1993 MC # 18

1. $f(x) = x^{3/2}$
   $f'(x) = \frac{3}{2}x^{\frac{1}{2}}$
   $f'(4) = \frac{3}{2}\sqrt{4}$
   $= 3$

2. top curve: $y=d$
   bottom curve: $y=f(x)$
   $A = \int_a^b (d - f(x))\,dx$

3. $\lim_{n\to\infty} \frac{3n^3-5n}{n^3-2n^2}$
   concentrating on highest degree terms
   $\lim_{n\to\infty} \frac{3n^3}{n^3} = 3$

7. $y = \frac{2x+3}{3x-2}$
   $y' = \frac{(3x-2)2 - (2x+3)3}{(3x-2)^2}$
   $y' = \frac{-13}{(3x-2)^2}$
   $y'(1) = -13 = \text{slope}$
   $y-5 = -13(x-1)$
   $y-5 = -13x+13$
   $13x+y = 18$

8. $y = \tan x - \cot x$
   $y' = \sec^2 x - (-\csc^2 x)$
   $= \sec^2 x + \csc^2 x$

10. $f(x) = (x-1)^2 \sin x$
    $f'(x) = (x-1)^2(\cos x) + (\sin x)(2(x-1))$
    $f'(0) = \cos 0 + (\sin 0)(-2)$
    $= 1 + 0$
    $= 1$

11. $a(t) = 6t-2$
    $v(t) = 3t^2-2t+C_1$
    using $v=25$ @ $t=3$
    $25 = 27-6+C_1$
    $4 = C_1$
    $v(t) = 3t^2-2t+4$
    $x(t) = t^3-t^2+4t+C_2$
    using $x=10$ @ $t=1$
    $10 = 1-1+4+C_2$
    $6 = C_2$
    $x(t) = t^3-t^2+4t+6$

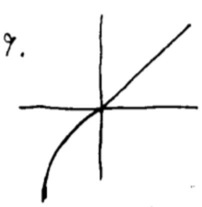
14. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx$
    $\int 3x^2(x^3+1)^{-\frac{1}{2}}\,dx$
    $2(x^3+1)^{\frac{1}{2}} + C$
    $2\sqrt{x^3+1} + C$

15. $f(x) = (x-2)(x-3)^2$
    $f'(x) = (x-2)2(x-3) + (x-3)^2$
    $= (x-3)(2x-4+x-3)$
    $= (x-3)(3x-7)$
    f': + | − | + at $\frac{7}{3}$, 3
    max at $x = \frac{7}{3}$

16. $y = 2\ln(\sec x)$
    $y' = 2\frac{1}{\sec x}\sec x\tan x$
    $y' = 2\tan x$
    $y'(\frac{\pi}{4}) = 2\tan\frac{\pi}{4} = 2$
    slope (norm) $= -\frac{1}{2}$

17. $\int (x^2+1)^2\,dx$
    $\int (x^4+2x^2+1)\,dx$
    $\frac{x^5}{5} + \frac{2}{3}x^3 + x + C$

18. **MVT** $f'(c) = \frac{f(b)-f(a)}{b-a}$
    $f(x) = \sin\frac{x}{2}$
    $f'(x) = \frac{1}{2}\cos\frac{x}{2}$
    $\frac{1}{2}\cos\frac{x}{2} = \frac{\sin\frac{3\pi}{4} - \sin\frac{\pi}{4}}{\frac{3\pi}{2} - \frac{\pi}{2}}$
    $\frac{1}{2}\cos\frac{x}{2} = 0$
    $\cos\frac{x}{2} = 0$
    $\frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$
    $x = \pi$, ~~$3\pi, \ldots$~~

19.


only E is true
it is always increasing
(Note $f'(0)$ DNE – sharp turn)

24. $f(x) = (x^2-2x-1)^{2/3}$
    $f'(x) = \frac{2}{3}(x^2-2x-1)^{-\frac{1}{3}}(2x-2)$
    $f'(0) = \frac{2}{3}(-1)^{-\frac{1}{3}}(-2)$
    $= \frac{4}{3}$

25. $\frac{d}{dx}(2^x) = 2^x \ln 2$
    (you could use log. diff. if you haven't memorized this deriv.)

27. $f(x) = x^3+12x-24$
    $f'(x) = 3x^2+12$
    $f'(x)$ is always positive
    so $f(x)$ is always increasi[ng]

30.


disc Method
$V = \pi\int_0^3 (\sqrt{x})^2\,dx$
$= \pi\int_0^3 x\,dx$
$= \pi \left.\frac{x^2}{2}\right]_0^3$
$= \pi(\frac{9}{2}-0)$

31. $f(x) = e^{3\ln(x^2)}$
    $f(x) = e^{\ln(x^2)^3}$
    $f(x) = e^{\ln x^6}$
    $f(x) = x^6$
    $f'(x) = 6x^5$

32. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$
    $\left.\text{Arcsin}\frac{x}{2}\right]_0^{\sqrt{3}}$
    $\text{Arcsin}\frac{\sqrt{3}}{2} - \text{Arcsin}\,0$
    $\frac{\pi}{3} - 0$

(cont)

33. $\frac{dy}{dx} = 2y^2$

Separate Variables: $\frac{dy}{2y^2} = dx$

$\int \frac{1}{2} y^{-2} dy = \int dx$

$-\frac{1}{2} y^{-1} = x + C$

(if $y = -1$ and $x = 1$)

$-\frac{1}{2}(-1)^{-1} = 1 + C$

$\frac{1}{2} = 1 + C$

$-\frac{1}{2} = C$

$-\frac{1}{2} y^{-1} = x - \frac{1}{2}$

let $x = 2$ and find y

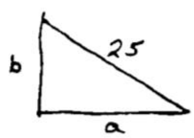
$-\frac{1}{2} y^{-1} = 2 - \frac{1}{2}$

$-\frac{1}{2} y^{-1} = \frac{3}{2}$

$y^{-1} = -3$

$y = -\frac{1}{3}$

34.



$\frac{db}{dt} = -3 \frac{ft}{min}$

find $\frac{da}{dt}$

$a^2 + b^2 = 25^2$

$2a \frac{da}{dt} + 2b \frac{db}{dt} = 0$

when $b = 7ft$ $a = 24ft$ (pyth. thm.)

$2 \cdot 24 \frac{da}{dt} + 2(7)(-3) = 0$

$\frac{da}{dt} = \frac{42}{48} = \frac{7}{8} \frac{ft}{min}$

37. I. True (this is the definition of the derivative with a in place of x)

II True (this is the alt. form of the def. of the deriv.)

III False

38. $f''(x) = 2x - \cos x$

$f'(x) = x^2 - \sin x + C_1$ Integrating

$f(x) = \frac{x^3}{3} + \cos x + C_1 x + C_2$ only letter A fits

41. $\frac{d}{dx} \int_0^x \cos(2\pi u)\, du = \cos(2\pi x)$

44. $f(x) = x \ln x$

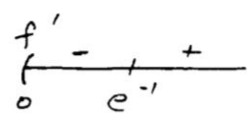
$f'(x) = \ln x + x \cdot \frac{1}{x}$

$f'(x) = \ln x + 1$

$0 = \ln x + 1$

$\ln x = -1$

$x = e^{-1}$



min when $x = e^{-1}$

$f(e^{-1}) = e^{-1} \ln e^{-1}$

$= -\frac{1}{e}$