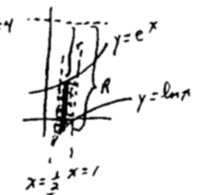
Assignment 15 2002 Free Response Solutions



1

Mean
4.08

(2 points)
a) $A = \int_{.5}^{1} (e^x - \ln x)\,dx = \boxed{1.223}$

(4 points)
b) washers $r = 4 - e^x$ $R = 4 - \ln x$

$V = \pi \int_{.5}^{1} \left((4-\ln x)^2 - (4-e^x)^2 \right) dx = \boxed{23.609}$

(3 points)
c) $h(x) = e^x - \ln x$

For absolute extr. check endpts and CN

$h'(x) = e^x - \frac{1}{x}$

$e^x - \frac{1}{x} = 0$

CN $x = .567$

$h(.5) = 2.342$
$h(.567) = 2.330$
$h(1) = 2.718$

$\boxed{\text{obs. min. is } 2.330 \quad \text{abs. max. is } 2.718}$

2 Mean 3.13

(3pts) a) $\int_{9}^{17} E(t)\,dt = \boxed{6004 \text{ people}}$

(1pt) b) $15 \int_{9}^{17} E(t)\,dt + 11 \int_{17}^{23} E(t)\,dt = \boxed{\$104{,}048}$

(3pts) c) $H'(t) = E(t) - L(t)$

$H'(17) = E(17) - L(17) = -380.281 \approx \boxed{-380}$

$\boxed{H(17) = 3725 \text{ means that at 5 PM there are 3725 people in the park.}}$

$\boxed{H'(17) = -380 \text{ means that at 5PM the number of people in the park is decreasing at the rate of 380 people per hour}}$

(2pts) d) $H'(t) = E(t) - L(t)$

$E(t) - L(t) = 0$

$E(t) = L(t)$

$\boxed{t = 15.795 \text{ hr.}}$ (about 3:48 PM)

3 Mean 3.12

(1pt) a) with calc.

$a(4) = v'(4) = \boxed{-.524}$

without calc.

$a(t) = v'(t)$

$= \cos\left(\frac{\pi}{3}t\right) \cdot \frac{\pi}{3}$

$a(4) = \boxed{\cos \frac{4\pi}{3} \cdot \frac{\pi}{3}}$

$= -\frac{\pi}{6}$

(3pts) b) $a = v'$ — on $(3, 4.5)$: $-$

Statement I is true since $a(t) < 0$

Statement II is true since $v(t) < 0$ and $a(t) < 0$.

(3pts) c) $T.D. = \int_0^4 \left| \sin\left(\frac{\pi}{3}t\right) \right| dt = 2.387$

d) $x(t) = \int \sin\left(\frac{\pi}{3}t\right) dt$

$x(t) = -\frac{3}{\pi} \cos\left(\frac{\pi}{3}t\right) + C$

since $x(0) = 2$

$2 = -\frac{3}{\pi} \cos 0 + C$

$2 + \frac{3}{\pi} = C$

$C = 2.955$

$x(t) = -\frac{3}{\pi} \cos\left(\frac{\pi}{3}t\right) + 2.955$

$\boxed{x(4) = -\frac{3}{\pi} \cos \frac{4\pi}{3} + 2.955} = 3.432$

4. (3pts) a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = \boxed{-\frac{3}{2}}$ ← neg. area of Δ

$g'(-1) = f(-1) = \boxed{0}$

$g''(-1) = f'(-1) = \boxed{3}$ ← slope on f graph

2pts b) $g'(x) = f(x)$

g': $-$ on $(-2,-1)$, $+$ on $(-1,1)$, $-$ on $(1,2)$

$\boxed{g \text{ is increasing on } (-1,1)}$ because $g' = f$ is positive

2pts c) $g''(x) = f'(x)$

g'': $+$ on $(-2,0)$, $-$ on $(0,2)$

$\boxed{g \text{ is concave down on } (0,2)}$ because $g' = f$ is decreasing

d) $g(-2) = \int_0^{-2} f(t)\,dt = 0$

$g(0) = \int_0^{0} f(t)\,dt = 0$

$g(2) = \int_0^{2} f(t)\,dt = 0$

5 Mean 2.29

$\frac{r}{h} = \frac{5}{10}$ similar triangles $r = \frac{1}{2}h$

(1pt) a) $h = 5,\ r = \frac{5}{2}$

$\boxed{V = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 \cdot 5 \text{ cm}^3} = \frac{125\pi}{12} \text{ cm}^3$

(1 point for units on parts a and b)

(5pts) b) $V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$

$\boxed{\frac{dV}{dt} = \frac{\pi}{12} \cdot 3 \cdot 5^2 \cdot \frac{-3}{10} \frac{\text{cm}^3}{\text{hr}}} = \frac{-15\pi}{8} \frac{\text{cm}^3}{\text{hr}}$

(2pts) (c) since $r = \frac{1}{2}h$, $h = 2r$

$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$ from part b

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3(2r)^2\left(-\frac{3}{10}\right)$

$\frac{dV}{dt} = \pi r^2\left(-\frac{3}{10}\right)$

$\boxed{\frac{dV}{dt} = A\left(-\frac{3}{10}\right)}$

$\boxed{\text{The constant of proportionality is } -\frac{3}{10}.}$

6 Mean 2.33

(2pts) a) $\int_0^{1.5} (3f'(x) + 4)\,dx =$

$(3f(x) + 4x)\Big|_0^{1.5} =$

$(3f(1.5) + 4 \cdot 1.5) - (3f(0) + 4 \cdot 0) =$

$\boxed{3(-1) + 4(1.5) - 3(-7)} =$

24

(3pts) b) $f(1) = -4 \rightarrow$ point $(1,-4)$

$f'(1) = 5 \rightarrow m = 5$

$y + 4 = 5(x-1)$

$y = 5x - 9$

$f(1.2) \approx y(1.2) = \boxed{5(1.2) - 9} = -3$

This is *less* than $f(1.2)$ because the graph of f is concave upward on the interval $[-1.5, 1.5]$

(2pts) c) $f''(c) = \frac{f'(.5) - f'(0)}{.5 - 0} = r$

$= \boxed{\frac{3-0}{.5} = r}$

$6 = r$

$\boxed{\text{by the Mean Value Theorem}}$

(2pts) d) $g'(x) = \begin{cases} 4x - 1, & x < 0 \\ 4x + 1, & x > 0 \end{cases}$

$g'(0)$ does not exist but $f'(0) = 0$

$\boxed{g \text{ and } f \text{ are } \underline{\text{not}} \text{ the same function}}$