Mixed Review B Assignment 7

1992 AB4

(a) $y' - \sin y\, y' = 1$

$y'(1-\sin y) = 1$

$\boxed{y' = \frac{1}{1-\sin y}}$

(b) y' must be undefined

$1-\sin y = 0$

$\sin y = 1$

$y = \frac{\pi}{2}$

$\frac{\pi}{2} + \cos\frac{\pi}{4} = x+1$

$x = \frac{\pi}{2} - 1$

vert. tan. line $\boxed{x = \frac{\pi}{2} - 1}$

(c) $y' = \frac{1}{1-\sin y} = (1-\sin y)^{-1}$

$y'' = -1(1-\sin y)^{-2}(-\cos y)y'$

$\boxed{y'' = -(1-\sin y)^{-2}(-\cos y)\frac{1}{1-\sin y}}$

2002 AB2 Form B

a. $p'(9) = -.646$

The amt. of pollutant is not inc. since $p'(9) < 0$.

b. $1 - 3e^{-.2t} = 0$

$t = 30.173$ days

p': $-$ on $(0, 30.173)$, $+$ on $(30.173, \infty)$

abs. min. at $t = 30.173$ days

because p' is neg. on $[0, 30.173)$ and pos. on $(30.173, \infty)$.

c. $p(30.173) = 50 + \int_0^{30.173} p'(t)\,dt$

$= 35.104$ gal.

Yes, the lake is safe since $p(30.173) < 40$ gal.

d. $p(0) = 50 \quad p'(0) = -2$

Tan. Line $p - 50 = -2(t-0)$

$p = -2t + 50$

$-2t + 50 = 40$

$t = 5$ days

2003 AB4 Form B

a. $a(t) = e^{1-t}(-1)$

$a(3) = -e^{-2} = -\frac{1}{e^2}$

b. $v(3) = -1 + e^{-2} = -1 + \frac{1}{e^2}$

The speed is increasing at $t=3$ since $a(3)$ and $v(3)$ are both negative.

c. $-1 + e^{1-t} = 0$

$e^{1-t} = 1$

$1 - t = 0$

$t = 1$

v: $+$ on $(0,1)$, $-$ on $(1, \infty)$

direction change at $t=1$ since v changes from pos. to neg.

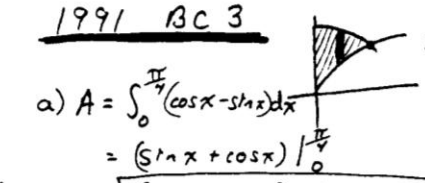
d. $TD = \int_0^3 (-1+e^{1-t})\,dt - \int_1^3 (-1+e^{1-t})\,dt$

$= (-t - e^{1-t})\Big|_0^1 - (-t - e^{1-t})\Big|_1^3$

$= (-1-1) - (-e) - (-3 - e^{-2} - (-1-1))$

$= e + \frac{1}{e^2} - 1$

1991 BC 3



pt of inters. $\sin x = \cos x$, $x = \frac{\pi}{4}$

a) $A = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$

$= (\sin x + \cos x)\Big|_0^{\frac{\pi}{4}}$

Stop here $\rightarrow = \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0)$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0+1)$

$= \frac{2}{\sqrt{2}} - 1$

$= \sqrt{2} - 1$

b) $V = \pi \int_0^{\frac{\pi}{4}} \left((\cos x)^2 - (\sin x)^2\right)dx$

$= \boxed{1.571}$

c) $V = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)^2\,dx$

side of square $s = \cos x - \sin x$

$= \boxed{.285}$

1996 AB1

(a) f': $+$ on $(-3,-2)$, $-$ on $(-2,1)$, $-$ on $(1,4)$, $+$ on $(4,5)$

rel. max. at $x = -2$ — f' changes from pos. to neg.

(b) rel. min. at $x = 4$ — f' changes from neg. to pos.

(c) f'': $-$ on $(-3,-1)$, $+$ on $(-1,1)$, $-$ on $(1,3)$, $+$ on $(3,5)$

his conc. up on $(-1,1)$ and $(3,5)$

f' is increasing

(d)