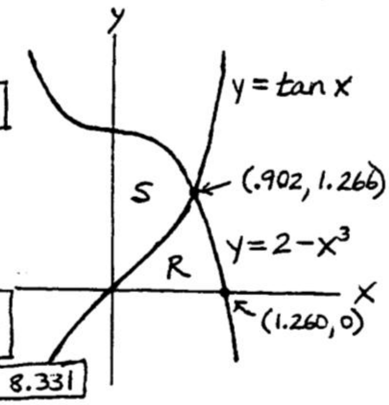
2001 FREE RESPONSE SOLUTIONS (AB) Assignment 14

1) ⓐ $\tan x = 2 - x^3$ WHEN $x = .902$ AND $y = 1.266$

$2 - x^3 = 0$ WHEN $x^3 = 2 \rightarrow x = \sqrt[3]{2}$ OR $x = 1.260$

AREA (OF REGION R) $= \int_0^{.902} \tan x\,dx + \int_{.902}^{1.260} (2 - x^3)\,dx = \boxed{.729}$



ⓑ AREA (OF REGION S) $= \int_0^{.902} (2 - x^3 - \tan x)\,dx = \boxed{1.161 \text{ OR } 1.160}$

ⓒ VOLUME (WASHER METHOD) $= \pi \int_0^{.902} \left((2 - x^3)^2 - (\tan x)^2\right) dx = \boxed{8.332 \text{ OR } 8.331}$

2) ⓐ YOU ARE USING A SECANT LINE SLOPE (AVG. RATE OF CHANGE) TO APPROXIMATE A TANGENT LINE SLOPE (INSTANTANEOUS R. OF C.)

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

$W'(12) \approx \dfrac{24 - 21}{9 - 15} = -\dfrac{3}{6} = \boxed{-\tfrac{1}{2}\,(°C) \text{ PER DAY}}$ OR

$W'(12) \approx \dfrac{22 - 21}{12 - 15} = \boxed{-\tfrac{1}{3}\,(°C) \text{ PER DAY}}$ OR $W'(12) \approx \dfrac{24 - 22}{9 - 12} = \boxed{-\tfrac{2}{3}\,(°C) \text{ PER DAY}}$

ⓑ $W_{AVG.} = \dfrac{\int_0^{15} W(t)\,dt}{15} \approx \boxed{\dfrac{\frac{1}{2} \cdot 3\,(20 + 2\cdot 31 + 2\cdot 28 + 2\cdot 24 + 2\cdot 22 + 21)}{15}}$ (°C)

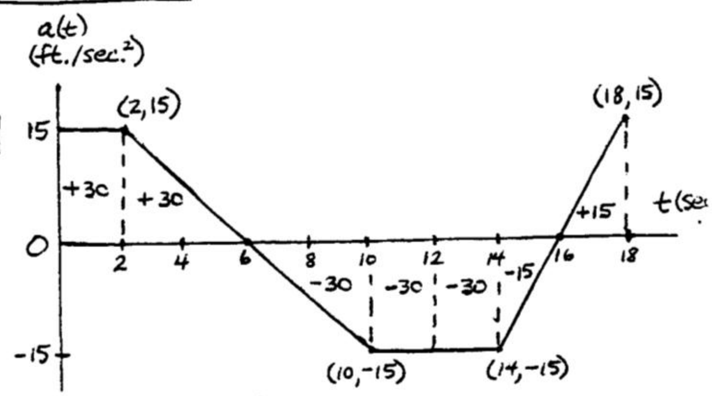
$= 25.1$ (°C)

ⓒ $P'(12) = \boxed{-.549}$ At time 12 days, the water temp. was decreasing at the approximate rate of .549 degrees per day.

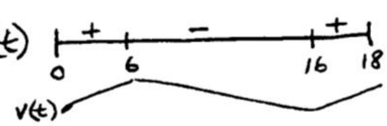
ⓓ $P_{AVG} = \dfrac{\int_0^{15} (20 + 10te^{(-t/3)})\,dt}{15} = \boxed{25.757\,(°C)}$

3) ⓐ $\boxed{\text{YES.}}$ (AT THE RATE OF 15 ft/sec²) BECAUSE $v'(t) = a(t) > 0$

ⓑ The velocity is 55 ft/sec at $\boxed{t = 12 \text{ sec}}$ because the gain in velocity from $t = 0$ to $t = 6$ (area above the axis) equals the loss in velocity from $t = 6$ to $t = 12$ (area below the axis).



