Mixed Review C Assignment 9

2003 AB 5 Form B

a. $g(3) = \int_2^3 f(t)\,dt$

area of trapezoid $\rightarrow = \frac{1}{2}\cdot 1(4+2)$

$= \boxed{3}$

$g'(3) = f(3) = \boxed{2}$

$g''(3) = f'(3) = \boxed{-2}$ (slope of graph)

b. $g(0) = \int_2^0 f(t)\,dt$

$= -\frac{1}{2}\cdot 2\cdot 4 = -4$

$\text{ARC} = \dfrac{g(3)-g(0)}{3-0}$

$= \boxed{\dfrac{3+4}{3}} = \dfrac{7}{3}$

c. $g'(c) = f(c) = \frac{7}{3} = 2\frac{1}{3}$

There are $\boxed{\text{2 c-values.}}$ There are 2 points on f with y-coordinate equal to $2\frac{1}{3}$.

d. $g''(x) = f'(x)$

g'': + on (0, 2), − on (2, 4), − on (4, 5), + on (5, 7); marks at 0, 2, 4, 5, 7

$\boxed{\text{g has P.I. at } x = 2 \text{ because } f = g' \text{ changes from inc. to dec. and also at } x = 5 \text{ where } f \text{ changes from inc. to dec.}}$

1999 AB 2

$x^2 = 4$

a. $A = \int_{-2}^{2}(4-x^2)\,dx \quad x = \pm 2$

$= \left(4x - \frac{1}{3}x^3\right)\Big|_{-2}^{2}$

$= \boxed{\left(8-\frac{8}{3}\right) - \left(-8+\frac{8}{3}\right)} = \dfrac{32}{3}$

b. $V = \pi\int_{-2}^{2}\left(4^2 - (x^2)^2\right)dx$

$= \pi\int_{-2}^{2}\left(16 - x^4\right)dx$

$= \pi\left(16x - \frac{1}{5}x^5\right)\Big|_{-2}^{2}$

$= \boxed{\pi\left(\left(32-\frac{32}{5}\right) - \left(-32+\frac{32}{5}\right)\right)}$

$= \dfrac{256\pi}{5}$

c. $\pi\int_{-2}^{2}\left((k-x^2)^2 - (k-4)^2\right)dx = \dfrac{256\pi}{5}$

1992 AB 2

(a) $v(t) = 3(t-1)(t-3)$

$v(t) = 3t^2 - 12t + 9$

$a(t) = v'(t) = 6t - 12$

$a'(t) = 6$

a': + on [0, 5]

$a(t)$ is always increasing on $[0,5]$ since $a'(t) > 0$

minimum is at $t = 0$

$a(0) = -12$

$\boxed{\text{min. accel. is } -12}$

(b) $\text{T.D.} = \int_0^5 |v(t)|\,dt$

$= 28.000$

(c) $v_{avg} = \dfrac{\int_0^5 v(t)\,dt}{5}$

$= 4.000$

1997 AB 4

a) $f'(x) = 3x^2 - 12x$

$0 = 3x^2 - 12x$

$0 = 3x(x-4)$

CN

$x = 0, 4$

f': + , 0, − , 4, +

rel max when $x = 0$ since f' switches from pos. to neg.

rel min when $x = 4$ since f' switches from neg. to pos.

$\boxed{\text{rel. maximum value of } f \text{ is } f(0) = p}$

$\boxed{\text{rel. minimum value of } f \text{ is } f(4) = 64 - 96 + p = p - 32}$

c) $f_{avg} = \dfrac{\int_{-1}^{2}(x^3 - 6x^2 + p)\,dx}{2+1} = 1$

$\dfrac{\left(\frac{x^4}{4} - 2x^3 + px\right)\Big]_{-1}^{2}}{3} = 1$

$\dfrac{(4 - 16 + 2p) - \left(\frac{1}{4} + 2 - p\right)}{3} = 1$

$-14\frac{1}{4} + 3p = 3$

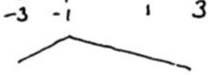
$3p = 17\frac{1}{4}$

$\boxed{p = \dfrac{17\frac{1}{4}}{3}} = 5.75$

1984 AB 4

a. f': + on (−3, −1), − on (−1, 1), − on (1, 3); marks at −3, −1, 1, 3

f



$\boxed{\text{abs. max at } x = -1}$

since $f' > 0$ on $[-3,-1)$ and $f' \le 0$ on $(-1, 3)$

abs. min. at $x = -3$ or $x = 3$

$f(-3) = 4$ and $f(3) = 1$

$\boxed{\text{abs. min. at } x = 3}$

b. f'': + , −1, + , 1, −

$\boxed{\text{P.I. at } x = 1}$

since f'' changes sign at $x = 1$.

c.