Assignment 11 Other Rate Problems

Question 1

(a) $\int_0^4 E(t)\,dt = \boxed{3981 \text{ gallons}}$

(b) Let $S(t)$ = amt. of sewage in the tank

$S'(t) = E(t) - 645 = 0$

CN

$t = 2.309,\ 3.559$

Abs. max is at CN or at endpoint:

$S(0) = 0$

$S(2.309) = \int_0^{2.309} E(t)\,dt - 645(2.309)$

$= 1637.178$

$S(3.559) = \int_0^{3.559} E(t)\,dt - 645(3.559)$

$= 1228.520$

$S(4) = 3981.022 - 645(4)$

$= 1401.022$

$\boxed{\text{Max at } t = 2.309 \text{ hours, Amt. is } 1637 \text{ gallons}}$

(c) Total Cost $= \int_0^4 (.15 - .02t)E(t)\,dt$

$= \boxed{\$474}$

1996 AB 3

a. $2C = Ce^{k\cdot 5}$ $\qquad \boxed{C = 6 \ \frac{\text{billion gallons}}{\text{year}}}$

$2 = e^{5k}$

$\ln 2 = 5k$

$\boxed{\frac{\ln 2}{5} = k \text{ or } k = .139}$

b. $S_{avg} = \dfrac{\int_3^{13} 6e^{.139t}\,dt}{13-3}$

$= \boxed{19.680 \ \frac{\text{billion gallons}}{\text{year}}}$

c. $\int_5^7 S(t)\,dt \approx$

$\boxed{\frac{1}{2}\cdot\frac{1}{2}\left(6e^{(.139)5} + 2\cdot 6e^{(.139)5.5} + 2\cdot 6e^{(.139)6} + 2\cdot 6e^{(.139)6.5} + 6e^{(.139)7}\right)}$

$= 27.668$

d. $\int_5^7 S(t)\,dt$ is the total amount of cola consumed in billions of gallons in the U.S. during 1985 and 1986.

1989 AB6

a) $\frac{dy}{dt} = ky$

$\frac{dy}{y} = k\,dt$

$\int \frac{dy}{y} = \int k\,dt$

$\ln|y| = kt + C_1$

Skip these steps if you recognise it is exponential decay.

$y = Ce^{kt}$

C = initial amt. $= 1{,}000{,}000$

when $t = 6$ $y = 500{,}000$

$500{,}000 = 1{,}000{,}000\, e^{k\cdot 6}$

$\frac{1}{2} = e^{6k}$

$\ln\frac{1}{2} = 6k$

$\frac{1}{6}\ln\frac{1}{2} = k$ or $k = -.116$

$\boxed{y = 1{,}000{,}000\, e^{\frac{1}{6}\ln\frac{1}{2}\, t}} = 1{,}000{,}000\, e^{-.116t}$

b) $\frac{dy}{dt} = ?$ $\quad y = 600{,}000$

$\frac{dy}{dt} = ky$

$\frac{dy}{dt} = \frac{1}{6}\ln\frac{1}{2}(600{,}000)$

$\boxed{\frac{dy}{dt} = 100{,}000 \ln\frac{1}{2} \ \frac{\text{gal}}{\text{yr}}}$

$\frac{dy}{dt} = -69314.718 \ \text{gal/yr}$

c) $y \le 50{,}000$

$50{,}000 = 1{,}000{,}000\, e^{-.116t}$

$.05 = e^{-.116t}$

$\ln .05 = -.116t$

$\boxed{25.932 \text{ yrs} = t}$ $\qquad$ Warning: don't round off until the last step!

$t = \dfrac{\ln\frac{1}{20}}{\frac{1}{6}\ln\frac{1}{2}}$ yrs.

Other Rate Problems (cont.)

1999 AB 3

a. $\int_0^{24} R(t)dt \approx 6(10.4 + 11.2 + 11.3 + 10.2)$

The total water flow from the pipe in the 24-hour period was approximately 258.6 gallons.

b. Yes. $R'(t) = 0$ at some time since $R(t)$ switches from increasing to decreasing and is differentiable. (This is the Mean Value Theorem.)

c. $R_{avg} \approx g_{avg} = \dfrac{\int_0^{24} \frac{1}{79}(768 + 23t - t^2)dt}{24 - 0}$

$= 10.784$ gal/hr. or 10.785 gal/hr

1997 AB 6

Method 1 (Hard Way)

a) $\frac{dv}{dt} = -2v - 32$

$\frac{dv}{-2v-32} = dt$

$\int \frac{dv}{-2v-32} = \int dt$

$-\frac{1}{2} \ln|-2v-32| = t + C_1$

$\ln|-2v-32| = -2t + C_2$

$|-2v-32| = e^{-2t+C_2}$

$-2v - 32 = C_3 e^{-2t}$

$-2v = C_3 e^{-2t} + 32$

$v = C_4 e^{-2t} - 16$

Since $v(0) = -50$

$-50 = C_4 e^0 - 16$

$-50 = C_4 - 16$

$-34 = C_4$

$\boxed{v = -34e^{-2t} - 16}$

Method 2

$\frac{dv}{dt} = -2v - 32 = -2(v + 16)$

$\frac{dv}{v+16} = -2dt$

$\int \frac{dv}{v+16} = \int -2dt$

$\ln|v+16| = -2t + C_1$

$|v+16| = e^{-2t+C_1}$

$|v+16| = C_2 e^{-2t}$

$v + 16 = C_3 e^{-2t}$

$v = C_3 e^{-2t} - 16$

Since $v(0) = -50$

$-50 = C_3 e^0 - 16$

$-34 = C_3$

$\boxed{v = -34e^{-2t} - 16}$

6pts

Mean 1.19

1pt b) $\lim_{t\to\infty} (-34e^{-2t} - 16) = \lim_{t\to\infty} \left(\frac{-34}{e^{2t}} - 16\right) = 0 - 16 = \boxed{-16 \ \frac{ft}{sec}}$

c) $v = -20$

$-20 = -34e^{-2t} - 16$

2pts $\boxed{t = 1.070 \text{ sec}}$ or $t = -\frac{1}{2} \ln \frac{2}{17}$ sec