ⓒ $v'(t) = a(t)$



$v(6) = 55 + 60 = 115$ ft./sec.

$v(18) = 55 + 60 - 105 + 15 = 25$ ft./sec.

THE CAR'S MAXIMUM VELOCITY ON $0 \le t \le 18$ IS $\boxed{115 \text{ ft./sec. AT } t = 6 \text{ sec.}}$

ⓓ $v(0) = 55$ ft./sec.

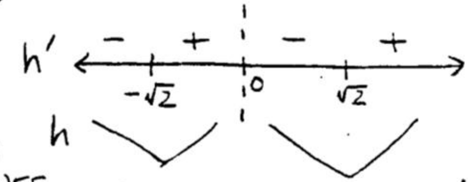
$v(16) = 55 + 60 - 105 = 10$ ft./sec. = MINIMUM VELOCITY (SEE NUMBER LINE FROM PART c)

THEREFORE, THE CAR'S VELOCITY IS $\boxed{\text{NEVER ZERO}}$ ON THE INTERVAL $0 \le t \le 18$

2001 FREE RESPONSE SOLUTIONS (AB)

4) (a) FOR HORIZONTAL TANGENTS, $h'(x)=0$

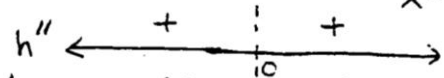
$\frac{x^2-2}{x}=0$ WHEN $x^2=2$, $x=\pm\sqrt{2}$



THE GRAPH OF h HAS HORIZONTAL TANGENTS WHEN $\boxed{x=\pm\sqrt{2}}$. AT EACH OF THESE X-VALUES h HAS A LOCAL MINIMUM because h' switches from negative to positive.

(b) $h''(x)=\frac{x(2x)-(x^2-2)}{x^2}$ OR USING $h'(x)=x-\frac{2}{x}$

$=\frac{2x^2-x^2+2}{x^2}=\frac{x^2+2}{x^2}$ $\qquad h''(x)=1+\frac{2}{x^2}=\frac{x^2+2}{x^2}$



THE GRAPH OF h IS CONCAVE UP ON THE INTERVALS $\boxed{(-\infty,0) \text{ AND } (0,\infty)}$ because h'' is positive.

(c) $h(4)=-3$, SO A POINT ON THE TANGENT LINE IS $(4,-3)$.

$h'(4)=\frac{4^2-2}{4}=\frac{14}{4}$ OR $\frac{7}{2}$ = THE SLOPE OF THE TANGENT LINE.

TANGENT LINE EQUATION: $\boxed{y+3=\frac{7}{2}(x-4)}$

(d) FOR $x>4$, THE LINE TANGENT TO THE GRAPH OF h AT $x=4$ LIES BELOW THE GRAPH OF h BECAUSE $h''(x)>0$ for $x>4$ (the graph of h is concave up for $x>4$)

5) (a) $f(x)=4x^3+ax^2+bx+k$

$f'(x)=12x^2+2ax+b$ $\qquad f''(x)=24x+2a$

$f'(-1)=12-2a+b=0$ $\qquad f''(-2)=-48+2a=0$

$12-48+b=0$ $\qquad 2a=48$

$-36+b=0$ $\qquad \leftarrow \boxed{a=24}$

$\boxed{b=36}$

(b) $\int_0^1 f(x)\,dx=\int_0^1(4x^3+24x^2+36x+k)\,dx=32$

$x^4+8x^3+18x^2+kx\Big|_0^1=32$

$1+8+18+k=32$

$\boxed{k=5}$

6) (a) $\frac{dy}{dx}=y^2(6-2x)$

$\frac{d^2y}{dx^2}=y^2(-2)+(6-2x)\left(2y\frac{dy}{dx}\right)\overset{\text{AT }(3,\frac{1}{4})}{=}\boxed{\left(\frac{1}{4}\right)^2(-2)+0}$ OR $-2\left(\frac{1}{16}\right)$ OR $-\frac{1}{8}$

(b) $\frac{1}{y^2}dy=(6-2x)dx$

$\int y^{-2}dy=\int(6-2x)dx$

$-y^{-1}=6x-x^2+C$

$-\frac{1}{y}=6x-x^2+C$

AT $(3,\frac{1}{4})$: $\frac{-1}{\frac{1}{4}}=6(3)-3^2+C$

$-4=18-9+C$

$C=-13$

$-\frac{1}{y}=6x-x^2-13$

$-y=\frac{1}{6x-x^2-13}$

$y=f(x)=\boxed{\frac{1}{x^2-6x+13}}$