$
Neighbors who have children but no pets: $0.25 \times 0.65 n=0.1625 n$
Neighbors who have children and pets: $0.6 \times 0.35 n=0.21 n$
Percent of neighbors who have children: $0.1625 n+0.21 n=0.3725 n$
37.25\% of Mina's neighbors have children.

## Exit Ticket Sample Solutions

A $\mathbf{2 5} \%$ vinegar solution is combined with triple the amount of a $\mathbf{4 5} \%$ vinegar solution and a 5\% vinegar solution resulting in $\mathbf{2 0}$ milliliters of a 30\% vinegar solution.

1. Determine an equation that models this situation, and explain what each part represents in the situation.

Let $s$ represent the number of milliliters of the first vinegar solution.

$$
(0.25)(s)+(0.45)(3 s)+(0.05)(20-4 s)=(0.3)(20)
$$

$(0.25)(s)$ represents the amount of the $25 \%$ vinegar solution.
$(0.45)(3 s)$ represents the amount of the $45 \%$ vinegar solution, which is triple the amount of the $25 \%$ vinegar solution.
$(0.05)(20-4 s)$ represents the amount of the $5 \%$ vinegar solution, which is the amount of the remainder of the solution.
(0.3)(20) represents the result of the mixture, which is 20 mL of a $\mathbf{3 0} \%$ vinegar solution.
2. Solve the equation, and find the amount of each of the solutions that were combined.

$$
\begin{aligned}
& 0.25 s+1.35 s+1-0.2 s=6 \\
& 1.6 s-0.2 s+1=6 \\
& 1.4 s+1-1=6-1 \\
& 1.4 s \div 1.4=5 \div 1.4 \\
& s \approx 3.57 \\
& 3 s \approx 3(3.57)=10.71 \\
& 20-4 s \approx 20-4(3.57)=5.72
\end{aligned}
$$

Around 3.57 mL of the $25 \%$ vinegar solution, 10.71 mL of the $45 \%$ vinegar solution and 5.72 mL of the $5 \%$ vinegar solution were combined to make 20 mL of the $30 \%$ vinegar solution.

## Problem Set Sample Solutions

1. A 5 -liter cleaning solution contains $\mathbf{3 0} \%$ bleach. A 3 -liter cleaning solution contains $\mathbf{5 0} \%$ bleach. What percent of bleach is obtained by putting the two mixtures together?

Let $x$ represent the percent of bleach in the resulting mixture.

$$
\begin{aligned}
0.3(5)+0.5(3) & =x(8) \\
1.5+1.5 & =8 x \\
3 \div 8 & =8 x \div 8 \\
x & =0.375
\end{aligned}
$$

The percent of bleach in the resulting cleaning solution is $\mathbf{3 7 . 5} \%$.
2. A container is filled with $\mathbf{1 0 0}$ grams of bird feed that is $\mathbf{8 0} \%$ seed. How many grams of bird feed containing $5 \%$ seed must be added to get bird feed that is $\mathbf{4 0} \%$ seed?

Let $x$ represent the amount of bird feed, in grams, to be added.

$$
\begin{aligned}
0.8(100)+0.05 x & =0.4(100+x) \\
80+0.05 x & =40+0.4 x \\
80-40+0.05 x & =40-40+0.4 x \\
40+0.05 x & =0.4 x \\
40+0.05 x-0.05 x & =0.4 x-0.05 x \\
40 \div 0.35 & =0.35 x \div 0.35 \\
x & \approx 114.3
\end{aligned}
$$

About 114.3 grams of the bird seed containing 5\% seed must be added.
3. A container is filled with $\mathbf{1 0 0}$ grams of bird feed that is $\mathbf{8 0} \%$ seed. Tom and Sally want to mix the $\mathbf{1 0 0}$ grams with bird feed that is $5 \%$ seed to get a mixture that is $\mathbf{4 0} \%$ seed. Tom wants to add 114 grams of the $5 \%$ seed, and Sally wants to add 115 grams of the $5 \%$ seed mix. What will be the percent of seed if Tom adds 114 grams? What will be the percent of seed if Sally adds 115 grams? How much do you think should be added to get $\mathbf{4 0} \%$ seed?
If Tom adds 114 grams, then let $x$ be the percent of seed in his new mixture. $214 x=0.8(100)+0.05(114)$. Solving, we get the following:

