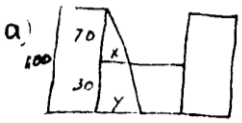
Assignment 10 Related Rates

1991 AB6

a) 

$\frac{dx}{dt} = 2\frac{ft}{sec}$ $\frac{dy}{dt} = ?$

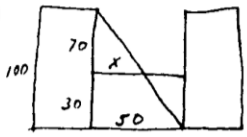
similar triangles → proportional sides

$\frac{x}{y} = \frac{70}{100}$

$y = \frac{10}{7}x$

$\frac{dy}{dt} = \frac{10}{7}\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{10}{7} \cdot 2 = \boxed{\frac{20}{7}\frac{ft}{sec}}$

b) 

$x = ?$

$\frac{x}{50} = \frac{70}{100}$

$\boxed{x = 35 \text{ feet}}$

2002 AB6 Form B

a. $d^2 = 3^2 + 4^2$

$\boxed{d = 5 \text{ km}}$

b. $d^2 = x^2 + y^2$

$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$2 \cdot 5\frac{dd}{dt} = 2 \cdot 4(-15) + 2 \cdot 3 \cdot 10$

$\boxed{\frac{dd}{dt} = \frac{2 \cdot 4(-15) + 2 \cdot 3 \cdot 10}{2 \cdot 5}\frac{Km}{hr} = -6\frac{Km}{hr}}$

c. $\tan\theta = \frac{y}{x}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$

$\left(\frac{5}{4}\right)^2\frac{d\theta}{dt} = \frac{4(10) - (3)(-15)}{4^2}$

$\boxed{\frac{d\theta}{dt} = \frac{40 + 45}{\frac{25}{16}}\frac{rad}{hr}}$

$= \frac{17}{5}\frac{rad}{hr}$

1984 AB5

a. $\frac{dr}{dt} = \frac{1}{2}$, $\frac{dA}{dt} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\boxed{\frac{dA}{dt} = 2\pi \cdot 3 \cdot \frac{1}{2}}$

b. $\frac{dV}{dt} = 28\pi$, $\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$

$12\pi = \frac{1}{3}\pi \cdot 9h$

$4 = h$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi r^2\frac{dh}{dt} + h \cdot \frac{2}{3}\pi r\frac{dr}{dt}$

$28\pi = \frac{1}{3}\pi \cdot 9 \cdot \frac{dh}{dt} + 4 \cdot \frac{2}{3}\pi \cdot 3 \cdot \frac{1}{2}$

$28\pi = 3\pi\frac{dh}{dt} + 4\pi$

$24\pi = 3\pi\frac{dh}{dt}$

$\boxed{8 = \frac{dh}{dt}}$

c. $\frac{dA}{dh} = ?$

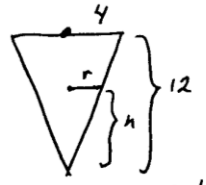
$\frac{dA}{dh} = \frac{dA}{dt} \cdot \frac{dt}{dh}$

$= \boxed{3\pi \cdot \frac{1}{8}}$

Assignment 10 continued

1995 AB 5

a.



$\frac{r}{4} = \frac{h}{12}$

$r = \frac{h}{3}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$\boxed{V = \frac{1}{27}\pi h^3}$

b. $\frac{dh}{dt} = h - 12 \qquad \frac{dV}{dt} = ?$

$\frac{dh}{dt}(3) = 3 - 12$

$= -9$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$\boxed{\frac{dV}{dt} = \frac{1}{9}\pi \cdot 3^2(-9)\ \frac{ft^3}{min}}$

$= -9\pi\ \frac{ft^3}{min}$

$= -28.274\ \frac{ft^3}{min}$

c. for the cylindrical tank

$\frac{dV}{dt} = 9\pi$ (note the sign change)

$V = 400\pi y$

$\frac{dV}{dt} = 400\pi \frac{dy}{dt}$

$9\pi = 400\pi \frac{dy}{dt}$

$\boxed{\frac{9\pi}{400\pi}\ \frac{ft}{min} = \frac{dy}{dt}} = \frac{9}{400}\ \frac{ft}{min}$

$= .0225\ \frac{ft}{min}$

1996 AB 5

$y = \frac{9}{625}x^4 \rightarrow x = \sqrt[4]{\frac{625}{9}y}$

a. $V = \pi \int_0^9 \left(\sqrt[4]{\frac{625}{9}y}\right)^2 dy$

$= \boxed{471.239\ ft^3}$ watch units!

b. $\frac{471.239}{8} = \boxed{59 \text{ minutes}}$

c. $\frac{dV}{dt} = 8, \quad \frac{dh}{dt} = ?$

$V = \pi \int_0^h \left(\sqrt[4]{\frac{625}{9}y}\right)^2 dy$

$\frac{dV}{dt} = \frac{d}{dt}\left(\pi \int_0^h \left(\sqrt[4]{\frac{625}{9}y}\right)^2 dy\right)$

$\frac{dV}{dt} = \pi\left(\sqrt[4]{\frac{625}{9}h}\right)^2 \frac{dh}{dt}$

$8 = \pi\left(\sqrt[4]{\frac{625}{9}\cdot 4}\right)^2 \frac{dh}{dt}$

$\boxed{\frac{8}{\pi\left(\sqrt[4]{\frac{625}{9}\cdot 4}\right)^2}\ \frac{ft}{min} = \frac{dh}{dt}} = .153\ \frac{ft}{min}$ watch units