$$
x=\frac{80+5.7}{214}=\frac{85.7}{214} \approx 0.4005=40.05 \%
$$

If Sally adds 115 grams, then let $y$ be the percent of seed in her new mixture. $215 y=0.8(100)+0.05(115)$. Solving, we get the following:

$$
y=\frac{80+5.75}{215}=\frac{85.75}{215} \approx 0.3988=39.88 \%
$$

The amount to be added should be between 114 and 115 grams. It should probably be closer to 114 because 40.05\% is closer to $\mathbf{4 0} \%$ than $\mathbf{3 9 . 8 8} \%$.
4. Jeanie likes mixing leftover salad dressings together to make new dressings. She combined 0.55 L of a $90 \%$ vinegar salad dressing with 0.45 L of another dressing to make 1 L of salad dressing that is $\mathbf{6 0} \%$ vinegar. What percent of the second salad dressing was vinegar?

Let c represent the percent of vinegar in the second salad dressing.

$$
\begin{aligned}
0.55(0.9)+(0.45)(c) & =1(0.6) \\
0.495+0.45 c & =0.6 \\
0.495-0.495+0.45 c & =0.6-0.495 \\
0.45 c & =0.105 \\
0.45 c \div 0.45 & =0.105 \div 0.45 \\
c & \approx 0.233
\end{aligned}
$$

The second salad dressing was around 23\% vinegar.
5. Anna wants to make 30 mL of a $\mathbf{6 0} \%$ salt solution by mixing together a $\mathbf{7 2} \%$ salt solution and a $\mathbf{5 4} \%$ salt solution. How much of each solution must she use?

Let s represent the amount, in milliliters, of the first salt solution.

$$
\begin{aligned}
0.72(s)+0.54(30-s) & =0.60(30) \\
0.72 s+16.2-0.54 s & =18 \\
0.18 s+16.2 & =18 \\
0.18 s+16.2-16.2 & =18-16.2 \\
0.18 s & =1.8 \\
s & =10
\end{aligned}
$$

Anna needs 10 mL of the $72 \%$ solution and 20 mL of the $54 \%$ solution.
6. A mixed bag of candy is $\mathbf{2 5} \%$ chocolate bars and $\mathbf{7 5} \%$ other filler candy. Of the chocolate bars, $\mathbf{5 0} \%$ of them contain caramel. Of the other filler candy, $\mathbf{1 0} \%$ of them contain caramel. What percent of candy contains caramel? Let c represent the percent of candy containing caramel in the mixed bag of candy.

$$
\begin{aligned}
0.25(0.50)+(0.75)(0.10) & =1(c) \\
0.125+0.075 & =c \\
0.2 & =c
\end{aligned}
$$

In the mixed bag of candy, 20\% of the candy contains caramel.
7. A local fish market receives the daily catch of two local fishermen. The first fisherman's catch was $84 \%$ fish while the rest was other non-fish items. The second fisherman's catch was $76 \%$ fish while the rest was other non-fish items. If the fish market receives $\mathbf{7 5} \%$ of its catch from the first fisherman and $\mathbf{2 5} \%$ from the second, what was the percent of other non-fish items the local fish market bought from the fishermen altogether?

Let $n$ represent the percent of non-fish items of the total market items.

$$
\begin{aligned}
0.75(0.16)+0.25(0.24) & =n \\
0.12+0.06 & =n \\
0.18 & =n
\end{aligned}
$$

The percent of non-fish items in the local fish market is $18 \%$.

## Exit Ticket Sample Solutions

There are a van and a bus transporting students on a student camping trip. Arriving at the site, there are 3 parking spots. Let $v$ represent the van and $b$ represent the bus. The chart shows the different ways the vehicles can park.
a. In what percent of the arrangements are the vehicles separated by an empty parking space?

$$
\frac{2}{6}=33 \frac{1}{3} \%
$$

b. In what percent of the arrangements are the vehicles parked next to each other?

$$
\frac{4}{6}=66 \frac{2}{3} \%
$$

c. In what percent of the arrangements does the left or right parking space remain vacant?

$$
\frac{4}{6}=66 \frac{2}{3} \%
$$

|  | Parking | Parking | Parking |
| :---: | :---: | :---: | :---: |
| Space 1 | Space2 | Space 3 |  |
| Option 1 | V | B |  |
| Option 2 | V |  | B |
| Option 3 | B | V |  |
| Option 4 | B |  | V |
| Option 5 |  | V | B |
| Option 6 |  | B | V |
|  |  |  |  |

## Problem Set Sample Solutions

1. A six-sided die (singular for dice) is thrown twice. The different rolls are as follows:

1 and 1,1 and 2,1 and 3,1 and 4,1 and 5,1 and 6,
2 and 1, 2 and 2, 2 and 3,2 and 4,2 and 5, 2 and 6,
3 and 1,3 and 2,3 and 3,3 and 4,3 and 5,3 and 6 ,
4 and 1,4 and 2,4 and 3,4 and 4,4 and 5,4 and 6 ,
5 and 1,5 and 2,5 and 3,5 and 4,5 and 5,5 and 6 ,
6 and 1,6 and 2,6 and 3,6 and 4,6 and 5,6 and 6.
a. What is the percent that both throws will be even numbers?
$\frac{9}{36}=25 \%$
b. What is the percent that the second throw is a $\mathbf{5}$ ?
$\frac{6}{36}=16 \frac{2}{3} \%$
c. What is the percent that the first throw is lower than a 6 ?
$\frac{30}{36}=83 \frac{1}{3} \%$
2. You have the ability to choose three of your own classes, art, language, and physical education. There are three art classes (A1, A2, A3), two language classes (L1, L2), and two P.E. classes (P1, P2) to choose from. The order does not matter and you must choose one from each subject.

| A1, L1, P1 | A2, L1, P1 | A3, L1, P1 |
| :---: | :---: | :---: |
| A1, L1, P2 | A2, L1, P2 | A3, L1, P2 |
| A1, L2, P1 | A2, L2, P1 | A3, L2, P1 |
| A1, L2, P2 | A2, L2, P2 | A3, L2, P2 |

Compare the percent of possibilities with A1 in your schedule to the percent of possibilities with L1 in your schedule.
A1: $\frac{4}{12}=33 \frac{1}{3} \% \quad$ L1: $\frac{6}{12}=50 \%$
There is a greater percent with L1 in my schedule.
3. Fridays are selected to show your school pride. The colors of your school are orange, blue, and white, and you can show your spirit by wearing a top, a bottom, and an accessory with the colors of your school. During lunch, 11 students are chosen to play for a prize on stage. The table charts what the students wore.

| Top | W | O | W | O | B | W | B | B | W | W | W |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bottom | B | O | B | B | O | B | B | B | O | W | B |
| Accessory | W | O | B | W | B | O | B | W | O | O | O |

a. What is the percent of outfits that are one color?

$$
\frac{2}{11}=18 \frac{2}{11} \%
$$

b. What is the percent of outfits that include orange accessories?

$$
\frac{5}{11}=45 \frac{5}{11} \%
$$

Lesson 18:
4. Shana wears two rings ( $G$ represents gold, and $S$ represents silver) at all times on her hand. She likes fiddling with them and places them on different fingers (pinky, ring, middle, index) when she gets restless. The chart is tracking the movement of her rings.

|  | Pinky Finger | Ring Finger | Middle Finger | Index Finger |
| :---: | :---: | :---: | :---: | :---: |
| Position 1 |  | G | S |  |
| Position 2 |  |  | S | G |
| Position 3 | G |  | S |  |
| Position 4 |  |  |  | S, G |
| Position 5 | S | G |  |  |
| Position 6 | G | S |  |  |
| Position 7 | S |  | S |  |
| Position 8 | G |  | S |  |
| Position 9 |  | S, G | G | S |
| Position 10 |  | G |  |  |
| Position 11 |  | S |  |  |
| Position 12 |  |  | S, G |  |
| Position 13 | S, G |  |  |  |
| Position 14 |  |  |  |  |

a. What percent of the positions shows the gold ring on her pinky finger?
$\frac{4}{14} \approx 28.57 \%$
b. What percent of the positions shows both rings on the same finger?
$\frac{4}{14}=28 \frac{4}{7} \%$
5. Use the coordinate plane below to answer the following questions.

a. What is the percent of the 36 points whose quotient of $\frac{x \text {-coordinate }}{y \text {-coordinate }}$ is greater than one?
$\frac{15}{36}=41 \frac{2}{3} \%$
b. What is the percent of the 36 points whose coordinate quotient is equal to one?
$\frac{6}{36}=16 \frac{2}{3} \%$

